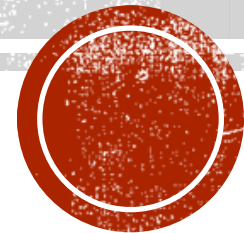


# CHINNECK CHAPTER 4: THE MECHANICS OF THE SIMPLEX METHOD

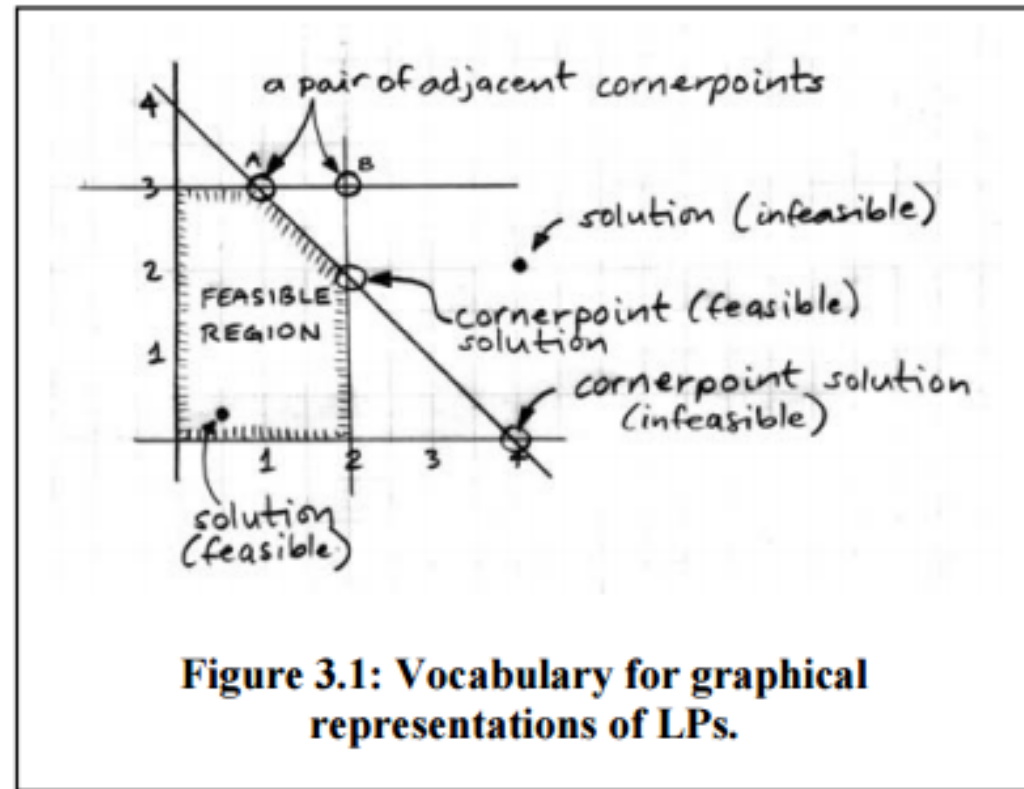
CSE 40/60236

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# THE SIMPLEX METHOD: IMPORTANT TERMINOLOGY

- Solution: any point in the variable space (both feasible and infeasible)
- Cornerpoint solution: anywhere two or more constraints intersect; could be feasible or infeasible
- Feasible cornerpoint solution: a cornerpoint solution that is feasible
- Adjacent cornerpoint solutions: two cornerpoint solutions that are connected by a single constraint line segment; could be feasible or infeasible



# PHASES

- Phase 1
  - Find an initial cornerpoint feasible solution (basic feasible solution). If none is found, then the model is infeasible, so exit.
- Phase 2
  - Iterate until the stopping conditions are met.



# PHASE 2 BREAKDOWN

- Phase 2.1
  - *Are we optimal yet?* Look at the current version of the objective function to see if an entering basic variable is available. If none is available, then exit with the current basic feasible solution as the optimum solution.
- Phase 2.2
  - *Select entering basic variable:* choose the nonbasic variable that gives the fastest rate of increase in the objective function value.



# PHASE 2 BREAKDOWN (CONTINUED)

- Phase 2.3
  - *Select the leaving basic variable* by applying the Minimum Ratio Test.
- Phase 2.4
  - *Update the equations* to reflect the new basic feasible solution.
- Phase 2.5
  - Go to Step 2.1.



# THE SIMPLEX TABLEAU

- A table representation of the basis at any cornerpoint
- Contains all information needed to decide on the exchange of variables that drives the movement between cornerpoints as the simplex method advances
- Can be used to solve simple LPs by hand
  - This can be tedious and error-prone, though



# EXAMPLE: ACME BICYCLE PROBLEM (ABC)

basic variable	eqn. no.	Z	$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	RHS	MRT
Z	0	1	-15	-10	0	0	0	0	<i>never</i>
$s_1$	1	0	1	0	1	0	0	2	
$s_2$	2	0	0	1	0	1	0	3	
$s_3$	3	0	1	1	0	0	1	4	

**Tableau 4.1: ABC at the origin.**

- Equation 0: objective function
- Equations 1-3: constraints
- RHS: right hand side (of the equation)
- MRT: minimum ratio test



# STANDARD FORM VS. PROPER FORM

- Standard form refers to linear problems, but proper form refers to tableaus
- Proper form characteristics
  - Exactly 1 variable per equation
  - Coefficient of the basic variable is always exactly +1, and the coefficients above and below it in the same column are all 0
  - Z is treated as the basic variable for the objective function row (equation 0)





# STEP 2.1: ARE WE OPTIMAL YET?

- If no entering variable is available, then yes
- Entering variable is the nonbasic variable that gives the fastest rate of increase in the objective function value
- In tableaus, this changes a bit to become *the variable in the objective function row that has the most negative value as the entering basic variable*



# STEP 2.2: SELECT THE ENTERING BASIC VARIABLE

- Entering basic variable is the nonbasic variable in the objective function row that has the most negative coefficient
- The tableau column for the entering basic variable is called the pivot column



# STEP 2.3: SELECT THE LEAVING BASIC VARIABLE

- The MRT is used to determine the leaving basic variable
  - This determines which constraint most limits the increase in the value of the entering basic variable
  - The most limiting constraint is the one whose basic variable is driven to zero first as the basic variable increases in value



# STEP 2.3: SELECT THE LEAVING BASIC VARIABLE (CONTINUED)

- To determine MRT values, look only at entries in the pivot column for the constraint rows, and calculate the following:

$$\text{(RHS)} / \text{(coefficient of entering basic variable)}$$

- Two special cases:
  - If coefficient of the basic entering variable = 0, enter *no limit* in the MRT column
  - If coefficient of the basic entering variable < 0, enter *no limit* in the MRT column
- The MRT is *never* applied to the objective function row



# STEP 2.3: SELECT THE LEAVING BASIC VARIABLE (CONTINUED)

- After all rows have been calculated, the leaving basic variable is associated with the row with the smallest MRT value
  - This row is called the pivot row
  - The intersection of the pivot row and pivot column is known as the pivot element

basic variable	eqn. no.	Z	$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	RHS	MRT
Z	0	1	-15	-10	0	0	0	0	<i>never</i>
$s_1$	1	0	1	0	1	0	0	2	$2/1 = 2$
$s_2$	2	0	0	1	0	1	0	3	<i>no limit</i>
$s_3$	3	0	1	1	0	0	1	4	$4/1 = 4$

Tableau 4.2: Updated ABC at the origin.



# STEP 2.4: UPDATE THE TABLEAU

- Since updating the entering and leaving basic variables, the tableau is now not in proper form, and must be put back in form
- This can be done with the following steps:
  - 2.4.1: In the *basic variable* column, replace the leaving basic variable for the pivot row by the entering basic variable
  - 2.4.2: The pivot element becomes the new coefficient associated with the new basic variable. If it is not already +1, divide all elements in the pivot row by the pivot element to obtain +1 in the pivot element position



# STEP 2.4: UPDATE THE TABLEAU (CONTINUED)

- 2.4.3: All of the coefficients in the pivot column except the pivot element must be set to zero. This can be done for any row  $k$  by using the following equation:

$$(\text{new row } k) = (\text{row } k) - (\text{pivot column coefficient in row } k) \times (\text{pivot row})$$

For example, to clean out the -15 that appears in the objective function row,

- new  $Z$  coefficient =  $1 - (-15) \times 0 = 1$
- new  $x_1$  coefficient =  $-15 - (-15) \times 1 = 0$
- new  $x_2$  coefficient =  $-10 - (-15) \times 0 = -10$
- new  $s_1$  coefficient =  $0 - (-15) \times 1 = 15$
- new  $s_2$  coefficient =  $0 - (-15) \times 0 = 0$
- new  $s_2$  coefficient =  $0 - (-15) \times 0 = 0$
- new RHS =  $0 - (-15) \times 2 = 30$



# STEP 2.4: UPDATE THE TABLEAU (CONTINUED)

basic variable	eqn. no.	Z	$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	RHS	MRT
Z	0	1	0	-10	15	0	0	30	<i>never</i>
$x_1$	1	0	1	0	1	0	0	2	
$s_2$	2	0	0	1	0	1	0	3	
$s_3$	3	0	0	1	-1	0	1	2	

**Tableau 4.3: The second basic feasible solution.**





# FINISHING THE OPTIMIZATION

basic variable	eqn. no.	Z	$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	RHS	MRT
Z	0	1	0	-10	15	0	0	30	<i>never</i>
$x_1$	1	0	1	0	1	0	0	2	no limit
$s_2$	2	0	0	1	0	1	0	3	$3/1 = 3$
$s_3$	3	0	0	1	-1	0	1	2	$2/1 = 2$

**Tableau 4.4: The second iteration.**

basic variable	eqn. no.	Z	$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	RHS	MRT
Z	0	1	0	0	5	0	10	50	<i>never</i>
$x_1$	1	0	1	0	1	0	0	2	
$s_2$	2	0	0	0	1	1	-1	1	
$x_2$	3	0	0	1	-1	0	1	2	

**Tableau 4.5: The optimal tableau.**



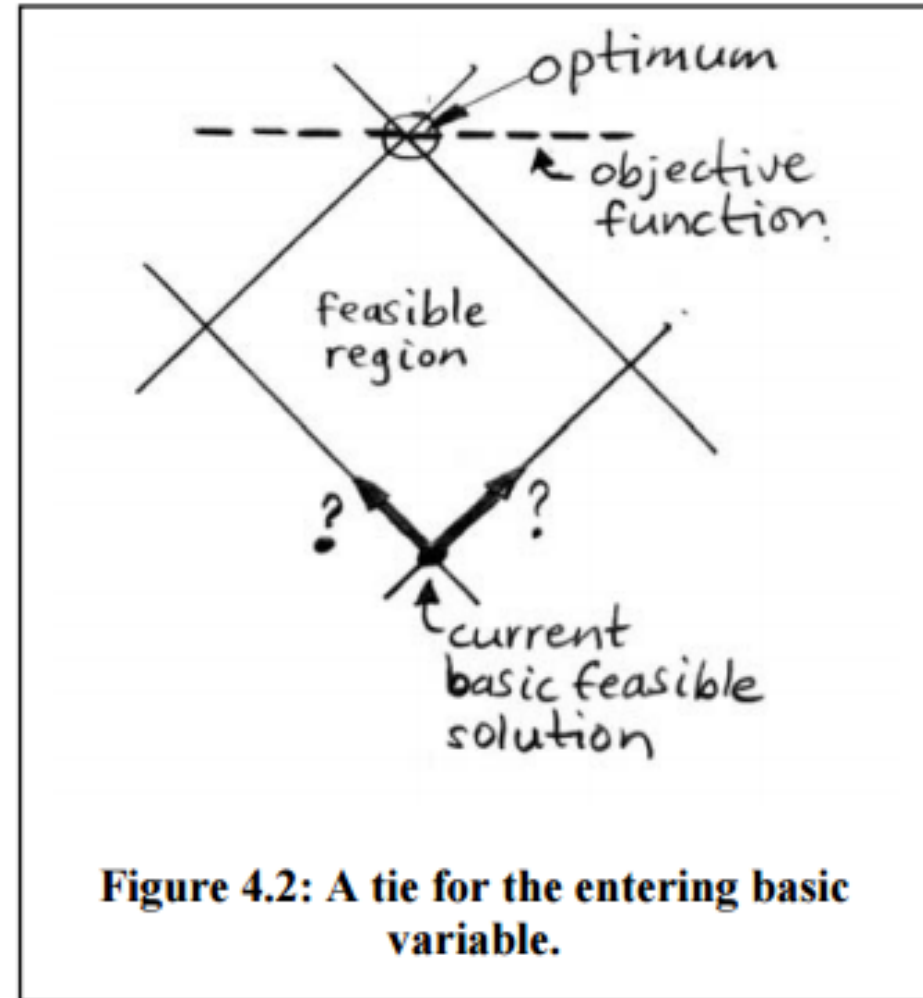
# SPECIAL CASES IN TABLEAU MANIPULATION

- Tie for the entering basic variable
- Tie for the leaving basic variable
- At the optimum, the coefficients of some nonbasic variables are zero in the objective function row

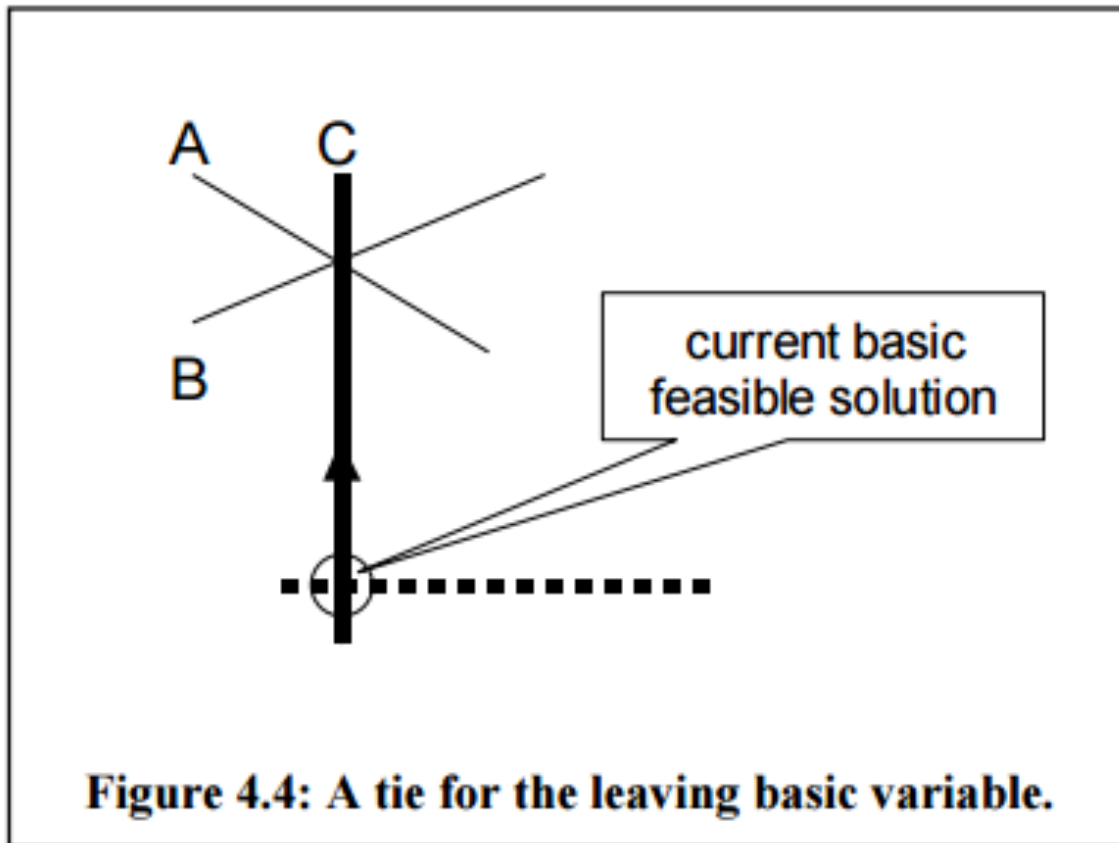


# ENTERING BASIC VARIABLE TIE

- What if there is a tie for the most negative value in the objective function?
- Solution: pick one arbitrarily and start there
- There is no way to know ahead of time which one will be more efficient in finding an optimal solution, so choosing one over the other doesn't matter



# LEAVING BASIC VARIABLE TIE

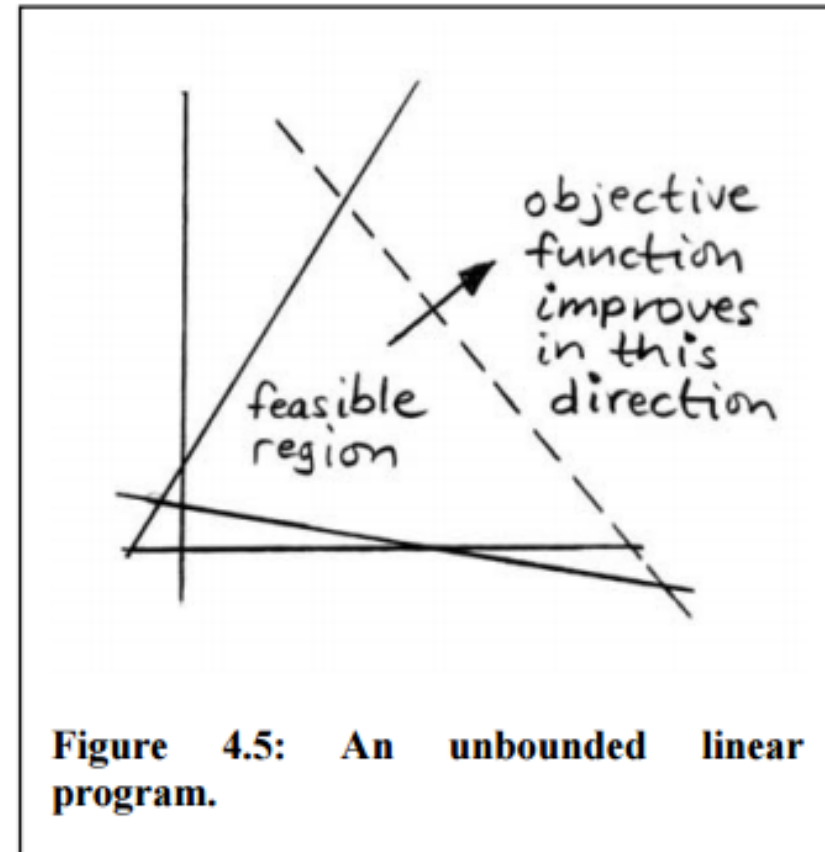


- What if there is more a tie during the MRT?
- Solution: pick one arbitrarily and go from there
- The variable not chosen will remain basic, but will have a calculated value of zero
- The variable chosen will be forced to zero by simplex
- Both variables will become zero simultaneously because both constraints are active at that point
  - The simplex just only needs one of them to define the basic feasible solution



# LEAVING BASIC VARIABLE TIE (CONTINUED)

- What if the MRT values are tied at *no limit*?
  - This means no constraint puts a limit on the increase in the value of the entering basic variable
  - There is, then, no limit on the increase in the value of the objective function
- This, unfortunately, usually means that you forgot a constraint
- These problems are *unbounded*, and have *unbounded solutions*



# NONBASIC VARIABLE COEFFICIENTS ARE ZERO IN OBJECTIVE FUNCTION ROW AT THE OPTIMUM

- In this case, choosing one of these variables as the entering basic variable has *no effect* on  $Z$
- Basically, you will pivot to a *different* basic feasible solution, which will have the same value of  $Z$
- This means that this problem has *multiple optimum solutions*
- To see these other optimum solutions, choose one of the nonbasic variables whose objective function coefficient is zero as the entering basic variable, and pivot to another basic feasible solution as you normally would

