Supplementary Material to "Cash Flow, Consumption Risk, and the Cross-Section of Stock Returns"

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This document contains supplementary material to the paper titled "Cash Flow, Consumption Risk, and the Cross Section of Stock Returns." It contains six sections. Section A details why the two-factor cash flow model captures the relation between risk premium and cash flow characteristics in the simple economy discussed in the paper. Section B solves the risk premium on an asset using the usual return-based beta representation, reinforcing the intuition behind the two-factor cash flow model and also relating the cash flow characteristics directly to the standard return-based consumption beta. Section C examines a slightly modified simple economy where the cash flow of an asset is also exposed to long-run consumption risk and shows that the two-factor cash flow model is still valid. Section D shows that the empirical long-run earnings-based measures identify their theoretical counterparts up to scaling factors in the simple economy. Section E examines the performance of the cash flow models on industry portfolios. Section F contrasts two related variables: cash flow duration and the book-to-market ratio.

A.   *Two-factor Cash Flow Model as an Approximation*

This section provides the details for Section I.D in the paper and explains why the two-factor cash flow model captures the relation between risk premium and cash flow characteristics in the simple economy discussed in the paper. Proposition 1 in the paper shows that the risk premium on an equity strip with a maturity n in the simple economy is

$$RP^i(n) = (1 + \phi^{n-1}\lambda^i)\left[1 + (\gamma - 1)\frac{1 - \rho_2\delta}{1 - \rho_1\delta}\right]\sigma_w^2.$$

The risk premium on a stock is just the value-weighted average of risk premia of all equity strips and can be approximated as

$$E_t\left[R_{t+1}^i - Rf_t\right] \approx \sum_{n=1}^{\infty} w^i(n)RP^i(n).$$

Consider a linear approximation of the risk premium on individual equity strip $RP^i(n)$ around some fixed maturity n^*:

$$RP^i(n) \approx RP^i(n^*) + RP_n^{i'}(n^*)(n - n^*),$$

where $RP_n^{i'}(n^*)$ denotes the first derivative of RP_n^i with respect to n, evaluated at n^*; the risk premium on a stock then becomes

$$E_t\left[R_{t+1}^i - Rf_t\right] \approx RP^i(n^*) + RP_n^{i'}(n^*)\left[\sum_{n=1}^{\infty}w^i(n)n - n^*\right]. \tag{1}$$

Direct computation shows that

$$RP^i(n^*) = a_0 + a_1\lambda^i, \tag{2}$$

$$RP_n^{i'}(n^*) = a_2\lambda^i, \tag{3}$$

$$\sum_{n=1}^{\infty}w^i(n)n - n^* \approx a_3 z_t^i, \tag{4}$$

$$a_0 = \left[1 + (\gamma - 1)\frac{1 - \rho_2\delta}{1 - \rho_1\delta}\right]\sigma_w^2,$$

$$a_1 = \phi^{n^*-1}\left[1 + (\gamma - 1)\frac{1 - \rho_2\delta}{1 - \rho_1\delta}\right]\sigma_w^2,$$

$$a_2 = \phi^{n^*-1}\log\phi\left[1 + (\gamma - 1)\frac{1 - \rho_2\delta}{1 - \rho_1\delta}\right]\sigma_w^2.$$

Given a risk-averse agent with $\gamma > 1$ and $n^* > 1$, it can easily be verified that $a_0 > 0$, $a_1 > 0$, and $a_2 < 0$.

To understand a_3, define a function $f(z_t^i)$ as

$$f(z_t^i) = \sum_{n=1}^{\infty}w^i(n)n.$$

Consider a linear approximation of $f(z_t^i)$ around $z_t^i = 0$:[1]

$$f(z_t^i) \approx f(0) + f'(0)z_t^i.$$

Choosing $n^* = f(0)$, which can be interpreted as the Macaulay duration of an asset with $z_t^i = 0$ (for example, the aggregate consumption portfolio), then

$$\sum_{n=1}^{\infty}w^i(n)n - n^* \approx a_3 z_t^i,$$

$$a_3 = f'(0).$$

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[1]The cash flow covariance measure λ^i also enters $w^i(n)$ through the convexity adjustment terms in $A^i(n)$. Its impact on $\sum w^i(n)n$, however, is relatively small.

Finally, it has to be verified that $a_3 = f'(0) > 0$. Direct calculation of $f'(0)$ shows that

$$
\begin{aligned}
f'(0) \quad &> \quad 0 \Leftrightarrow \\
\sum n \exp\left[A^i(n)\right](1 - \phi^n) \sum \exp\left[A^i(n)\right] \quad &> \quad \sum n \exp\left[A^i(n)\right] \sum (1 - \phi^n) \exp\left[A^i(n)\right] \Leftrightarrow \\
\frac{\sum n \exp\left[A^i(n)\right]}{\sum \exp\left[A^i(n)\right]} \quad &> \quad \frac{\sum n \exp\left[A^i(n)\right] \phi^n}{\sum \exp\left[A^i(n)\right] \phi^n}.
\end{aligned}
\tag{5}
$$

Define function $g(x)$ as

$$
g(x) = \frac{\sum n \exp\left[A^i(n)\right] \exp(nx)}{\sum \exp\left[A^i(n)\right] \exp(nx)}.
$$

Then inequality (5) is equivalent to $g(0) > g(\log\phi)$, which will be true if $g(x)$ is increasing in x or $g'(x) > 0$. Direct calculation of $g'(x)$ shows that

$$
\begin{aligned}
g'(x) \quad &> \quad 0 \Leftrightarrow \\
\sum n^2 a(n) \quad &> \quad \left[\sum n a(n)\right]^2, \\
\text{where } a(n) \quad &= \quad \frac{\exp\left[A^i(n)\right] \exp(nx)}{\sum \exp\left[A^i(n)\right] \exp(nx)}.
\end{aligned}
\tag{6}
$$

Given $a(n) > 0$, $\sum a(n) = 1$, and $h(n) = n^2$ is a convex function, inequality (6) then follows from Jensen's inequality. The intuition behind the positive a_3 is clear. The term z_t^i can be interpreted as the expected cash flow growth rate (relative to aggregate consumption growth). A higher z_t^i means that more cash flow will occur in the future, thus increasing the present value-weighted time as in $\sum w^i(n)n$.

Substituting (2), (3), and (4) into (1) gives us the two-factor cash flow model:

$$
E_t\left[R_{t+1}^i - Rf_t\right] \approx \gamma_0 + \gamma_1\lambda^i + \gamma_2(z_t^i\lambda^i),
$$

$$
\gamma_0 = a_0 > 0, \gamma_1 = a_1 > 0 \text{ and } \gamma_2 = a_2 a_3 < 0.
$$

B.   Beta Representation in a Return Log-linearization Framework

In this section, I provide an alternative derivation of the risk premium using the usual return based beta-representation framework to achieve three objectives: (1) to reinforce the intuition behind the two-factor cash flow model; (2) to relate the cash flow characteristics directly to the standard return-based consumption beta; and (3) to relate the cash flow characteristics to the "cash

flow risk" and "discount rate risk".

Under the usual assumption that the return and the Stochastic Discount Factor (SDF) are jointly log-normally distributed (conditionally), the conditional expected excess return of an asset can be expressed as[2]

$$E_t\left[r^i_{t+1} - rf_t\right] + \frac{\sigma^2_t\left[r^i_{t+1}\right]}{2} = -cov_t(r^i_{t+1}, m_{t+1}). \tag{7}$$

I then use a log-linear approximation to write the return on asset i as

$$
\begin{aligned}
r^i_{t+1} &= \kappa^i_0 + \kappa^i_1 x^i_{t+1} - x^i_t + d^i_{t+1} - d^i_t, \\
\kappa^i_1 &= \frac{\exp(\overline{x}^i)}{1 + \exp(\overline{x}^i)}, \\
\kappa^i_0 &= \log\left[1 + \exp(\overline{x}^i)\right] - \kappa^i_1 \overline{x}^i,
\end{aligned}
$$

where x^i_t is the log price to cash flow ratio $\left(\log \frac{P_t}{D_t}\right)$ at time t and $\overline{x}^i$ is its time-series average.

Conjecture $x^i_{t+1} = a^i + b^i z_{t+1}$ and use the cash flow process, consumption growth process and the SDF specified in the simple economy to evaluate the following relation:

$$E_t\left[\exp(m_{t+1} + r^i_{t+1})\right] = 1.$$

After collecting the terms on z^i_t, we have

$$\phi\kappa^i_1 b^i - b^i + (1 - \phi) = 0,$$

which implies

$$b^i = \frac{1 - \phi}{1 - \phi\kappa^i_1}.$$

Then,

$$
\begin{aligned}
r^i_{t+1} &= E_t\left[r^i_{t+1}\right] + \beta^i w_{t+1} + \frac{1 - \kappa^i_1}{1 - \phi\kappa^i_1}\varepsilon^i_{t+1}, \text{ where} \\
\beta^i &= 1 + \frac{1 - \kappa^i_1}{1 - \phi\kappa^i_1}\lambda^i.
\end{aligned}
$$

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[2]See Campbell (1993), for example.

Equation (7) can also be rewritten using a beta representation in my economy:

$$E_t\left[r_{t+1}^i - rf_t\right] + \frac{\sigma_t^2\left[r_{t+1}^i\right]}{2} = \beta^i \lambda_c, \tag{8}$$

$$\beta^i = 1 + \frac{1-\kappa_1^i}{1-\phi\kappa_1^i}\lambda^i,$$

$$\lambda_c = \left[1 + (\gamma-1)\frac{1-\rho_2\delta}{1-\rho_1\delta}\right]\sigma_w^2,$$

where β^i denotes the beta of asset i, and λ_c denotes consumption risk premia. The term $\kappa_1^i = \frac{\exp(\overline{x}^i)}{1+\exp(\overline{x}^i)}$ is a log-linearization constant where $\overline{x}^i$ is usually chosen to be equal to the average log price-to-cash flow ratio. The term β^i can be rewritten using $\overline{x}^i$ as

$$\beta^i = 1 + \lambda^i - (1-\phi)\exp(\overline{x}^i)\lambda^i. \tag{9}$$

Since the average log price-to-cash-flow ratio should be directly related to cash flow duration (see Proposition 1), (9) also gives us the two-factor cash flow model. On the other hand, the usual practice of assuming a constant κ_1^i across all stocks effectively eliminates the impact of cash flow duration when examining cross-sectional variation in risk premia.

Campbell and Shiller (1988) decompose the return on an asset into a component ($N_{CFi,t+1}$) related to cash flow news and a component ($N_{DRi,t+1}$) related to discount rate news, written as:

$$r_{t+1}^i - E_t\left[r_{t+1}^i\right] = N_{CFi,t+1} - N_{DRi,t+1}, \text{ where} \tag{10}$$

$$N_{CFi,t+1} = (E_{t+1} - E_t)\sum_{j=0}^{\infty}\left(\kappa_1^i\right)^j \Delta d_{t+1+j}^i,$$

$$N_{DRi,t+1} = (E_{t+1} - E_t)\sum_{j=1}^{\infty}\left(\kappa_1^i\right)^j r_{t+1+j}^i.$$

We may have the impression that $N_{CFi,t+1}$ is related to cash flow covariance risk and that $N_{DRi,t+1}$ is related to cash flow duration. By substituting (10) into (7), the beta can also be decomposed into two parts: a cash flow beta (β_{CF}^i) and a discount rate beta (β_{DR}^i) similar to those in Campbell and Mei (1993) and Campbell, Polk, and Vuolteenaho (2003). Specifically,

$$\beta^i = \beta_{CF}^i + \beta_{DR}^i,$$

$$\beta_{CF}^i = \frac{\left(1-\kappa_1^i\right)\lambda^i}{1-\phi\kappa_1^i} + \frac{1-\rho_2\kappa_1^i}{1-\rho_1\kappa_1^i},$$

$$\beta_{DR}^i = 1 - \frac{1-\rho_2\kappa_1^i}{1-\rho_1\kappa_1^i}.$$

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In my model, discount rate news is driven by risk-free rate dynamics, which are in turn driven by innovations in consumption growth. This, together with ψ equaling one, explains why the second terms in the cash flow beta and in the discount rate beta offset each other. The cash flow covariance measure λ^i enters the expression of β^i directly whereas the cash flow duration measure z_t^i enters only indirectly through κ_1^i. The impact of the duration on beta and expected return is therefore difficult to examine. Although the return decomposition approach has many theoretical advantages (e.g, economically intuitive, allows for time-varying risk premium), it does not allow us to see clearly the linkage between risk/return and fundamental cash flow characteristics. This is why I choose to examine each equity strip separately in this paper.

C. Cash Flow Models with Long-run Risk

In the paper, I model the cash flow covariance as the contemporaneous covariance between innovations in cash flow share and innovations in the aggregate consumption growth. This simple specification allows for both analytical tractability and easy economic interpretation. In this section, I model the cash flow covariance as the exposure of cash flow share to both long-run and short-run consumption risk and show that a similar two-factor cash flow model can still be derived in the simple economy.

In the simple economy considered in the paper, the log aggregate consumption growth in the economy follows an ARMA(1,1) process:

$$
\begin{aligned}
\Delta c_{t+1} &= \mu_c(1 - \rho_1) + \rho_1 \Delta c_t + w_{t+1} - \rho_2 w_t, \\
w_t &\sim N(0, \sigma_w^2).
\end{aligned}
$$

Define $x_t = E_t\left[\Delta c_{t+1}\right] - \mu_c$. The ARMA(1,1) process can be rewritten as

$$
\begin{aligned}
\Delta c_{t+1} &= \mu_c + x_t + w_{t+1}, \\
x_{t+1} &= \rho_1 x_t + (\rho_1 - \rho_2)w_{t+1}.
\end{aligned}
$$

As a result, x_t, which captures the conditional expected consumption growth rate, can be interpreted as the "long-run" consumption risk. The term w_{t+1}, which measures the contemporaneous innovation in consumption growth, can be interpreted as the "short-run" consumption risk.

I next model the cash flow growth rate on a stock (portfolio) i as

$$\Delta d_{t+1}^i = \Delta c_{t+1} + (1 - \phi)z_t^i + \lambda^i(\Delta c_{t+1} - \mu_c) + \varepsilon_{t+1}^i$$

$$= \Delta c_{t+1} + (1 - \phi)z_t^i + \lambda^i(x_t + w_{t+1}) + \varepsilon_{t+1}^i,$$

$$z_{t+1}^i = \phi z_t^i - \lambda^i(x_t + w_{t+1}) - \varepsilon_{t+1}^i.$$

The only difference between this specification and the specification used in the paper is that cash flow covariance (λ) is now modeled as the exposure of cash flow share to both long-run and short-run consumption risk, rather than the exposure to short-run consumption risk alone. To keep the algebra simple, I assume the cash flow share has the same exposure to both long-run and short-run consumption risk. This assumption is also made implicitly in the co-integration specification of Bansal, Dittmar, and Lundblad (2002).

Having specified the cash flow and aggregate consumption growth process, I proceed to solve for the risk premium expression on an equity strip as a function of its maturity (n). The risk premium is

$$RP^i(n) = \left\{ 1 + \lambda^i \left[\phi^{n-1} + (\rho_1 - \rho_2)A(n-1) \right] \right\} \left[1 + (\gamma - 1)\frac{1 - \rho_2\delta}{1 - \rho_1\delta} \right] \sigma_w^2, \tag{11}$$

where $A(n)$ evolves according to the following difference equation with an initial condition that $A(0) = 0$:

$$\phi^{n-1} + \rho_1 A(n-1) = A(n).$$

The risk premium expression is obtained by following the exact same procedure as described in Appendix A of the paper and the details are thus omitted. Due to the presence of the long-run risk, the algebra is more complicated and an analytical expression cannot be obtained. Compared to the equity strip risk premium in the paper, the presence of long-run risk results in one additional term — $(\rho_1 - \rho_2)A(n-1)$.

It can be shown that for reasonable parameter values of ϕ and ρ_1 (close to but smaller than one), $A(n-1)$ first increases, then decreases in n. The term ϕ^{n-1}, on the other hand, always decreases in n. A typical long-run risk model sets ρ_1 to be close to one and slightly greater than ρ_2. Such parameter choice allows consumption growth to closely resemble an i.i.d. process empirically. At the same time, the persistent expected consumption growth rate leads to a larger risk premium

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and a potential solution to the equity risk premium puzzle. In this case, $\rho_1 - \rho_2$ will be very small and $\phi^{n-1} + (\rho_1 - \rho_2)A(n-1)$, dominated by the first term (ϕ^{n-1}), is likely to be decreasing in n when n is not too small. This pattern has been confirmed in Figure S1.

Place Figure S1 about here

Figure S1 plots ϕ^{n-1}, $(\rho_1 - \rho_2)A(n-1)$, and their sum separately as a function of n (in number of months). The ARMA(1,1) parameters ($\rho_1 = 0.965$ and $\rho_2 = 0.851$) are taken from Bansal and Yaron (2000) and ϕ is chosen as 0.98 for the plot. As shown in the figure, the value of $\phi^{n-1} + (\rho_1 - \rho_2)A(n-1)$ peaks around year 2 (month 24), and is decreasing in n after that. As in the simple economy, for an equity strip with infinite maturity ($n = \infty$), the risk premium becomes $\left[1 + (\gamma - 1)\frac{1-\rho_2\delta}{1-\rho_1\delta}\right]\sigma_w^2$, again due to the mean-reversion in cash flow share, such that the impact of cash flow covariance diminishes with maturity.

A similar two-factor cash flow model can be derived in this economy where cash flow is also exposed to long-run risk. Higher cash flow covariance (λ^i) should lead to higher risk premium as in (11). Since $\phi^{n-1} + (\rho_1 - \rho_2)A(n-1)$ is decreasing in n after year 2, the interaction between cash flow covariance and duration will be very similar to that in the simple economy. When cash flow covariance (λ^i) is positive, the risk premium of an individual equity strip generally decreases with maturity. When this happens, high duration assets will have lower returns since a long-maturity cash flow with lower return receives higher present-value weight and the weighted average is lower. The reverse logic holds for negative cash flow covariance, with a higher duration leading to a higher return. Consequently, the product of cash flow covariance and duration is negatively related to the risk premium on a stock.

D.   Theoretical and Empirical Cash Flow Characteristics

This section proves that the empirical long-run earnings-based measures (Cov and Dur) identify their theoretical counterparts (λ and z) up to scaling factors in the simple economy.

D.1 Cash Flow Duration

By definition,

$$\sum_{n=0}^{\infty}\rho^n\Delta s^i(t, n+1) = \sum_{n=0}^{\infty}\rho^n\Delta d^i(t, n+1) - \sum_{n=0}^{\infty}\rho^n\Delta c_{t+n+1}.$$

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Taking the expectation at each portfolio formation time t on both sides,

$$E_t \left\{ \sum_{n=0}^{\infty} \rho^n \Delta s(t, n+1) \right\} = E_t \left[\sum_{n=0}^{\infty} \rho^n (1-\phi) z(t, n) \right]$$
$$= \frac{1-\phi}{1-\rho\phi} z_t,$$

cash flow duration $E[z_t]$ can be identified (up to a scaling factor) by

$$Dur^i = E\left[Dur_t^i\right]$$
$$= E\left\{ \Sigma_t^{ei} - \frac{\kappa}{1-\rho} - \xi_t^i - E_t\left[\Sigma_t^{\Delta c}\right] \right\}$$
$$= E\left\{ E_t\left[\Sigma_t^{ei}\right] - \frac{\kappa}{1-\rho} - \xi_t^i - E_t\left[\Sigma_t^{\Delta c}\right] \right\}$$
$$= \frac{1-\phi}{1-\rho\phi} E[z_t],$$

where $\Sigma_t^{ei} = \sum_{n=0}^{\infty} \rho^n e^i(t, n+1)$ and $\Sigma_t^{\Delta c} = \sum_{n=0}^{\infty} \rho^n \Delta c_{t+n+1}$.

D.1 Cash Flow Duration

To estimate the cash flow covariance λ^i, consider

$$cov\left(\sum_{n=0}^{\infty} \rho^n \Delta s^i(t, n+1), \sum_{n=0}^{\infty} \rho^n w_{t+n+1} \right) = cov\left(\sum_{n=0}^{\infty} \rho^n \left[e^i(t, n+1) - \Delta c_{t+n+1} \right], \sum_{n=0}^{\infty} \rho^n w_{t+n+1} \right).$$

In my model specification, the LHS is

$$\frac{1}{(1-\phi\rho)(1+\rho)} \lambda^i \sigma_w^2.$$

Therefore, by regressing $\sum_{n=0}^{\infty} \rho^n \left[e^i(t, n+1) - \Delta c_{t+n+1} \right]$ on $\sum_{n=0}^{\infty} \rho^n w_{t+n+1}$, the regression coefficient Cov identifies cash flow covariance (λ^i) up to a scaling factor.

E. Performance of the Cash Flow Models on Industry Portfolios

This section examines the performance of the cash flow models on industry portfolios.

Every June, I sort all stocks of industrial firms (excluding financials and utilities) traded on NYSE, Amex, and NASDAQ into industry portfolios according to a 17 Fama-French industry classification.[3] The resulting 15 portfolios are: Food, Mines(mining and minerals), Oil(oil and petroleum

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[3]I do not consider a finer industry classification since that would result in too few stocks in certain industry

products), Clths(textiles, apparel, and footware), Durbl(consumer durables), Chems(chemicals), Cnsum(drugs, soap, perfumes, and tobacco), Cnstr(construction), Steel, FabPr(fabricated products), Machn(machinery and business equipment), Cars, Trans(transportation), Rtail(retail stores) and Other.

Table S1 presents various portfolio characteristics including the portfolio book-to-market ratio (BM), market equity (ME, measured in millions) at formation, and annual return during the first year after portfolio formation. All values are time-series averages across a sampling period from 1964 to 2002.

Place Table S1 about here

I also directly test the validity of the AR(1) assumption on the cash flow share for the 15 portfolios. I first fit an AR(1) process for the cash flow share and compute the residuals. I then test whether these residuals violate the white noise condition using the Ljung-Box Q test. Both the Ljung-Box (LB) Q test statistics and the associated p-values are reported. In addition, I also test the stationarity of the cash flow share using the Augmented Dickey-Fuller test with a constant and a lag of one. The t-values are reported (** means the hypothesis of a unit root can be rejected at the 99% confidence level and * means the hypothesis can be rejected at the 95% confidence level). The cash flow share in year t is computed as the log of the ratio between the portfolio cash flow (sum of common dividend and common share repurchase) and aggregate consumption during year t.

For seven out of the 15 industry portfolios, I am not able to reject the unit root hypothesis, indicating that the key assumption that cash flow share is mean-reverting might not be very appropriate for industry portfolios. As a result, the cash flow covariance defined in my paper might not be measuring the true consumption risk on the industry portfolio.

Table S1 also presents the cash flow covariance and duration estimates for the 15 portfolios. As expected, a "growing" industry such as Cnsum is associated with a higher cash flow duration while an industry with little "growth" potential such as Steel has a lower cash flow duration.

I then investigate the performance of cash flow models on industry portfolios in a cross-sectional analysis. The coefficient or the risk premium estimates on the cash flow models are obtained from OLS regressions. However, the robust t-values are computed using GMM standard errors that account for both cross-sectional and time-series error correlations with the Newey-West formula of

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portfolios, rendering the estimation of cash flow characteristics very imprecise.

seven lags. The one-stage GMM estimation is carried out by stacking moment conditions of both time-series regressions and cross-sectional regressions. The results are presented in Table S2. The cash flow models do a reasonably good job in describing the cross-sectional variation of average excess returns on the 15 industry portfolios. The one-factor cash flow model with only cash flow covariance has a R^2 of 42.5% (adjusted-R^2 is 38.1%). The two-factor cash flow model with only cash flow covariance and duration has a R^2 of 59.8% (adjusted-R^2 is 53.1%). Similar to the findings in the paper, the inclusion of cash flow duration improves the R^2 by about 17%.

Place Table S2 about here

The risk premium on cash flow covariance (Cov) is positive while the risk premium on the interaction term ($Cov \times Dur$) is negative, consistent with the prediction of the theory. However, once the time-series estimation errors on cash flow covariance and duration are accounted for, both Cov and $Cov \times Dur$ are associated with insignificant risk premia.

Overall, the performance of the cash flow models on industry portfolios are qualitatively similar although the associated statistical significance is much weaker, possible due to mis-specification of the cash flow share process, large time-series estimation errors, and small sample size in the cross-section.

F.   *Cash flow duration vs. book-to-market ratio*

This section contrasts the cash flow duration to the commonly studied book-to-market ratio. Empirically, book-to-market seems to be inversely related to cash flow duration. This pattern should not surprise us. As Lintner (1975) and Santa-Clara (2004) point out, any measure of cash flow duration will be related to book-to-market simply as a result of accounting identities. Making use of the accounting clean surplus identity and return-dividend-price relation, Vuolteenaho (2002) shows that the log book-to-market ratio (θ_t) can be approximated as

$$\theta_t = \sum_{j=0}^{\infty} \rho^j r_{t+j+1} - \sum_{j=0}^{\infty} \rho^j e_{t+j+1}, \tag{12}$$

where r denotes log returns. Therefore, an increase in future accounting earnings that increases cash flow duration measure will at the same time decrease the book-to-market ratio. In turn, cash flow duration is negatively correlated the with book-to-market. Lettau and Wachter (2007) study an economy in which stocks are only distinguished by the timing of their cash flows. In such an

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economy, they show that stocks with cash flows weighted more to the future (high duration) have low price ratios (book-to-market ratio, for example) and earn low return. Therefore, cash flow duration can potentially explain value premium. My results, on the topical level, seem to support their hypothesis since value stocks indeed have lower duration than growth stocks. However, I would require further analysis to answer a more interesting question: can the cash flow duration alone explain value premium? If the cash flow duration alone perfectly explains value premium, we would expect further sorting on book-to-market to generate no spread in returns once we control for cash flow duration.

To control for cash flow duration, I first sort all stocks according to the "ex-ante" cash flow duration measure – $\widehat{Dur}_t$ into three groups: Low Duration, Medium Duration, and High Duration. Within each group, I further sort stocks according to BM into three subgroups. To make sure that such portfolio construction is implementable, at each year, I reestimate duration using data from 1965 through the current year, so the duration measure $\widehat{Dur}_t$ is only computed using information available at year t. For this reason, I start my portfolio construction at year 1975. Table S3 contains the results of the double sort. Since BM and duration are negatively correlated, sorting on BM within each duration group will likely induce a spread in cash flow durations. This is particularly true for stocks in Low Duration groups in which low BM stocks have a cash flow duration measure of 1.28 but high BM stocks have a cash flow duration measure of only -0.05. In contrast, the spread is much smaller for stocks in Medium and High Duration groups. Therefore, if cash flow duration alone explains the value premium, I should expect that further sorting on BM generates no significant spread on returns for these stocks with similar cash flow duration. This is not the case. Value stocks still earn much higher returns than growth stocks in the same cash flow duration group. This finding is not necessarily inconsistent with a duration-based explanation of value premium if we interpret price-based BM as a less noisy measure of cash flow duration. However, under the hypothesis that duration risk alone explains value premium, we wouldn't expect the return spread induced by the second sort on BM to be systematically related to cash flow covariance. This is not what we find in the data. Table S3 shows that return spread can be explained by the cash flow covariance risk – value stocks have indeed higher cash flow covariance risk than growth stocks. This last finding suggests that cash flow covariance rather than duration is more important in explaining value premium.

Place Table S3 about here

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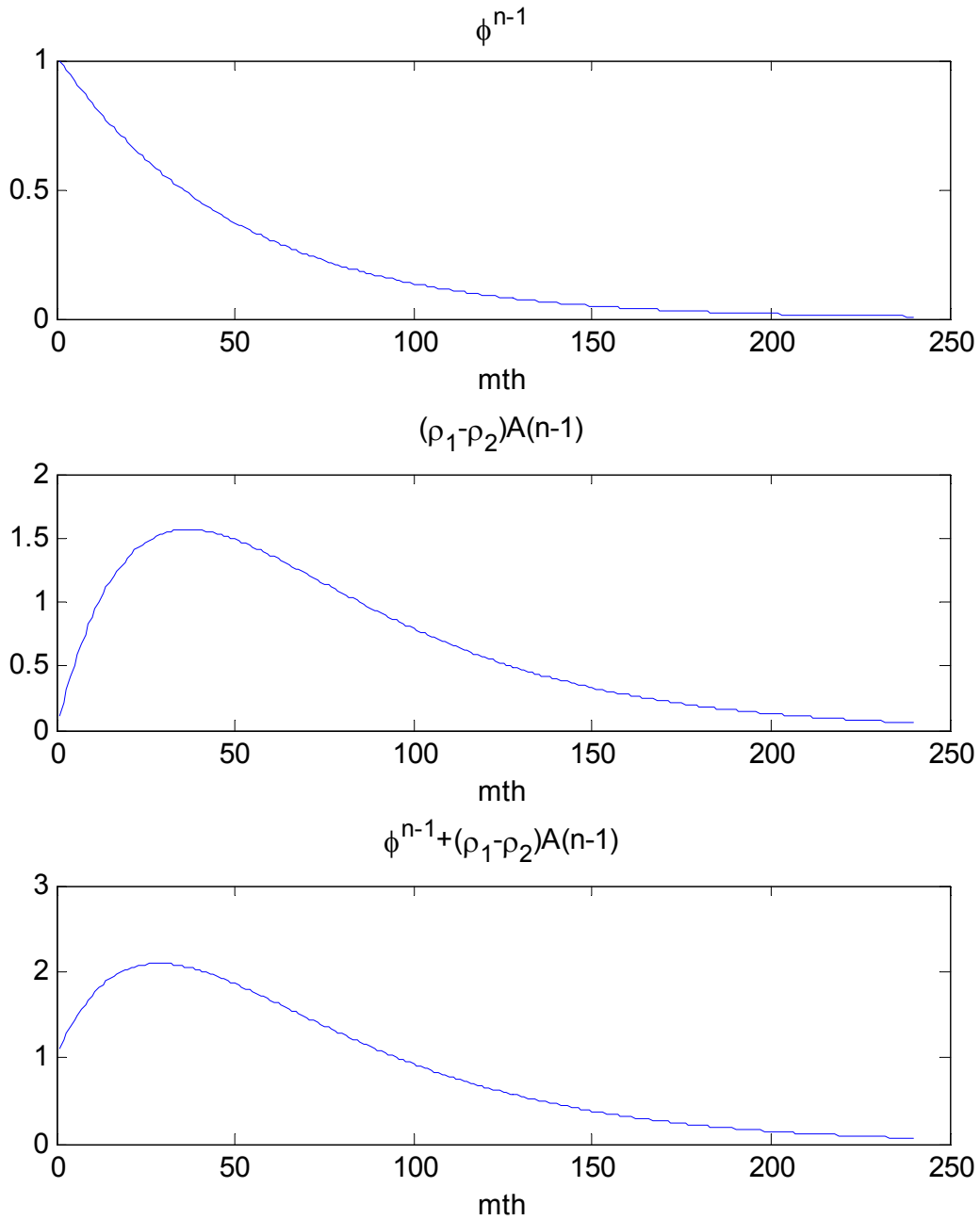Vuolteenaho, Tuomo, 2002, What drives firm-level stock returns, *Journal of Finance* 57, 233-264.

Figure S1. Equity strip risk premium terms as functions of maturities. This figure plots various terms as functions of maturities (n) in the equity strip risk premium expression. The top plot corresponds to ϕ^{n-1}. The middle plot corresponds to $(\rho_1 - \rho_2)A(n-1)$. The bottom plot corresponds to their sum. The parameters (at the monthly frequency) used in this figure are: $\rho_1 = 0.965$, $\rho_2 = 0.851$, and $\phi = 0.98$.

Table S1

Characteristics of Industry Portfolios

Every June, I sort all stocks of industrial firms (excluding financials and utilities) traded on NYSE, AMEX, and NASDAQ into industry portfolios according to the 17 Fama-French industry classification. The resulting 15 portfolios are: Food, Mines(mining and minerals), Oil(oil and petroleum products), Clths(textiles, apparel, and footware), Durbl(consumer durables), Chems(chemicals), Cnsum(drugs, soap, perfumes, and tobacco), Cnstr(construction), Steel, FabPr(fabricated products), Machn(machinery and business equipment), Cars, Trans(transportation), Rtail(retail stores), and Other.

This table presents various portfolio characteristics. The portfolio book-to-market ratio (BM), market equity (ME, measured in millions) at formation, annual return during the first year after portfolio formation are reported. All values are time-series averages across a sampling period from 1964 to 2002. I also directly test the validity of the AR(1) assumption on the cash flow share for the 15 portfolios. I first fit an AR(1) process for the cash flow share and compute the residuals. I then test whether these residuals violate the white noise condition using the Ljung-Box (LB) Q test. Both the Ljung-Box (LB) Q test statistics and the associated p-values are reported. In addition, I also test the stationarity of the cash flow share using the Augmented Dickey-Fuller test with a constant and a lag of one. The t-values are reported (** means the hypothesis of a unit root can be rejected at the 99% confidence level and * means the hypothesis can be rejected at the 95% confidence level). The cash flow share in year t is computed as the log of the ratio between the portfolio cash flow (sum of common dividend and common share repurchase) and aggregate consumption during year t. Finally, the cash flow covariance (Cov) and average duration (Dur) for each portfolio are also reported.

	Food	Mines	Oil	Clths	Durbl	Chems	Cnsum	Cnstr	Steel	FabPr	Machn	Cars	Trans	Rtail	Other
nobs	131	54	165	108	132	71	147	166	62	49	524	64	120	228	915
BM	0.84	0.71	0.71	1.11	0.89	0.77	0.56	0.97	1.05	0.91	0.77	0.93	0.95	0.86	0.74
ME	1210.26	316.78	1344.75	213.10	752.07	1135.84	1734.29	529.32	493.69	276.10	651.29	1140.77	672.75	732.14	641.65
Return	0.141	0.115	0.133	0.135	0.135	0.109	0.148	0.128	0.081	0.113	0.130	0.111	0.130	0.134	0.111
LB Q test stat	12.08	3.35	2.55	4.33	8.00	18.15	20.78	11.28	16.73	16.35	9.20	8.05	5.23	9.89	7.09
p-value	0.280	0.972	0.990	0.931	0.629	0.053	0.023	0.336	0.080	0.090	0.513	0.624	0.876	0.450	0.717
ADF t-value	-6.12**	-5.71**	-2.15	-1.64	-5.68**	-2.79	-4.40**	-6.00**	-0.04	-3.86**	-1.15	-5.45**	0.50	-2.45	-4.06**
Dur	0.90	0.27	0.41	0.49	0.75	0.47	1.45	0.56	-0.25	0.32	0.58	0.35	0.42	0.85	0.34
Cov	0.40	-0.83	-0.40	0.13	-0.36	-0.62	0.11	0.03	-1.27	0.12	-0.79	-2.09	-0.07	-0.06	-0.68

Table S2
Performance of Cash Flow Models on Industry Portfolios

This table reports the results of cross-sectional regressions of average excess returns on the 15 portfolios on cash flow duration and covariance measures. The coefficient estimates are obtained from OLS regressions. However, the robust t-values are computed using GMM standard errors which account for both cross-sectional and time-series error correlations, with Newey-West formula of seven lags. The one-stage GMM estimation is carried out by stacking moment conditions of both time-series regressions and cross-sectional regressions. Finally, both R^2s adjusted-R^2s of the regressions are reported. The sampling period is from 1964 to 1995.

	intercept	Cov	$Dur \times Cov$	R^2 / adj R^2
One factor:				
Coefficient	0.066	0.020		0.425
Robust t-value	*2.08*	*1.76*		*0.381*
Two Factors:				
Coefficient	0.067	0.031	-0.038	0.598
Robust t-value	*2.08*	*1.90*	*-1.26*	*0.531*

Table S3
Duration and BM-sorted Portfolios

Each year from 1975 to 1996, I sort all stocks first into three groups according to a rolling-window "ex-ante" cash flow duration measure, and within each group, I further sort stocks into three subgroups according to their book-to-market ratio. The book-to-market-ratio, annual excess returns, point estimates of cash flow duration, and covariance are reported in the table.

	BM			Excess Return		
	growth		value	growth		value
Low Dur	0.502	1.063	2.269	0.063	0.077	0.123
Med Dur	0.365	0.794	1.716	0.05	0.089	0.127
High Dur	0.24	0.539	1.184	0.058	0.115	0.131

	Dur			*Cov*		
	growth		value	growth		value
Low Dur	1.28	0.22	-0.05	-3.83	-0.43	0.65
Med Dur	0.67	0.92	0.71	-0.05	-0.22	4.14
High Dur	2.41	2.77	2.76	-3.52	-1.3	0.18