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No-relationship Between Impossibility of Faster-Than-Light Quantum Communication and Distinction of Ensembles with the Same Density Matrix*

WANG Chuan,[1] LONG Gui-Lu,[1,2] and SUN Yang[1,3]

[1]Department of Physics and Key Laboratory for Quantum Information and Measurements, Tsinghua University, Beijing 100084, China

[2]Key Laboratory for Atomic and Molecular NanoSciences, Tsinghua University, Beijing 100084, China

[3]Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556, USA

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Abstract *It has been claimed in the literature that impossibility of faster-than-light quantum communication has an origin of indistinguishability of ensembles with the same density matrix. We show that the two concepts are not related. We argue that even with an ideal single-atom-precision measurement, it is generally impossible to produce two ensembles with exactly the same density matrix; or to produce ensembles with the same density matrix, classical communication is necessary. Hence the impossibility of faster-than-light communication does not imply the indistinguishability of ensembles with the same density matrix.*

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The study on quantum information and quantum computation has become one of the research focuses worldwide. The combination of quantum mechanics with information and computer sciences has produced many fruitful results, which may become an advanced technology in the future. A quantum computer can offer additional computing power, which can greatly speed up the solution for a prime factorization of a large number,[1] and for an unsorted database search.[2] The quantum key distribution offers unconditional security in secret communication.[3] With quantum entanglement, users at distant sites may share particles that are part of an entangled system to fulfill certain communication task, as for instance in sharing a secret.[4] Thus it is tempting to look for more applications of quantum mechanics.

One such search is to look for the faster-than-light communication using shared entangled particles. However, any faster-than-light motion is in an obvious violation of special relativity. By using ensembles of qubits that are parts of entangled pairs, a scheme was proposed in Ref. [5] to show that a faster-than-light communication is impossible. The impossibility is claimed to relate to the indistinguishability of ensembles with the same density matrix.[5,6] On the other hand, however, it has recently been pointed out that ensembles with the same density matrix can be distinguished physically,[7,8] and this conclusion is closely linked to the quantum nature of NMR quantum computing.[9] There is apparently an contradiction between these two results.

One uses Einstein–Podolsky–Rosen (EPR) pairs separated in space as means of communication. Then a measurement of two different observables collapses the EPR pairs and produces two ensembles having the same density matrix. The two different measurements transmit one bit of information if one can distinguish these two ensembles having the same density matrix. In this paper, we will show that the conclusion of distinction of ensembles with the same density matrix bears no relationship with the impossibility of faster-than-light communication. We show that there is a flaw in Preskill's argument in Ref. [5]: the two ensembles produced by measuring EPR pairs using either σ_x or σ_z bases will generally NOT produce ensembles with identical density matrix. The conclusion drawn in Ref. [5] should be rephrased into that it is impossible to produce two ensembles with the same density matrix by measuring particles from N EPR pairs without classical communication.

First we briefly review the Preskill Scheme. As shown in Fig. 1, Alice and Bob share N pairs of qubits in state

$$|\Phi_{AB}\rangle = \frac{1}{\sqrt{2}}(|\uparrow_Z\rangle_A|\uparrow_Z\rangle_B + |\downarrow_Z\rangle_A|\downarrow_Z\rangle_B)\,. \qquad (1)$$

Alice and Bob are separated by a large distance. Bob can send Alice a one-bit message by measuring his particles with either σ_x as shown in Fig. 2 or σ_z as shown in Fig. 3, thus preparing Alice's spins in either $(|\uparrow_Z\rangle_A, |\downarrow_Z\rangle_B)$ or $(|\uparrow_X\rangle_A, |\downarrow_X\rangle_B)$. The two ensembles produced by these

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σ_x or σ_z measurements have the same density matrix,

$$\rho_A = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} . \tag{2}$$

If Alice could tell the difference between these two ensemble preparations, then she would be able to read Bob's information immediately, accomplishing a faster-than-light communication.

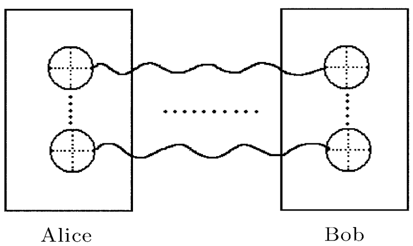

Fig. 1 Each pair of qubits connected by a wave line represents an EPR pair.

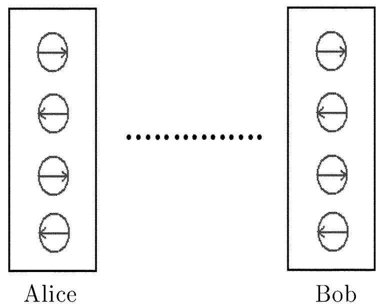

Fig. 2 By measuring σ_x on each qubit, each pair collapses into $|\uparrow_x\rangle$ or $|\downarrow_x\rangle$. This action represents the value 0.

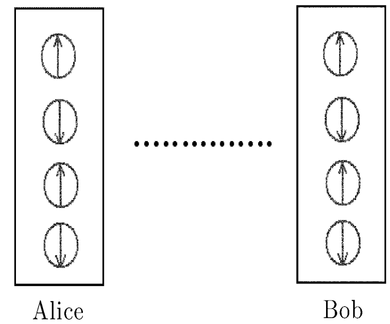

Fig. 3 By measuring σ_z on each qubit, each pair collapses into $|\uparrow_z\rangle$ or $|\downarrow_z\rangle$. This action, the σ_z measurement, represents the value 1.

However, this is impossible. Preskill stressed in Ref. [5] that "though the two preparation methods are surely different, both ensembles are described by precisely the same density matrix ρ_A. Thus there is no conceivable measurement Alice can make that will distinguish the two ensembles, and no way for Alice to tell what action Bob performed. The 'message' is unreadable".

Preskill then provided a method for Alice and Bob to distinguish the two ensemble preparations with some additional help. Alice can choose a measuring device, say σ_x, to measure each of her particles and compare with the result of Bob, which was transmitted to her through a telephone line. If her results have perfect agreement with Bob's, then she knows that Bob's action is σ_x, otherwise Bob's action is σ_z. Here ideal conditions are assumed for simplicity. Apparently, the two ensemble preparations can be distinguished. However, Preskill stressed that faster-than-light communication is not possible because telephone calls are needed for the distinction, and signal in a telephone line travels at the speed of light.

Preskill has related the impossibility of faster-than-light communication to the indistinguishability of ensembles having the same density matrix.

There is a Flaw in the Preskill analysis. Apparently Preskill's analysis ignored the fluctuation in the measured result. It is true that in both measurements, each particle has 1/2 probability to collapse into $|\uparrow_Z\rangle$ $(|\uparrow_X\rangle)$ or $|\downarrow_Z\rangle(|\downarrow_X\rangle)$, but the number of particles in the $|\uparrow_Z\rangle$ $(|\uparrow_X\rangle)$ direction is not exactly the same as that in the $|\downarrow_Z\rangle(|\downarrow_X\rangle)$ direction. Hence the density matrix of one ensemble in the σ_z-basis or σ_x-basis is

$$\rho_A = \begin{pmatrix} \dfrac{1}{2} - \dfrac{N_\delta}{N} & 0 \\ 0 & \dfrac{1}{2} + \dfrac{N_\delta}{N} \end{pmatrix} , \tag{3}$$

where N_δ is a random number that can be positive or negative and is proportional to $\sqrt{N}$. N_δ indicates the difference between the number of particles in the $|\uparrow_Z\rangle$ $(|\uparrow_X\rangle)$ state and the $N/2$. We stress that the two different ensemble preparations in general leads to different density matrices if the number of particles is finite, which is usually true in real physical circumstances. Because one can measure the particles in the ensembles one by one, the absolute value of N_δ increases with N. Thus as N goes large, the fluctuation becomes large too. This makes the distinction of the ensembles more easily. Even if Bob repeats the same kind of measurement, say σ_x, he would not be able to produce exactly two identical ensembles if the particle number is finite. Thus strictly speaking, the density matrices of the two ensembles produced by σ_x and σ_z measurement are not identical. Though the two ensembles have different density matrices, one cannot use them for communication as the fluctuation is uncontrollable. Hence there is no question of distinction of ensembles having the same density matrix at all in this problem.

Now we show that even if one can distinguish ensembles with the same density matrix, it is still impossible to perform faster-than-light communication. We make the following modifications to Preskill's scheme. While preparing the ensemble by measuring each qubit with $\sigma_z(\sigma_x)$, Bob can make the number of particles in $|\uparrow_z\rangle$ $(|\uparrow_x\rangle)$ and $|\downarrow_z\rangle(|\downarrow_x\rangle)$ exactly the same by dropping some qubits. He then tells Alice which qubits should be excluded from her ensemble, so that Alice's ensemble is prepared with exactly equal numbers of qubits in opposite polarization. Of course, Bob and Alice can produce several such copies with equal or near equal total number of particles. But they all have equal number of particles in opposite directions. Of course, the communication in the preparation is classical, hence it is not faster-than-light communication. Now Alice's ensemble has exactly the density matrix $\rho_A = I_2/2$ and with two possible form of constituents: either polarized or anti-polarized along z-direction, or polarized or anti-polarized along x-direction. The question is whether Alice can distinguish the two cases with whatever methods that are available. Alice can determine which one of the two constructions the ensemble is by making a σ_z measurement on each of the particles in the ensemble. If the ensemble was prepared by the σ_z measurement with equal numbers of particles in opposite directions with the help of classical communication, then the sum of all the measured results is zero, namely

$$\Sigma_z = \sum_{i=1}^{N} \sigma_z(i) = 0\,. \tag{4}$$

However if the ensemble was prepared by the σ_x measurement instead, then the sum of all the measurement will be,

$$\Sigma'_z = \sum_{i=1}^{N} \sigma_z(i) \approx \pm\sqrt{N}\,. \tag{5}$$

In the first ensemble, the state of the individual particle is the eigenstate of σ_z, and it will give a definite result when σ_z is measured, while in the second case, the state of an individual particle is in the eigenstate of σ_x, there is fluctuations in the measured results though the average total sum is zero. Because Alice and Bob have several copies, these fluctuations can be easily found when Alice repeats the measurement on several such copies. By observing the fluctuation, Alice can easily determine what measurement Bob has performed. This has been suggested by d'Espagnat[7] and has been generalized into general ensembles in Ref. [8].

To summarize, it has been shown that the fact that ensembles with the same density matrix can be distinguished physically does not contradict the claim of impossibility of faster-than-light communication for two reasons. First, it is impossible to produce ensembles having the same density by measurements even if a single atom precision is available. This is because the collapse of state under measurement is random and the measured results have fluctuations. This makes the precise density matrix to fluctuate around the average value. Second, even if two ensembles are produced with exactly the same density matrix and they are distinguishable by observing the fluctuations of observables related to the whole ensemble, this cannot be used for information transmission because the classical communication is necessary to prepare the ensembles with the same density matrix.

Note added in proof: In Ref. [6], an ensemble is defined as having infinite molecules. Here in this article, an ensemble contains a fixed number of molecules. The scenario is somehow different between [6] and that in this article.

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