

---



---

NUCLEI  
Experiment

---



---

## High-Resolution Study of $0^+$ and $2^+$ Excitations in $^{168}\text{Er}$ with the $(p, t)$ Reaction\*

**D. Bucurescu<sup>1)\*\*</sup>, R. F. Casten<sup>2)</sup>, G. Graw<sup>3)</sup>, J. Jolie<sup>4)</sup>, N. Braun<sup>4)</sup>, P. von Brentano<sup>4)</sup>, T. Faestermann<sup>5)</sup>, S. Heinze<sup>4)</sup>, R. Hertzenberger<sup>3)</sup>, N. Lo Iudice<sup>6)</sup>, R. Krücken<sup>5)</sup>, M. Mahgoub<sup>5)</sup>, D. A. Meyer<sup>2)</sup>, O. Möller<sup>4)</sup>, D. Mücher<sup>4)</sup>, C. Scholl<sup>4)</sup>, N. Yu. Shirikova<sup>7)</sup>, Y. Sun<sup>8)</sup>, A. V. Sushkov<sup>9)</sup>, and H.-F. Wirth<sup>5)</sup>**

Received October 31, 2006

**Abstract**—Excited states in the deformed nucleus  $^{168}\text{Er}$  have been studied with high energy resolution in the  $(p, t)$  reaction, with the Munich Q3D spectrograph. A large number of excited  $0^+$  states (25) and  $2^+$  states (64) have been assigned up to 4.0-MeV excitation energy. This allows detailed investigations along two directions of current interest: first, an extension of microscopic model interpretations into the region of medium level density above the pairing gap; second, a first analysis of the statistical fluctuation (order/chaos) properties of pure sequences of levels, in one deformed nucleus. Predictions of two models (the quasiparticle–phonon model and the projected shell model) are compared to the data, and it is concluded that, in both cases, mixing of more configurations is required in the wave functions.

PACS numbers: PACS: 21.10.-k, 21.60.-n, 25.40.Hs, 24.60.Lz, 27.70.+q

DOI: 10.1134/S1063778807080030

### 1. INTRODUCTION

The direct transfer reactions with light projectiles have been traditionally one of the basic methods to investigate nuclear structure at low excitation energies. These reactions have specific selection rules, which are not, however, strictly obeyed owing to the mixing introduced in the structure of the nuclear states by the residual interactions. Therefore, an experiment with high energy resolution and high sensitivity, which

allows the observation and characterization of very weakly excited levels, may still provide information close to completeness. This study reports such a quasi-complete observation, with the  $(p, t)$  reaction, of the  $0^+$  and  $2^+$  states up to a relatively high excitation energy in a deformed nucleus. These states may result from mixing of many excitation modes, and for their understanding, one must use microscopic models.

A previous experiment using the  $(p, t)$  reaction disclosed 13 excited  $0^+$  states in the deformed nucleus  $^{158}\text{Gd}$  [1] up to 3.1-MeV excitation, a number which exceeded the expectations of most current theoretical estimations. This stirred considerable interest and several theoretical approaches attempted to explain these findings. Recent similar experiments were systematically performed on many nuclei between  $^{152}\text{Gd}$  and  $^{192}\text{Hg}$  and revealed a large number of excited  $0^+$  states, whose systematics is not yet understood in detail [2]. The experimental data reported here for  $^{168}\text{Er}$  are part of this  $(p, t)$  reaction campaign. In this nucleus, we observe a large number of  $0^+$  and  $2^+$  states, which, added to the previous rich experimental information [3], makes this nucleus remain one of the best known deformed nuclei.

A comparison of these detailed data is made (see also [4]) with two theoretical approaches: the

---

\*The text was submitted by the authors in English.

<sup>1)</sup>Hulubei National Institute of Physics and Nuclear Engineering, Bucharest, Romania.

<sup>2)</sup>Wright Nuclear Structure Laboratory, Yale University, New Haven, USA.

<sup>3)</sup>Sektion Physik, Ludwig Maximilians Universität München, Garching, Germany.

<sup>4)</sup>Institut für Kernphysik, Universität zu Köln, Köln, Germany.

<sup>5)</sup>Physik Department, Technische Universität München, Garching, Germany.

<sup>6)</sup>Dipartimento di Scienze Fisiche, Università di Napoli Federico II and Istituto Nazionale di Fisica Nucleare, Napoli, Italy.

<sup>7)</sup>Laboratory of Information Technologies, Joint Institute for Nuclear Research, Dubna, Russia.

<sup>8)</sup>Department of Physics and Joint Institute for Nuclear Astrophysics, University of Notre Dame, Notre Dame, USA.

<sup>9)</sup>Bogolyubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Russia.

\*\*E-mail: [bucurescu@tandem.nipne.ro](mailto:bucurescu@tandem.nipne.ro)

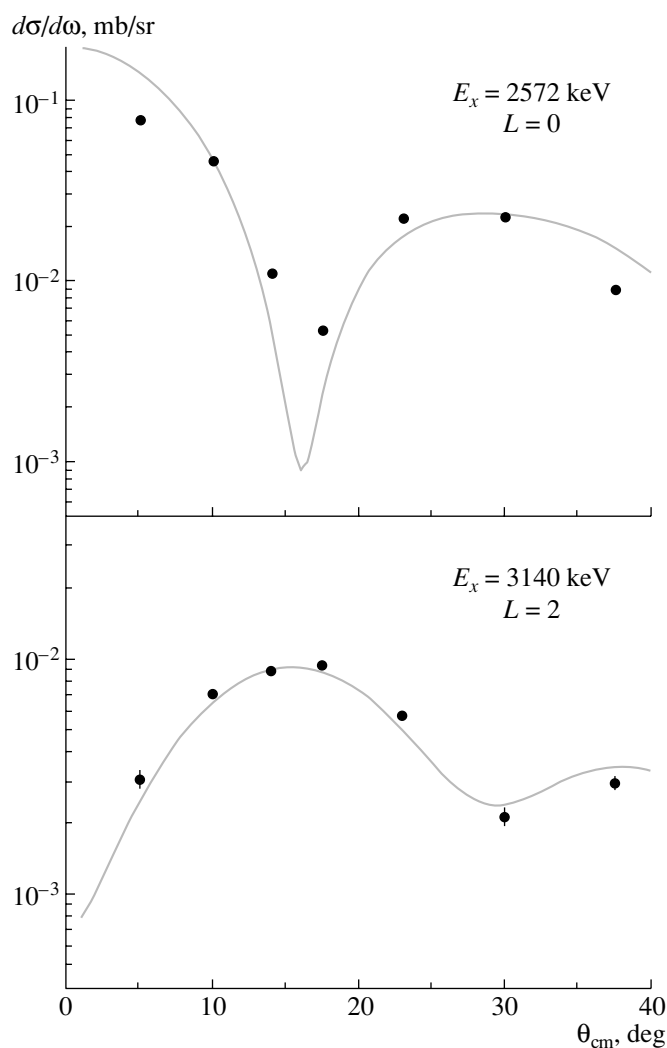


Fig. 1. Angular distributions representative for the  $L = 0$  and  $L = 2$  transfers, compared to DWBA calculations.

quasiparticle-phonon model (QPM) [5] and the projected shell model (PSM) [6]. Then, an investigation of the statistical fluctuation properties (chaoticity) of the two pure (unique  $J^\pi$ ) experimental ensembles of nuclear states ( $0^+$  and  $2^+$ , both experimental and theoretical) in this deformed nucleus is presented [7].

## 2. EXPERIMENT AND DATA ANALYSIS

The experiment used a proton beam of 25.0 MeV delivered by the Munich tandem accelerator. The reaction products were analyzed with the Munich Q3D spectrograph, and detected in a 1-m-long cathode strip focal plane detector which made  $\Delta E-E$  particle identification and position determination. Angular distributions were measured for levels up to  $\sim 4.1$ -MeV excitation. More than 200 excited states, some with intensity as low as 0.01% of that of the ground state, were determined. Experimental details are given in [4].

The value of the transferred angular momentum ( $L$ ) and spin ( $J = L$ ) for each final level is based on the comparison of the angular distributions with empirical shapes and with predictions of DWBA calculations. Unambiguous assignments could be made for  $0^+$  and  $2^+$  states, as illustrated in Fig. 1.

The DWBA calculations were performed with the code CHUCK3 [8], using a form factor which assumes one  $j^2$  neutron configuration. In principle, for each state, one should account for all neutron orbitals expected to contribute ( $1h_{9/2}$ ,  $1i_{13/2}$ ,  $2f_{7/2}$ ,  $2f_{5/2}$ ,  $3p_{3/2}$ ). The DWBA curves calculated with a one-orbit form factor do not show significant variation in shape, but the absolute cross sections and the kinematic ( $Q$  value) correction depend on the orbital. Since we did not know the contribution of each orbital, in order to minimize these effects and still get some spectroscopic information, we used a  $f_{7/2}^2$  form factor (which

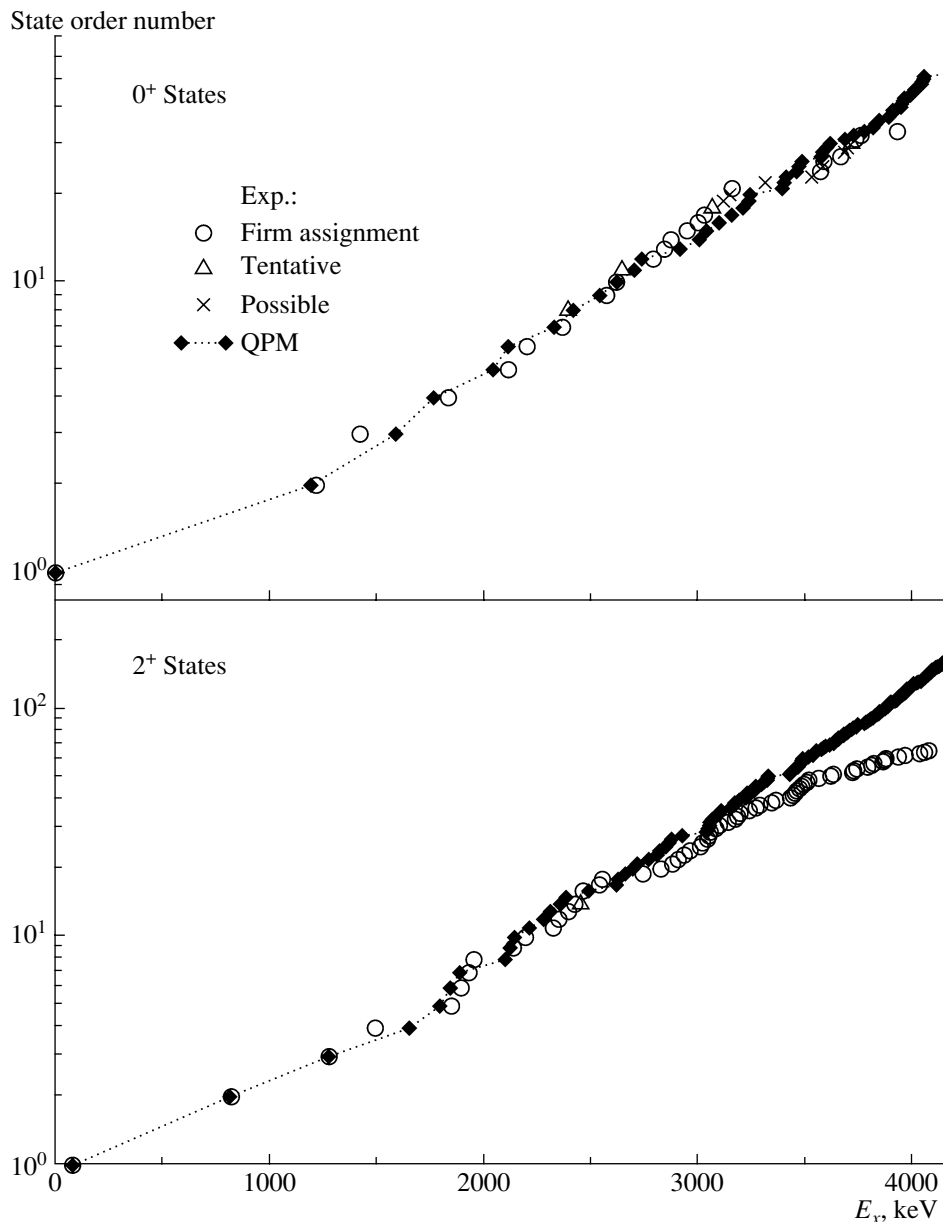


Fig. 2. Comparison of experimental and calculated (QPM)  $0^+$  and  $2^+$  levels in  $^{168}\text{Er}$  (note the logarithmic  $y$ -axis scale).

provides an average behavior) and extracted “spectroscopic strengths,” which are a rough indication of the structure of the states [4].

Up to 4 MeV, we have assigned the number of 25 excited  $0^+$  states and 64  $2^+$  states. A detailed discussion is given in [4]. We appreciate that the determination of the  $0^+$  and  $2^+$  levels up to 3.1 and 3.6 MeV, respectively, is essentially complete.

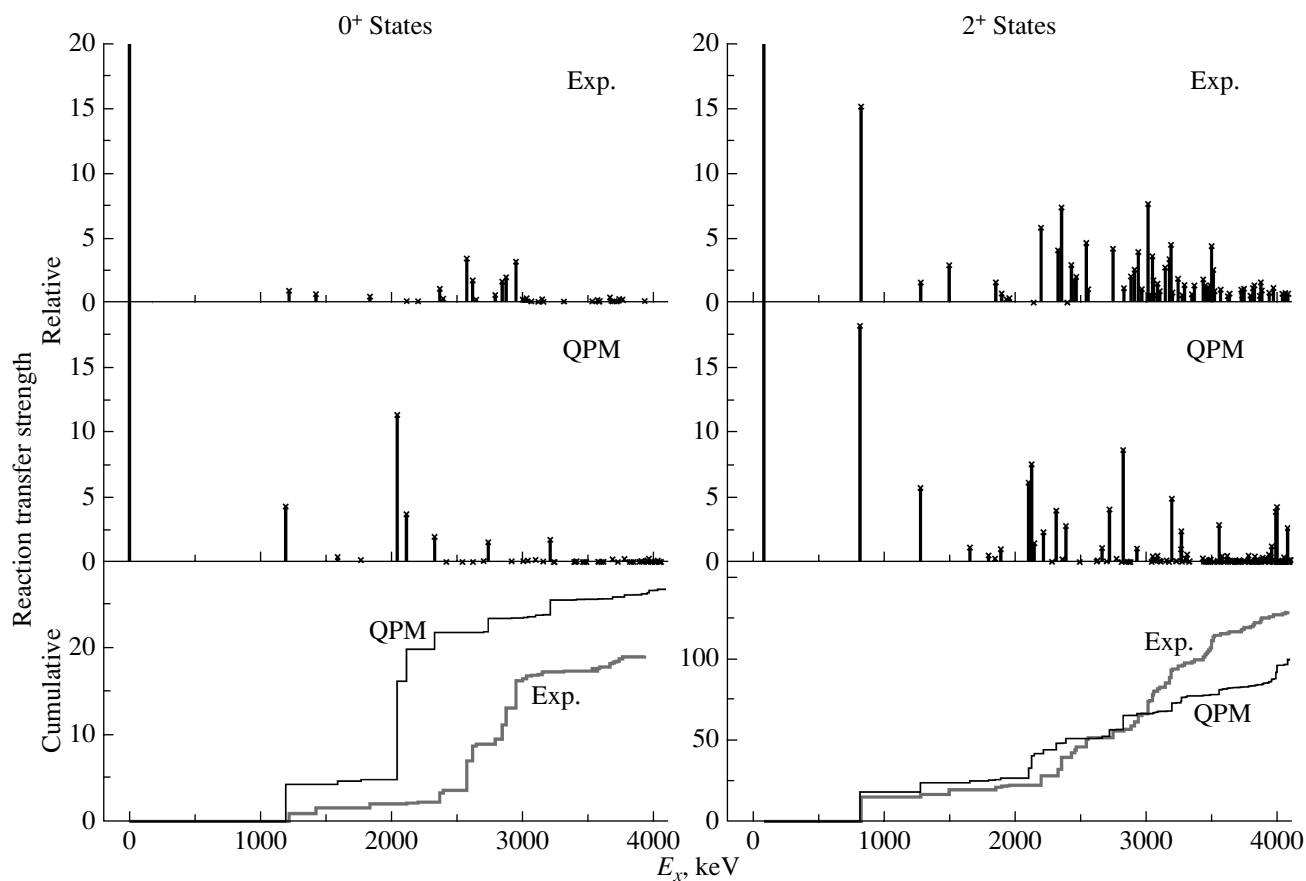
### 3. LEVEL-BY-LEVEL COMPARISON WITH MODEL PREDICTIONS

#### 3.1. The Quasiparticle-Phonon Model (QPM)

In the QPM [5], microscopic phonons are generated in the random phase approximation (RPA).

Then, the Hamiltonian composed of a sum of separable two-body potentials with different multiplicities is diagonalized in a basis of multiphonon states. Both collective and noncollective RPA phonons are included. Besides energy levels,  $(p, t)$  transfer spectroscopic factors normalized to the g.s. transition were calculated [4]. Figure 2 shows a comparison between the experimental and calculated levels.

The QPM yields more states than observed experimentally: up to 4 MeV, there are 44  $0^+$  and 125  $2^+$  states, respectively, compared to the observed 25, and 66 corresponding levels. The excess states, however,



**Fig. 3.** Comparison of experimental and calculated (QPM)  $0^+$  and  $2^+$  relative level reaction strengths for the  $(p, t)$  transfer. The values for the  $0_{g.s.}^+$  and  $2_1^+$  states are normalized to 100.

lie above 3 MeV and carry too little strength to be detected experimentally.

Figure 3 presents a similar comparison for the spectroscopic factors (with the caveat, as discussed above, that the experimental quantities are only crude approximations). The calculation reproduces fairly well the magnitude and distribution of the two-nucleon transfer strength for both  $0^+$  and  $2^+$  states [4, 9]. For the  $0^+$  states, there are discrepancies concerning the strength of the strongest excited state and the centroid of the strength distribution (which is predicted to be  $\sim 0.8$  MeV lower in energy than observed). For the  $2^+$  states, the distribution of the strength is rather well predicted up to about 3.0 MeV excitation, while above 3-MeV the calculation appears to underestimate the collected strength. The phonon structure of the  $0^+$  and  $2^+$  states is discussed in [4, 9]. We note that the structure of the predicted  $(p, t)$  spectra is the result of the phonon fragmentation and of the level of coherence of the two-quasiparticle amplitudes in each dominant RPA phonon. The comparison with the data shows that, at

least at higher excitation energies, the wave functions lack some of the required coherence.

### 3.2. The Projected Shell Model

The PSM [6] is a truncated shell model based on deformed bases. The PSM calculation uses a set of deformed Nilsson single-particle states with a given quadrupole deformation  $\varepsilon_2$  and pairing calculated with BCS. The Nilsson–BCS calculations define a set of quasiparticle (qp) states corresponding to the qp vacuum. Shell model bases are constructed by building multi-qp states from the Nilsson orbitals lying close to the Fermi levels. The broken rotational symmetry is recovered by exact angular momentum projection [6] to form a shell model basis in the laboratory frame. Finally a two-body shell model Hamiltonian (quadrupole plus pairing Hamiltonian with inclusion of quadrupole-pairing term) is diagonalized in the projected space. Particles in three major shells ( $N = 4, 5, 6$  for neutrons and  $N = 3, 4, 5$  for protons) were included in the present calculation, and the model space was truncated by excluding multi-qp states having energies higher than 4.4 MeV [4]. These

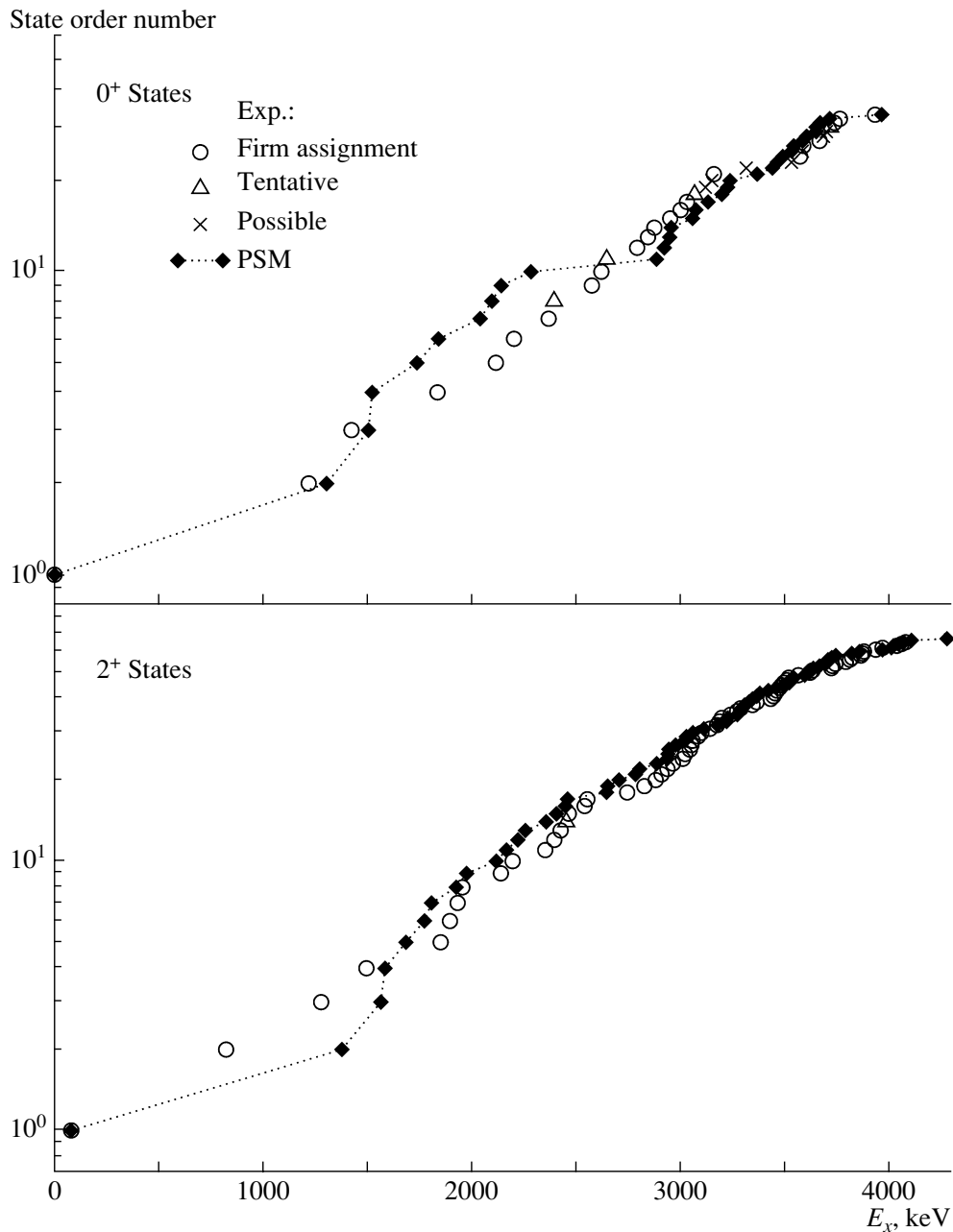


Fig. 4. Comparison of experimental and calculated (PSM)  $0^+$  and  $2^+$  levels in  $^{168}\text{Er}$ .

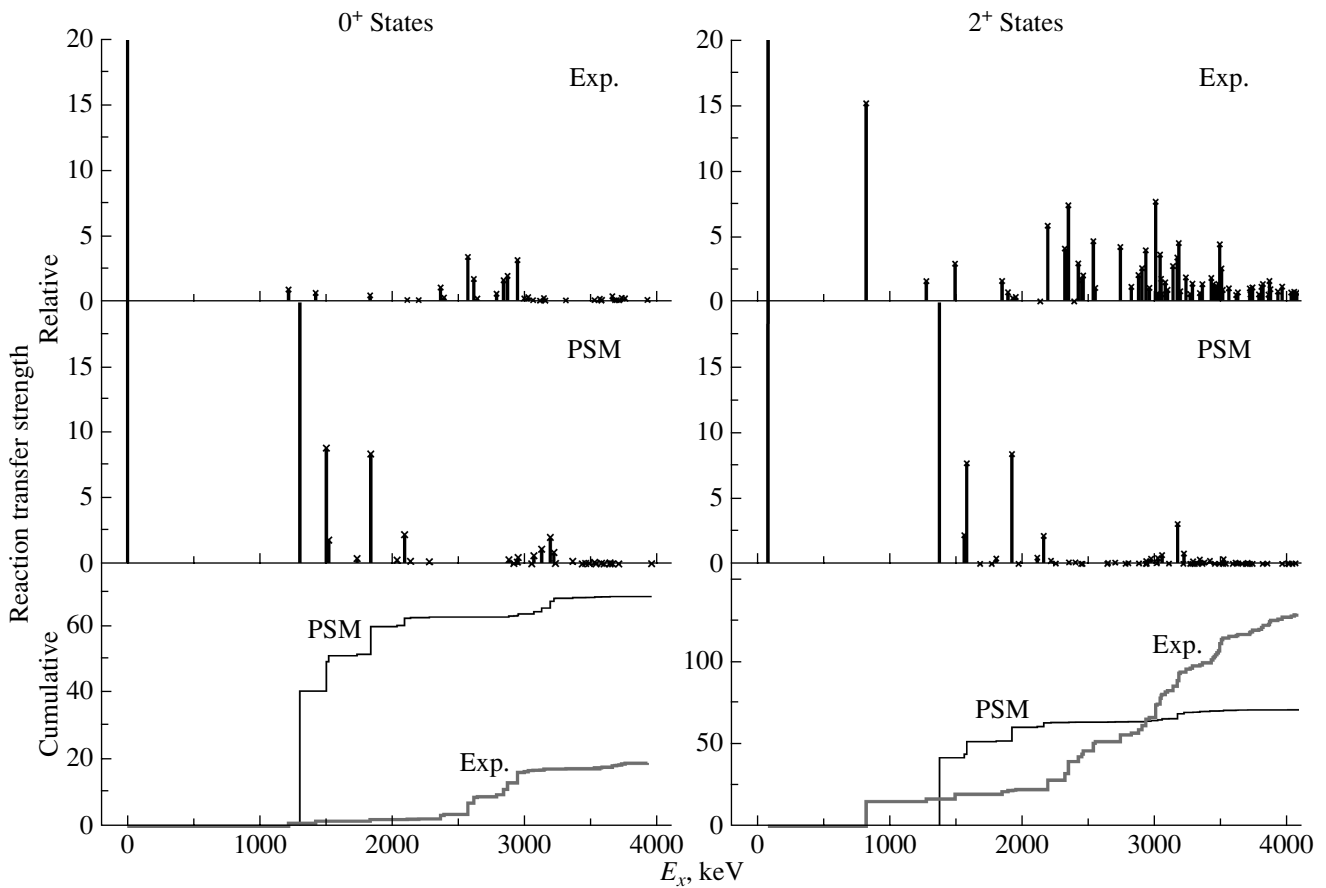
calculations did not include the vibrational degree of freedom (so that there is no  $\gamma$ -vibrational bandhead).

A comparison of the PSM with the experimental data is given in Figs. 4 and 5. The number of states predicted (the possible multi-qp states that can be constructed in the PSM model space) is rather close to the observed one, for both  $0^+$  and  $2^+$  states (Fig. 4). On the other hand, the spectroscopic strengths are not well described: the theoretical strengths concentrate incorrectly in a few low-lying states. This suggests that, although the PSM energy levels match the

data reasonably well, the content of the wave functions is incorrect. This may be due to the truncation of the model space which excludes configuration mixing with higher lying states.

#### 4. ORDER AND CHAOS PROPERTIES OF THE $0^+$ AND $2^+$ LEVELS

The complete determination of the  $0^+$  and  $2^+$  states up to excitation energies well above the pairing gap (1.75 MeV) gives the possibility to investigate, for the first time, the statistical fluctuation



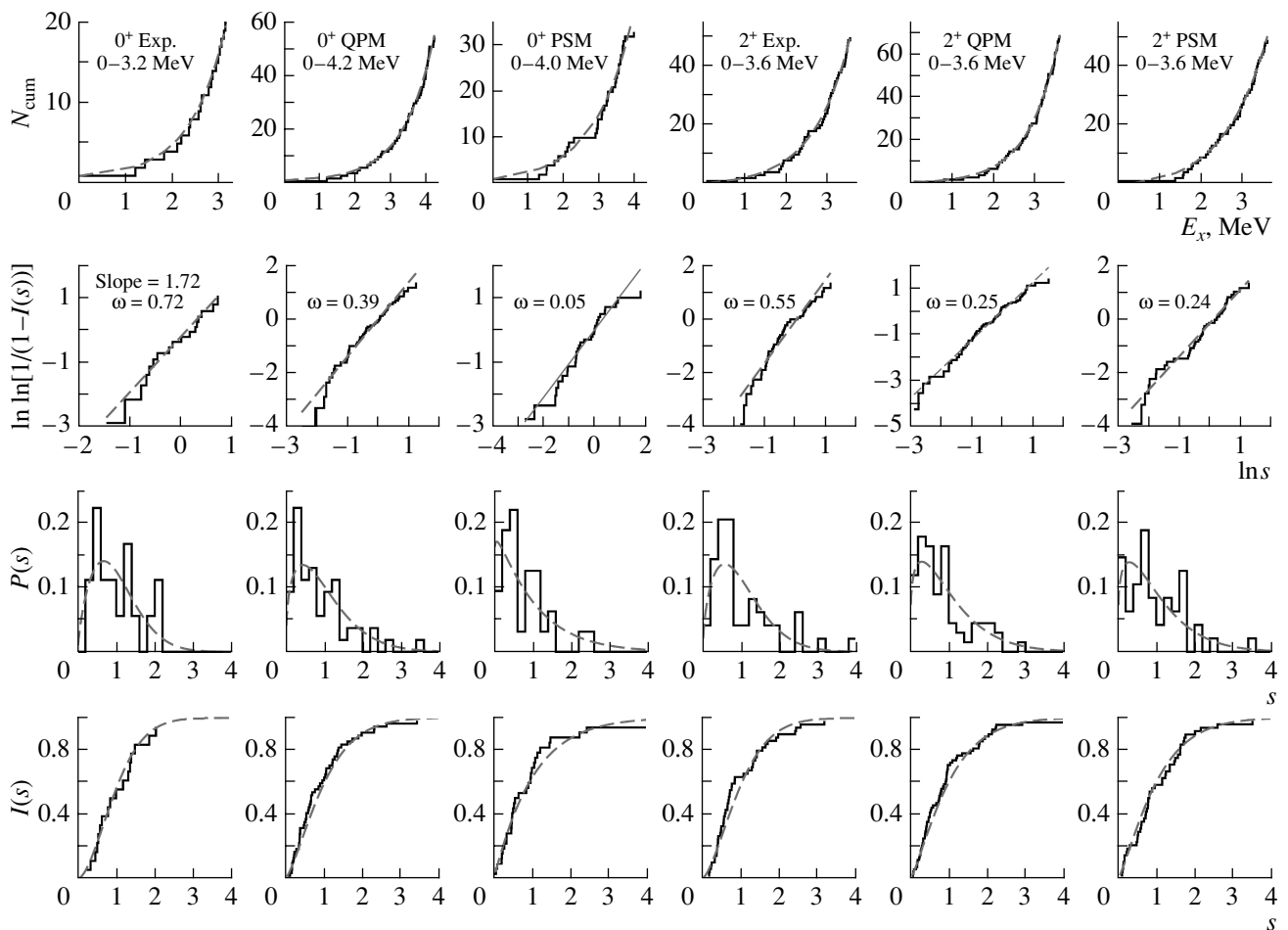
**Fig. 5.** Comparison of experimental and calculated (PSM)  $0^+$  and  $2^+$  relative level reaction strengths for the  $(p, t)$  transfer. The values for the  $0_{g.s.}^+$  and  $2_1^+$  states are normalized to 100.

(chaos/order) properties of ensembles of nuclear levels of unique quantum numbers in a single deformed nucleus. For each set of levels (experimental or calculated), we derived the nearest-neighbor spacing (NNS) distribution  $P(s)$  and compared it with the theoretical Brody distribution (see Fig. 6). The latter depends on one parameter,  $0 \leq \omega \leq 1$ , and interpolates between the Poisson distribution (for  $\omega = 0$ ), which, according to the conjecture of [10], describes

a regular system, and the Wigner distribution (for  $\omega = 1$ ), which, according to the same conjecture, describes a chaotic system. For the determination of the  $\omega$  value that best describes the data, we fit a straight line to the graph representing  $\ln \ln[1/(1 - I(s))]$  versus  $\ln s$  ( $I(s)$  being the total number of spacings below the value  $s$ ), whose slope is given by  $(1 - \omega)$  if the data are well represented by a Brody distribution [11].

**Table 1.** Values of  $\omega$  for the Brody distributions which describe best the specified ensemble of levels (Exp. is experimental; QPM, PSM are calculated)

	Exp. (0–3.2 MeV)	QPM (0–4.2 MeV)	PSM (0–4.1 MeV)
No. of $0^+$ states	19	55	33
$\omega$	0.72(22)	0.39(7)	0.05(20)
	Exp. (0–3.6 MeV)	QPM (0–3.6 MeV)	PSM (0–3.6 MeV)
No. of $2^+$ states	50	69	49
$\omega$	0.55(10)	0.25(6)	0.24(6)



**Fig. 6.** Statistical analysis of  $0^+$  and  $2^+$  levels in  $^{168}\text{Er}$ . For each of the three level sequences, experimental (Exp.) and calculated (QPM or PSM), in the energy range indicated, the graphs present the cumulative number of levels  $N_{\text{cum}}$  as a function of the excitation energy (histogram) and its fit with the three-parameter constant temperature formula; the double-log plot for the integrated probability  $I(s)$ , with linear fit and the resulting  $\omega$  value; the NNS  $P(s)$  distribution and its description with the Brody distribution for the  $\omega$  value determined above; and the integrated probability function  $I(s)$  compared to the calculated curve, as above.

The table shows some of the results of this analysis. The  $0^+$  levels are close to chaoticity ( $\omega = 0.72(22)$ ), while the  $2^+$  levels appear to be less chaotic ( $\omega = 0.55(10)$ ). Such a spin dependence was found also in [12] for deformed nuclei with mass  $150 < A \leq 180$  ( $\omega = 0.74(52)$  for the  $0^+$ ,  $3^+$  levels and  $\omega = 0.13(14)$  for  $2^+$ ,  $4^+$  levels). For both theoretical calculations, the  $\omega$  values are smaller; therefore, they predict less mixing (repulsion) than found for the experimental levels. This effect is especially strong for the  $0^+$  states in the PSM, although the low size of the ensemble of levels gives a large uncertainty. Since  $^{168}\text{Er}$  is a deformed nucleus, the  $K$  quantum number may also play a role. Only the  $0^+$  levels form a pure sequence since they can have only  $K = 0$ . The  $2^+$  levels have three possible values,  $K = 0, 1$ , and  $2$ , which can-

not be distinguished experimentally. If  $K$  is a good quantum number that was not considered when the sequences were analyzed, one should get a more Poisson-like statistic (regular behavior) owing to the superposition of uncorrelated sequences of levels. A breaking of the  $K$  quantum number can push the system towards chaoticity as discussed, e.g., in [13]. The value  $\omega = 0.55$  observed for the experimental  $2^+$  states, intermediate between the regular and chaotic regimes, may be due to some breaking of the  $K$  quantum number. Both theoretical results, closer to regularity (table), indicate insufficient mixing in the calculations.

## 5. CONCLUSIONS

The present study showed that a high-resolution  $(p, t)$  direct interaction experiment can provide a

quasi-complete determination of low-spin levels up to a high excitation energy in  $^{168}\text{Er}$ . This very rich knowledge of the  $0^+$  and  $2^+$  excitations allowed an extension of the data analysis in two directions: a detailed comparison with predictions of two microscopic structure models (the QPM and the PSM), up to 4-MeV excitation, well above the pairing gap, and a first analysis of the statistical fluctuation properties of two pure sequences of levels, with  $J^\pi$  of  $0^+$  and  $2^+$ , respectively, in a single nucleus.

The examination of both level distribution in energy and the fragmentation of the reaction strengths shows that, at higher excitation energies, an extension of the included configurations is needed. The statistical analysis showed that the  $0^+$  sequence is almost completely chaotic, while the  $2^+$  sequence represents an intermediate situation, between regular and chaotic, which confirms earlier findings of possible spin dependence of the fluctuation properties in deformed nuclei [12]. The statistical analysis is an alternate tool to the usual level-by-level analysis. It is found that both models, the QPM and PSM, predict less “chaoticity” and, therefore, less mixing of the levels than shown by the data, which strengthens the conclusions of the former type of analysis.

#### ACKNOWLEDGMENTS

D.B. acknowledges support within a DFG grant (II C4-GR894/1-3). This work was partly supported by the Romanian Ministry for Education and Research under contract no. CEX-05-D11-30, by NSF

under contract PHY-0140324, by the US DOE under grant no. DE-FG02-91ER-40609, and by the DFG grant 391/JO/2-3. We thank T. von Egidy for useful discussions and Y. Alhassid for extensive help on the statistical analysis.

#### REFERENCES

1. S. R. Leshner, A. Aprahamian, L. Trache, et al., *Phys. Rev. C* **66**, 051305(R) (2002).
2. D. A. Meyer et al., *J. Phys. G* **31**, S1399 (2005); *Phys. Lett. B* **638**, 44 (2006); and (in press).
3. V. S. Shirley, *Nucl. Data Sheets* **71**, 261 (1994).
4. D. Bucurescu et al., *Phys. Rev. C* **73**, 064309 (2006).
5. V. G. Soloviev, *Theory of Atomic Nuclei: Quasiparticles and Phonons* (Institute of Physics, Bristol, 1992).
6. K. Hara and Y. Sun, *Int. J. Mod. Phys. E* **4**, 637 (1997).
7. D. Bucurescu et al., (in press).
8. P. D. Kunz, Computer code CHUCK3, University of Colorado (unpublished).
9. N. Lo Iudice, A. V. Sushkov, and N. Yu. Shirikova, *Phys. Rev. C* **72**, 034303 (2005).
10. O. Bohigas, M. J. Giannoni, and C. Schmit, *Phys. Rev. Lett.* **52**, 1 (1984).
11. Y. Alhassid, private communication.
12. J. F. Shrinier, G. E. Mitchell, and T. von Egidy, *Z. Phys. A* **338**, 309 (1991).
13. V. Paar and D. Vorkapić, *Phys. Rev. C* **41**, 2397 (1990).