

Descartes, Mathematics and Music

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Descartes's influence in shaping the intellectual climate of the modern world is without parallel. More than any other figure of the seventeenth century, he marks the intellectual transition from the Middle Ages to the modern world. It is generally conceded that he is the "father of modern philosophy" as well as the "father of modern mathematics." In addition, his literary influence is inestimable. He is said to have been the first to have offered a perfect model of French prose.

Karl Stern refers to him as the Saint Augustine of the Age of Reason. And Alfred North Whitehead regards his marriage of algebra and geometry as one of the truly stirring moments in the history of human discovery:

It is impossible not to feel stirred at the thought of the emotions of men at certain historic moments of adventure and discovery—Columbus when he first saw the Western shore, Pizarro when he stared at the Pacific Ocean, Franklin when the electric spark came from the string to his kite, Galileo when he first turned his telescope to the heavens. Such moments are also granted to students in the abstract regions of thought, and high among them must be placed the morning when Descartes lay in bed and invented the method of co-ordinate geometry.¹

Pythagoras and Plato saw mathematics as a key to understanding the physical world. Aristotle, whose intellectual inclinations were far more naturalistic, did not share his predecessors enthusiasm for numbers. And since the mind of Aristotle dominated the intellectual temper of the Middle Ages, it was left to Descartes, together with a host of seventeenth century mathematical physicists, to revive mathematics as the key to understanding nature. But it was Descartes

¹ Alfred North Whitehead, *An Introduction to Mathematics* (New York: Oxford University Press, 1948), pp. 88-89.

who led the way, claiming that “Universal Mathematics” is the basis for unifying all human knowledge.

Descartes’s unique interest in mathematics was evident in his early years when he was a student at the Jesuit School at La Flèche. After completing the full curriculum of languages and humane letters, logic, ethics, mathematics, physics, and metaphysics, he later declared, “I found myself embarrassed with so many doubts and errors that it seemed to me that the effort to instruct myself had no effect other than the increasing discovery of my own ignorance.”² Mathematics alone appeared to be the exception “because of the certainty of its demonstrations and the evidence of its reasoning.”³

His first book, which was not published until after his death, was a *Compendium on Music (Compendium musicae)*. Although this work reveals the limitations of a mind that is youthful and inexperienced (Gilbert and Kuhn grade it as a “second-rate textbook,”⁴ while Descartes himself referred to it as a “baby-bear,”⁵) it nonetheless reveals his penchant for attempting to explain the physical by reducing it to the mathematical.

For Descartes, mathematical knowledge was the only knowledge worthy of the name. He did not hold that no other discipline should be studied. But he staunchly maintained that “in our search for the direct road towards truth, we should busy ourselves with no object about which we cannot attain a certitude equal to that of the demonstrations of arithmetic and geometry.”⁶ According to Gilson, the whole philosophy of Descartes is virtually contained in this outlook.⁷ The *Cogito* is the first principle of Descartes’s philosophy, but his insistence that every truth should have a certitude akin to the demonstrations of mathematics precedes and undergirds it. The absolute indubitability guaranteed in the act of thinking itself, which implies that the thinker exists, is a model of certainty and clarity that is characteristic of a mathematical proof. Yet the question must be raised—must all truth be grasped in like manner and with similar certitude and clarity? If anything can be truly said to express the innermost spirit of the Cartesian philosophy, Gilson goes on to say, it is this “Mathematicism” which

² René Descartes, “Discourse on the Method of Rightly Conducting the Reason and Seeking for Truth in the Sciences, Part I,” in Ralph M. Eaton (ed.), *Descartes Selections* (New York: Charles Scribner’s Sons, 1955), p. 4.

³ *Ibid.*, p. 7.

⁴ Katherine Gilbert and Helmut Kuhn, *A History of Aesthetics* (New York: Macmillan, 1939), p. 209.

⁵ René Descartes, *Compendium of Music*, trans. W. Robert (Rome: American Institute of Musicology, 1961), p. 53.

⁶ *Ibid.*, pp. 43-4.

⁷ Étienne Gilson, *The Unity of the Philosophical Experience* (New York: Charles Scribner’s Sons, 1965), p. 132.

is “nothing else than a recklessly conducted experiment to see what becomes of human knowledge when molded into conformity with the pattern of mathematical evidence.”⁸

The single most original and lasting achievement of Descartes is his unification of algebra and geometry. But this single insight by no means suggests that all sciences can be brought into a single unity or that the only truths worth having are those that have mathematical certainty. For Maritain, this is the “sin” of Descartes, the vain effort to “reduce all the speculative sciences to one same degree, one same method, one same type of intelligibility.”⁹

Mathematics does have something to say about all things in the physical universe because they are co-extensive with quantity. But this fact does not imply that the only truths that we should bother to learn are those of a mathematical type that are based on quantity. Descartes does not regard the intelligible object with sufficient respect. Rather than move from thing to mind so as to do justice to the thing known, he moves from idea to thing and thereby imposes arbitrary limitations on the thing he is trying to know. In this way his rationalism is most evident. A good illustration of this tendency to force mathematical forms onto objects rather than try to know them as objects in their own peculiar nature which, to a certain extent, may resist being reduced to mathematical structures, is his *Compendium of Music*. For example, he asserts that “The ear is more satisfied by an octave than by a fifth as final discord.”¹⁰ It is this type of remark that led Gilbert and Kuhn to appraise the *Compendium* as another “Illustration of his rational ideal for all knowledge.”¹¹

The rate of vibration of any musical note in relation to its octave is a perfectly rational 1:2. In music, however, aesthetic pleasure is a much more complex phenomenon than numerical rationality. In fact, as musicologists have explained, an essential feature of great music is the avoidance of pattern or predictability. The musical composer creates on the level of music, not mathematics. To put the matter in its simplest terms, as Mozart said, “Music must always be music.”¹²

Frédéric Chopin’s *Nocturnes* are generally regarded as lyrically pleasant, elegant, reposeful. Not one of the nineteen ends on an octave. Most of them end on a fifth. Robert Schumann’s *Kinderscenen* is a series of thirteen delightful piano pieces written for children. Seven of them end on a fifth, four on a fourth

⁸ *Ibid.*, p. 133.

⁹ Jacques Maritain, *Philosophy of Nature*, trans. Imelda C. Byrne (New York: Philosophical Library, 1951), p. 25.

¹⁰ René Descartes, *Compendium of Music*, p. 51.

¹¹ Katherine Gilbert and Helmut Kuhn, p. 208.

¹² Albert Leitzmann, ed., *Briefe* (Leipzig: Insel-Verlag, 1910), letter from Wolfgang Mozart to Leopold Mozart, dated Vienna 26 Sept. 1781.

and one on a sixth. Ironically, the only one that concludes on the octave is *Ritter vom Steckenpferd* (*The Knight of the Rockinghorse*), one of the wildest and most syncopated of the series, which ends abruptly in a manner that is decidedly not pleasant to the ear. Music, like life, is full of surprises. It frequently suspends gratification, changes keys and modes to avoid monotony or predictability, and continuously welcomes the mysterious meeting between sound and sensibility.

There is an evident Pythagorean flavor in Descartes's *Compendium*. The impression is given that the significance of music is to provide mathematical images for the mind. This view of music was later echoed by Gottfried Leibniz who held that "Music charms us, although its beauty consists only in the agreement of numbers and in the counting, which we do not perceive but which the soul nevertheless continues to carry out, of the beats or vibrations of sounding bodies which coincide at certain intervals."¹³

Descartes's *Compendium* is a link between a Pythagorean fascination with numbers and mathematical ratios, and a broader Cartesian view which adds to Pythagoreanism a universal method and a modern respectability. The spirit of Cartesian thinking has inspired a number of twentieth century American mathematicians to enlarge upon what Descartes began in his compendium. Of particular interest are the works of George David Birkhoff, Joseph Schillinger, and Harry Olson.

George David Birkhoff's *magnus opus*, *Aesthetic Measure*, is a concerted attempt to reduce the aesthetic to the mathematical. According to Birkhoff, the "feeling of value" or "aesthetic measure" in a work of art is determined by dividing the amount of "order" by the amount of "complexity" that it contains. In studying the famous choral melody of Beethoven's Ninth Symphony, Birkhoff uses "o" to represent the sum of the elements of order of harmonic, melodic, and rhythmic types, and "c" for the number of notes that appear as melodic constituents. By dividing "o" by "c", "m" becomes the quotient, or "aesthetic measure." Hence, his formula: $m = o/c$. After carefully analyzing Beethoven's choral melody, Birkhoff assigns it a value of slightly more than 5, the result of dividing 303 instances of order by 60 units of complexity.¹⁴ For Birkhoff, to achieve roughly 5 instances of order for each unit of complexity is to achieve a good score. Birkhoff is concerned about the net score rather than the particular moments in the music when order and complexity come into play.

Joseph Schillinger was convinced that a person with no musical ability could compose music solely through mathematical techniques. In his chief work, *The*

¹³ G. W. Leibniz, *Principes de la Nature et de la Grace*, ed. and trans. Leroy E. Loemker (Chicago: University of Chicago Press, 1956), p. 1042.

¹⁴ George David Birkhoff, *Aesthetic Measure* (Cambridge, Massachusetts: Harvard University Press, 1933), p. 156.

Mathematical Basis of the Arts, which is the fruit of twenty-five years of research and reflection, he predicts that the day will come when aesthetics will be reduced to mathematics:

We learn through music what we learn through astronomy and biology. We arrive at an idea. Music is one of the embodiments of the idea. In the remote future of human history through the continuous process of abstraction, this idea will emancipate itself from its functional associations in the way that a pentacle emancipated itself from a starfish or a sea urchin. This will be the logical end of music . . . As physiology becomes a branch of electrical engineering in the study of brain functioning, esthetics becomes a branch of mathematics.¹⁵

Schillinger, who at one time gave George Gershwin lessons in composition, was a composer in his own right. One of his compositions results from correlating musical notes with the rise and fall of the week's stock market. In this case the basis for his music is not mathematics, in the ordinary sense of the term, but an entirely random arrangement.

Harry Olson, in his book, *Musical Engineering*, describes a technique for computer-produced "songs" based on the fact that musical melodies exhibit low entropy. The task of the musical engineer is to determine the probability of certain notes following others in a melodic sequence and, after collecting a sufficient number of probability values, construct a melody. He offers, as an example of this technique, an experiment in which eleven songs of Stephen Foster were fed into a computer which calculated the statistical probability of one note following another. The computer then produced a new Stephen Foster song, so to speak, which sounded like an "average" song written by the composer of "Beautiful Dreamer" and "I Dream of Jeanie with the Light Brown Hair", which lacked any particular distinctiveness.¹⁶

Birkhoff, Schillinger, and Olson exemplify the Cartesian viewpoint that music is not really music, but mathematics. They, like Descartes, view music solely as an object for the intellect. The sensuous element of music, its material embodiment, is so much dross that it must be done away with so that the idea can be emancipated and made to conform to the intellect.

Maritain has pointed out that Descartes hated the concepts proper to the first degree of abstraction and refused them any objective value because they are not

¹⁵ Joseph Schillinger, *The Mathematical Basis of the Arts* (New York: Philosophical Library, 1948), p. 16.

¹⁶ Harry Olson, *Musical Engineering* (New York: McGraw-Hill, 1952), pp. 429-33.

pure concepts. Descartes wanted physics, as well as music, to be free from any relationship with the senses.¹⁷ Music, which is immersed in a sensuous medium, is dependent on matter to a greater extent than is physics. Music as music is not separable from sound. Though it is of an intelligible order, its intelligibility is not conceptual. Furthermore, it has a fundamental appeal to the senses and the passions. On this point, Maritain distinguishes the “ontological splendor” of a work of art from “conceptual clarity.” It is a Cartesian misconception, Maritain goes on to explain, to reduce a work of art that is clear and luminous *in itself* to something that has clarity *for us*. The clarity of the artwork, its intelligibility which is inseparable from its sensuous medium, belongs to the order of a mystery. And “mystery” exists where there is more to be known than is given to comprehension. To refer to the “ontological splendor” of a work of music is to relate it to the radiance of a mystery.¹⁸

Descartes’s insistence on the purely luminous idea, as the only kind of idea worthy of scientific pursuit, brings about a complete antinomy between intellection and mystery.¹⁹ For Descartes, music is not music, but a medium for surrendering concepts. Music is, as it were, frozen mathematics, a kind of congealed intelligibility. At the same time, man becomes a disembodied intellect who comprehends clear ideas, and no longer an embodied person whose emotions are aroused by the strains of a sensuous medium such as music. Thus, the corollary of the antinomy between intellection and mystery is the antinomy between reason and emotion.

Descartes did write a *Treatise on the Passions*. But his treatise serves to deepen the rift he established that alienated bodily passions from the conscious ego. He views emotions from the privileged position of reason. In this regard they are seen from the outside, objectively, so to speak, but not as states of being which we experience as living subjects. Philosopher William Barrett finds that Descartes’s treatise presents feelings as pets who occupy a “house that the master Reason must keep in order.”²⁰ Thus, the I that feels and suffers becomes an alien object that the reason views from a distance. This radical split between reason and feeling is a direct consequence of Descartes’s understanding of man as an isolated consciousness. In “Meditation II” of his *Meditations on First Philosophy*, he writes:

¹⁷ Jacques Maritain, *Philosophy of Nature*, p. 82.

¹⁸ Jacques Maritain, *Art and Scholasticism*, trans. Joseph W. Evans (New York: Charles Scribner’s Sons, 1962), p. 28.

¹⁹ Jacques Maritain, *The Dream of Descartes*, trans. Mabelle L. Andison (New York: Philosophical Library, 1944), p. 174.

²⁰ William Barrett, *Death of the Soul* (Garden City, New York: Doubleday, 1987), p. 20.

. . . to speak accurately I am not more than a thing which thinks, that is to say a mind or a soul, or an understanding, or a reason, . . . I am, however, a real thing and really exist; but what thing? I have answered: a thing which thinks.²¹

Cartesianism de-substantialises or de-materializes music, both in the music itself as well as in the auditor. But music is not music if it is completely spiritualized or conceptualized. Emotion plays as essential a role in the person listening to music as physical sound plays in the music itself. Descartes doubted the reliability of the senses as he passed to the indubitability of the *cogito's* existence. But he could never overcome the distrust he initially projected onto both his sense experience and his bodily existence. The logical end of music in the Cartesian scheme of things is the *cogito* entertaining pure concepts abstracted from musical phenomena. A passage from John Keats's "Ode on a Grecian Urn" comes to mind:

Heard melodies are sweet, but those unheard
Are sweeter; therefore, ye soft pipes, play on;
Not to the sensual ear, but, more endear'd,
Pipe to the spirit ditties of no tone.

Susanne Langer's *Philosophy in a New Key*, though it does not purport to be an original work, sounds an important key-change in the continuing evolution of modern philosophy. This change of key has important epistemological implications and stresses the meaningful symbolic form that is presentational, non-discursive, and undefinable. Music speaks directly to sense and is not reducible to clear concepts.²²

Carl Jung might also be associated with this "change of key." For Jung, who spent a good portion of his professional career studying the nature of a symbol, a symbol loses its value or its "magical" power once it becomes an object for intellectual analysis. As Jung writes, the form of a symbol must be "sufficiently remote from comprehension as to frustrate every attempt of the critical intellect to give any satisfactory account of it; and finally, its aesthetic appearance must have such a convincing appeal to feeling that no sort of argument can be raised against it on that score."²³

Antonio Damasio, as a neurologist, finds scientific evidence that reason and

²¹ René Descartes, *Descartes Selections*, p. 99.

²² Susanne Langer, *Philosophy in a New Key* (New York: The New American Library, 1962), pp. 88-90.

²³ C. G. Jung, *Psychological Reflections*, ed. Jolande Jacobi (New York: Harper & Row, 1953), p. 41.

emotion are not only not antagonistic to each other, but that reason may need emotion in order to maintain its own capacity for rational expression. In his book, *Descartes's Error: Emotion, Reason and the Human Brain*, he draws the reader's attention to research done on previously rational individuals who suffered neurological damage in specific brain systems which prevented them from professing emotions in a normal way. These people were still able to recruit the instruments of rationality, retain knowledge of the practical world in which they must function, and recognize the logic of a problem without evident rational impairment. Yet many of their personal decisions were irrational, and frequently disadvantageous to them and to others.²⁴

Damasio's conclusion is that the absence of emotion appears to be at least as pernicious for rational behavior as its excess. He believes that his neurological findings offer confirmation of the point Pascal was making when he said that reason must establish itself on a knowledge of the heart.

Maritain argued in *The Dream of Descartes* that reason alone is not sufficient to make people behave reasonably.²⁵ In fact, according to Maritain, rationalism is the death of spirituality.

In Damasio's view, Descartes's great error lies in separating the mind so completely from the body that feelings come to be regarded as nothing more than an alien intrusion in the processes of rational thinking. But Damasio's neurological research proves the contrary, that reason needs a support of emotion if the individual is to behave reasonably. "Emotion", he writes, "may well be the support system without which the edifice of reason cannot function properly and may even collapse."²⁶

Descartes's notion of the disembodied mind may have been the source of the contemporary metaphor of mind as a software program. The simple disjunction of software from hardware clearly parallels the Cartesian separation of mind from body. Nevertheless, the organization of the mind-body human being is not that of a computer. The human being is an organic unity. Music, then, although it affects the mind, due to its physical nature and due to the bodily nature of the listener, appeals to the senses and arouses feelings.

Knowledge, therefore, is analogous. There is conceptual knowledge of a mathematical type, and there is poetic knowledge that involves both the senses and the passions. Aquinas has referred to the latter as knowledge through union

²⁴ Antonio Damasio, *Descartes' Error: Emotion Reason and the Human Brain* (New York: G.P. Putnam's Sons, 1994), pp. 245-52.

²⁵ Jacques Maritain, *The Dream of Descartes*, p. 178.

²⁶ Antonio Damasio, "Descartes' Error and the Future of Human Life," *Scientific American* (October 1994), p. 144.

or inclination, connaturality or congeniality.²⁷ This knowledge is of the singular and is intimately bound up with the subjectivity of the artist. Music belongs to the order of poetic knowledge. Moreover, it is a *habitus* of the artist, not a method for the scientist. For this reason, music, which is profoundly personal, can never be reduced to impersonal mathematics.

It may be that the feelings which music is capable of stirring in us have a metaphysical depth beyond that of the rational intellect. Perhaps they grasp, however obscurely, some feature of human life, the world, or the cosmos, that is inaccessible to reason.²⁸ The distinguished symphony conductor Bruno Walter acknowledged this possibility when he confessed that “Music ever spoke to me of a mysterious world beyond, which moved my heart deeply and eloquently intimated its transcendental nature.”²⁹

Music produces a kind of aesthetic-affective satisfaction that we embodied human beings cannot do without. It affirms our nature, helps orient us in our relations with other human beings, and gives us an inspired sense of a numinous order that transcends our meager capacities to comprehend. In this regard, Descartes himself is not an exception.

In one of her letters to Descartes, Princess Elisabeth of Bohemia asked her illustrious correspondent to explain to her “how man’s soul, being only a thinking substance, can determine animal spirits so as to cause voluntary actions.”³⁰ Descartes’s answer is decidedly un-Cartesian. It is wiser to employ the better part of your time, he wrote to her, studying thoughts in which “the understanding acts with the imagination and the senses.”³¹

Perhaps no one is entirely deserving of the “ism” that posterity attaches to his name. Descartes, the person, was not the personification of Cartesianism. His life was not his philosophy put into practice. The inscription he chose for his tombstone was a quotation from Ovid’s *Tristitia* that he apparently had used frequently: *Bene qui latuit, bene vixit* (“He who hid well, lived well.”). The harmonious integration of philosophy and life that this epitaph implies stands as a curious disclaimer of his radical dualism. The notion that a certain disposition

²⁷ Jacques Maritain, *The Range of Reason* (New York: Charles Scribner’s Sons, 1952), pp. 22-29.

²⁸ William Barrett, *Death of the Soul: From Descartes to the Computer* (Garden City, New York: Doubleday, 1987), p. 16.

²⁹ Bruno Walter, *Of Music and Music-Making*, trans. Paul Hamburger (New York: Norton, 1961), p. 16.

³⁰ “Correspondence with Princess Elisabeth, Concerning the Union of Mind and Body,” in *The Essential Descartes*, ed. Margaret D. Wilson (New York: The New American Library, 1969), 6/16 May 1643, p. 373.

³¹ *Ibid.*, 28 June 1643, p. 380.

of mind is conducive to health and long-life has a holistic ring to it, one, in fact, that could have served as an epigraph for Norman Cousins's 1989 book, *Head First: The Biology of Hope*.

Descartes the man surpasses the philosopher, and most assuredly the spirit of Cartesianism, despite the unshakable hold it has on the intellectual consciousness of the modern world. Setting all his philosophical "sins" and methodological "errors" aside, Descartes himself remains elusive and mysterious, the very qualities he sought with all his strength to overcome. Descartes is ample testimony to a timeless truth we must learn again and again, namely, that the aim of philosophy is not to conquer life by comprehending it, but to comprehend it enough so that we can live better.