

Review: Expected Utility

ECON 40447
Fall 2009

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Intermediate Micro

- **Workhorse model of intermediate micro**
 - Utility maximization problem
 - Consumers Max $U(x,y)$ subject to the budget constraint, $I = P_x x + P_y y$
- **Problem is made easier by the fact that we assume all parameters are known**
 - Consumers know prices and income, simple mathematical problem for person

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- **Many cases, some of the parameters are unknown**
- **Uncertainty about prices? Income? State of the world? Will you have a job?**
- **This section, will review utility theory under uncertainty**
 - Hopefully, most of this will be a review of intermediate micro

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- **Will emphasize the special role of insurance in a generic sense**
 - Why insurance is 'good'
 - How much insurance should people purchase
- **Insurance markets may generate incentives that reduce the welfare gains of insurance**
 - Moral hazard
 - Adverse selection

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Definitions

- **Probability** - likelihood discrete event will occur
 - n possible events, $i=1,2,\dots,n$
 - P_i be the probability event i happens
 - $0 \leq P_i \leq 1$
 - $P_1 + P_2 + P_3 + \dots + P_n = 1$
- Probabilities can be 'subjective' or 'objective', depending on the model
- In our work, probabilities will be known with certainty

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- **Expected value** –
 - Weighted average of possibilities, weight is probability
 - Sum of the possibilities times probabilities

$$x = \{x_1, x_2, \dots, x_n\}$$

$$P = \{P_1, P_2, \dots, P_n\}$$

$$E(x) = P_1 x_1 + P_2 x_2 + P_3 x_3 + \dots + P_n x_n$$

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- Roll of a die, all sides have $(1/6)$ prob. What is expected roll?
- $E(x) = 1(1/6) + 2(1/6) + \dots + 6(1/6) = 3.5$
- Suppose you have: 25% chance of an A, 50% B, 20% C, 4% D and 1% F
- $E[\text{quality points}] = 4(.25) + 3(.5) + 2(.2) + 1(.04) + 0(.01) = 2.94$

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Expected utility

- Suppose income is random. Two potential values (Y_1 or Y_2)
- Probabilities are either P_1 or $P_2 = 1 - P_1$
- When incomes are realized, consumer will experience a particular level of income and hence utility
- But, looking at the problem beforehand, a person has a particular 'expected utility'

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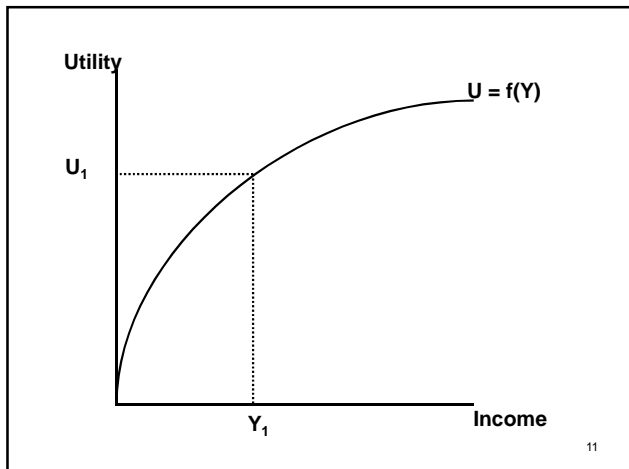
- However, suppose an agent is faced with choice between two different paths
 - Choice a: Y_1 with probability P_1 and Y_2 with P_2
 - Choice b: Y_3 with probability P_3 and Y_4 with P_4
- Example: you can accept a job with steady pay or a job with huge upside income potential (but one with a chance you will be looking for another job)
- How do you choose in the face of uncertainty?

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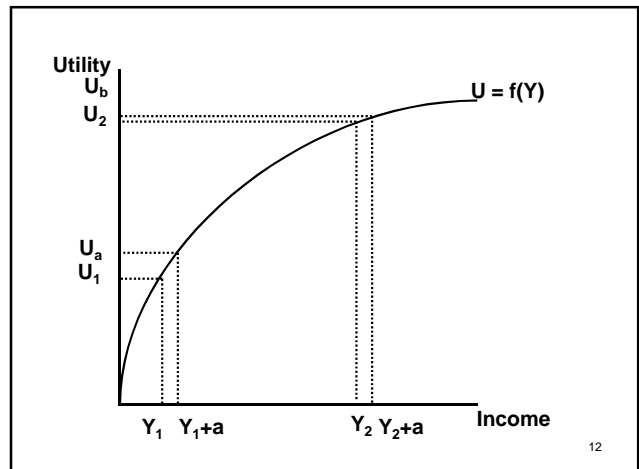
Assumptions about utility with uncertainty

- One element (income or wealth)
- $U = U(Y)$
- Marginal utility is positive
 - $U' = dU/dY > 0$
- Standard assumption, declining marginal utility $U'' < 0$
 - Implies risk averse but we will relax this later

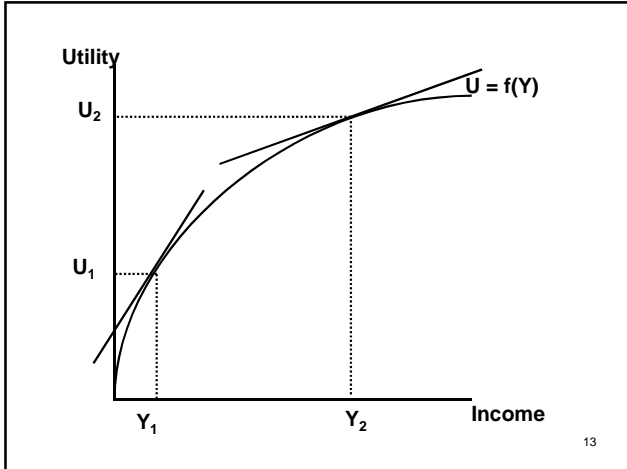
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Von Neumann-Morganstern Utility

- **N states of the world, with incomes defined as Y_1, Y_2, \dots, Y_n**
- **The probabilities for each of these states is P_1, P_2, \dots, P_n**
- **A valid utility function is the expected utility of the gamble**
- **$E(U) = P_1U(Y_1) + P_2U(Y_2) + \dots + P_nU(Y_n)$**

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- **$E(U)$ is the sum of the possibilities times probabilities**

- **Example:**

- 40% chance of earning \$2500/month
- 60% chance of \$1600/month
- $E(U) = P_1U(Y_1) + P_2U(Y_2)$
- $U(Y) = Y^{0.5}$
- $E(U) = 0.4(2500)^{0.5} + 0.6(1600)^{0.5}$
 $= 0.4(50) + 0.6(40) = 44$

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- **Note that expected utility in this case is very different from expected income**
 - $E(Y) = 0.4(2500) + 0.6(1600) = 1960$
- **Expected utility allows people to compare gambles**
- **Given two gambles, we assume people prefer the situation that generates the greatest expected utility**
 - People maximize expected utility

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- Expected income is not what we compare
- Two situations can have the same expected value but very different expected utilities
- Job A: certain income of \$50K
- Job B: 50% chance of \$10K and 50% chance of \$90K
- Expected income is the same (\$50K), one is much more certain?
- Which one is preferred?

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- $U = \ln(y)$
- $EU_a = \ln(50,000) = 10.82$
- $EU_b = 0.5 \ln(10,000) + 0.5 \ln(90,000) = 10.31$
- Job a is preferred

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Example

- Job 1
 - 40% chance of \$2500, 60% of \$1600
 - $E(Y) = 0.4 \cdot 2500 + .6 \cdot 1600 = \1960
 - $E(U_1) = (0.4)(2500)^{0.5} + (0.6)(1600)^{0.5} = 44$
- Job 2
 - 25% chance of \$5000, 75% of \$1000
 - $E(Y) = .25(5000) + .75(1000) = \2000
 - $E(U_2) = 0.25(5000)^{0.5} + 0.75(1000)^{0.5} = 41.4$
- Job 2 is preferred to 1, even though 1 has higher expected income

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The Important of Marginal Utility: The St Petersburg Paradox

- Pot starts at \$2. Flip a coin and if a head appears, the pot doubles. If tails appears, you win the pot and the game ends.
- So, if you get H, H, H T, you win \$16
- What would you be willing to pay to 'play' this game?

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- Probabilities?
- $\Pr(h)=\Pr(t) = 0.5$
- All events are independent
- $\Pr(h \text{ on } 2^{\text{nd}} | h \text{ on } 1^{\text{st}}) = \Pr(h \text{ on } 1^{\text{st}})$
- Recall definition of independence
- If A and B are independent events
 - $\Pr(A \cap B) = \Pr(A)\Pr(B)$

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- Note, $\Pr(\text{first tail on } k^{\text{th}} \text{ toss}) =$
- $\Pr(h \text{ on } 1^{\text{st}})\Pr(h \text{ on } 2^{\text{nd}} \dots)\dots\Pr(t \text{ on } k^{\text{th}})=$
- $(1/2)(1/2)\dots(1/2) = (1/2)^k$
- What is the expected pot on the k^{th} trial?
 - 2 on 1st or 2^1
 - 4 on 2nd or 2^2
 - 8 on 3rd, or 2^3
 - So the payoff on the k^{th} is 2^k

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- What is the expected value of the gamble
- $E = (1/2)\$2^1 + (1/2)^2\$2^2 + (1/2)^3\$2^3 + (1/2)^4\2^4

$$E = \sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k (2^k) = \sum_{k=1}^{\infty} (1) = \infty$$

- The expected payout is infinite

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Round	Winnings	Probability
5 th	\$32	0.03125
10 th	\$1,024	0.000977
15 th	\$32,768	3.05E-5
20 th	\$1,048,576	9.54E-7
25 th	\$33,554,432	2.98E-8

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Suppose Utility is $U=Y^{0.5}$

$$E = \sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k (2^k)^{1/2} = \sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^{k/2} \left(\frac{1}{2}\right)^{k/2} (2^k)^{1/2}$$

$$= \sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^{k/2} \left(\frac{1}{2}\right)^{k/2} (2)^{k/2} = \sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^{k/2} = \sum_{k=1}^{\infty} \left(\frac{1}{\sqrt{2}}\right)^k =$$

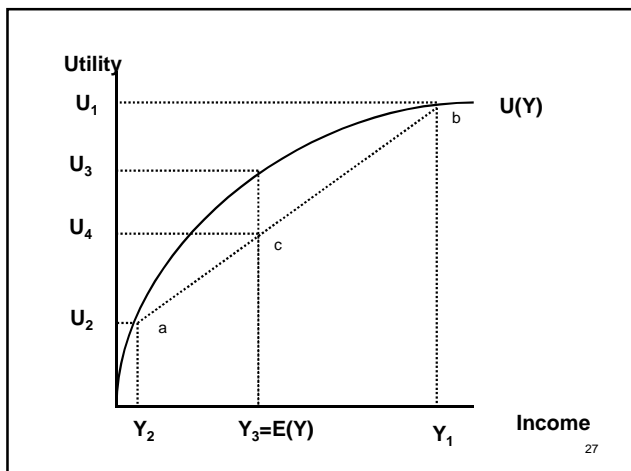
Can show that $= \sum_{k=1}^{\infty} \left(\frac{1}{\sqrt{2}}\right)^k = \left[\frac{1}{1 - \frac{1}{\sqrt{2}}} \right] - 1 = 2.414$

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How to represent graphically

- Probability P_1 of having Y_1
- $(1-P_1)$ of having Y_2
- U_1 and U_2 are utility that one would receive if they received Y_1 and Y_2 respectively
- $E(Y) = P_1 Y_1 + (1-P_1) Y_2 = Y_3$
- U_3 is utility they would receive if they had income Y_3 with certainty

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- Notice that $E(U)$ is a weighted average of utilities in the good and bad states of the world
- $E(U) = P_1 U(Y_1) + (1-P_1) U(Y_2)$
- The weights sum to 1 (the probabilities)
- Draw a line from points (a,b)
- Represent all the possible 'weighted averages' of $U(Y_1)$ and $U(Y_2)$
- What is the one represented by this gamble?

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- Draw vertical line from $E(Y)$ to the line segment. This is $E(U)$
- U_4 is Expected utility
- $U_4 = E(U) = P_1U(Y_1) + (1-P_1)U(Y_2)$

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- Suppose offered two jobs
 - Job A: Has chance of a high (Y_1) and low (Y_2) wages
 - Job B: Has chance of high (Y_3) and low (Y_4) wages
 - Expected income from both jobs is the same
 - P_a and P_b are the probabilities of getting the high wage situation

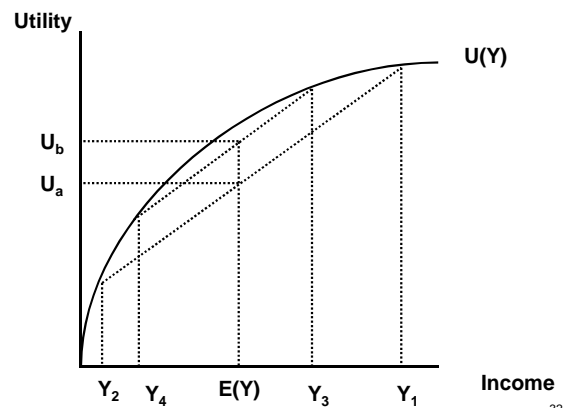
$$P_a Y_1 + (1-P_a)Y_2 = P_b Y_3 + (1-P_b)Y_4 = E(Y)$$

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Numeric Example

- Job A
 - 20% chance of \$150,000
 - 80% chance of \$20,000
 - $E(Y) = 0.2(150K) + 0.8(20K) = \$46K$
- Job B
 - 60% chance of \$50K
 - 40% chance of \$40K
 - $E(Y) = 0.6(50K) + 0.4(40K) = \$46K$

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- Notice that Job A and B have the same expected income
- Job A is riskier – bigger downside for Job A
- Prefer Job B (Why? Will answer in a moment)

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- The prior example about the two jobs is instructive. Two jobs, same expected income, very different expected utility
- People prefer the job with the lower risk, even though they have the same expected income
- People prefer to ‘shed’ risk – to get rid of it.
- How much are they willing to pay to shed risk?

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Example

- Suppose have \$200,000 home (wealth).
- Small chance that a fire will damage you house. If does, will generate \$75,000 in loss (L)
- $U(W) = \ln(W)$
- Prob of a loss is 0.02 or 2%
- Wealth in “good” state = W
- Wealth in bad state = W-L

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- $E(W) = (1-P) W + (1-P)(W-L)$
- $E(W) = 0.98(200,000) + 0.02(125,000) = \$198,500$
- $E(U) = (1-P) \ln(W) + P \ln(W-L)$
- $E(U) = 0.98 \ln(200K) + 0.02 \ln(200K-75K) = 12.197$

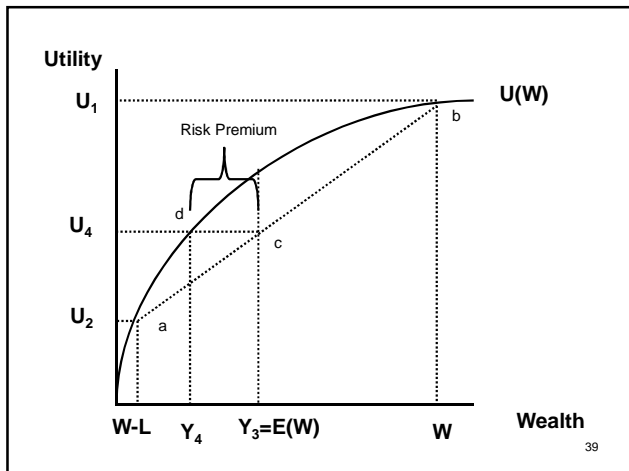
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- Suppose you can add a fire detection/prevention system to your house.
- This would reduce the chance of a bad event to 0 but it would cost you \$C to install
- What is the most you are willing to pay for the security system?
- $E(U)$ in the current situation is 12.197
- Utility with the security system is $U(W-C)$
- Set $U(W-C)$ equal to 12.197 and solve for C

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- $\ln(W-C) = 12.197$
- Recall that $e^{\ln(x)} = x$
- Raise both sides to the e
- $e^{\ln(W-C)} = W-C = e^{12.197} = 198,128$
- $W - 198,128 = 200,000 - 198,128 = C = \1872

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- Will earn Y_1 with probability p_1
– Generates utility U_1
- Will earn Y_2 with probability $p_2=1-p_1$
– Generates utility U_2
- $E(I) = p_1Y_1 + (1-p_1)Y_2 = Y_3$
- Line (ab) is a weighted average of U_1 and U_2
- Note that expected utility is also a weighted average
- A line from $E(Y)$ to the line (ab) give $E(U)$ for given $E(Y)$

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- Take the expected income, $E(Y)$. Draw a line to (ab). The height of this line is $E(U)$.
- $E(U)$ at $E(Y)$ is U_4
- Suppose income is known with certainty at I_3 . Notice that utility would be U_3 , which is greater than U_4
- Look at Y_4 . Note that $Y_4 < Y_3 = E(Y)$ but these two situations generate the same utility

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- The line segment (cd) is the “Risk Premium.” It is the amount a person is willing to pay to avoid the risky situation.
- If you offered a person the gamble of Y_3 or income Y_4 , they would be indifferent.
- Therefore, people are willing to sacrifice cash to ‘shed’ risk.

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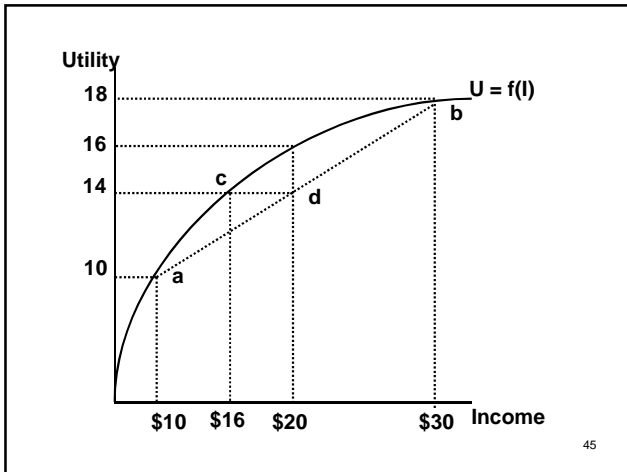
Some numbers

- Person has a job that has uncertain income
 - 50% chance of making \$30K, $U(30K) = 18$
 - 50% chance of making \$10K, $U(10K) = 10$
- $E(I) = (0.5)(\$30K) + (0.5)(\$10K) = \$20K$
- $E(U) = 0.5U(30K) + 0.5U(10K) = 14$

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- Expected utility. Weighted average of $U(30)$ and $U(10)$. $E(U) = 14$
- Notice that a gamble that gives expected income of \$20K is equal in value to a certain income of only \$16K
- This person dislikes risk.
 - Indifferent between certain income of \$16 and uncertain income with expected value of \$20
 - Utility of certain \$20 is a lot higher than utility of uncertain income with expected value of \$20

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- Although both jobs provide the same expected income, the person would prefer the guaranteed \$20K.
- Why? Because of our assumption about diminishing marginal utility
 - In the 'good' state of the world, the gain from \$20K to \$30K is not as valued as the 1st \$10
 - In the 'bad' state, because the first \$10K is valued more than the last \$10K, you lose lots of utils.

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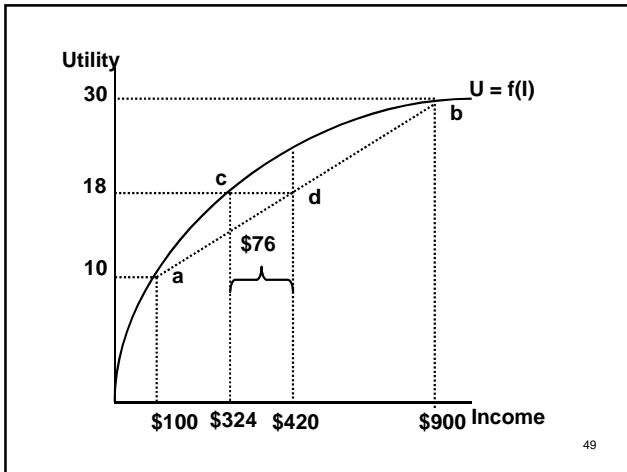
- Notice also that the person is indifferent between a job with \$16K in certain income and \$20,000 in uncertain
- They are willing to sacrifice up to \$4000 in income to reduce risk, risk premium

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Example

- $U = y^{0.5}$
- Job with certain income
 - \$400 week
 - $U = 400^{0.5} = 20$
- Can take another job that
 - 40% chance of \$900/week, $U = 30$
 - 60% chance of \$100/week, $U = 10$
 - $E(I) = 420, E(U) = 0.4(30) + 0.6(10) = 18$

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- Notice that utility from certain income stream is higher even though expected income is lower
- What is the risk premium??
- What certain income would leave the person with a utility of 18? $U=Y^{0.5}$
- So if $18 = Y^{0.5}$, $18^2 = Y = 324$
- Person is willing to pay $400-324 = \$76$ to avoid moving to the risky job

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Allais Paradox

- Which gamble would you prefer
 - 1A: \$1 million w/ certainty
 - 1B: (.89, \$1 million), (0.01, \$0), (0.1, \$5 million)
- Which gamble would you prefer
 - 2A: (0.89, \$0), (0.11, \$1 million)
 - 2B: (0.9, \$0), (0.10, \$5 million)

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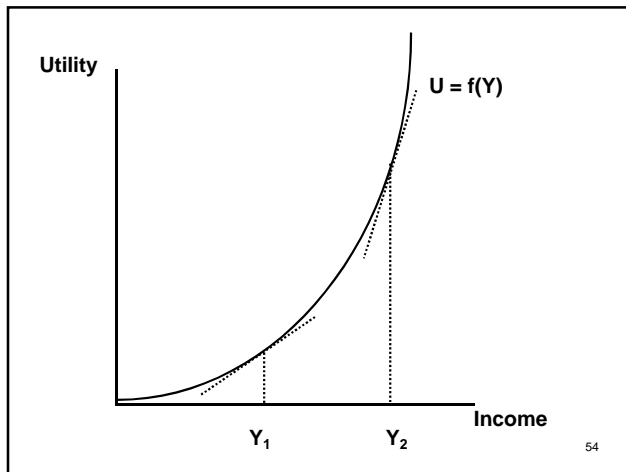
- 1st gamble:
 - $U(1) > 0.89U(1) + 0.01U(0) + 0.1U(5)$
 - $0.11U(1) > 0.01U(0) + 0.1U(5)$
- Now consider gamble 2
 - $0.9U(0) + 0.1U(5) > 0.89U(0) + 0.11U(1)$
 - $0.01U(0) + 0.1U(5) > 0.11U(1)$
- Choice of Lottery 1A and 2B is inconsistent with expected utility theory

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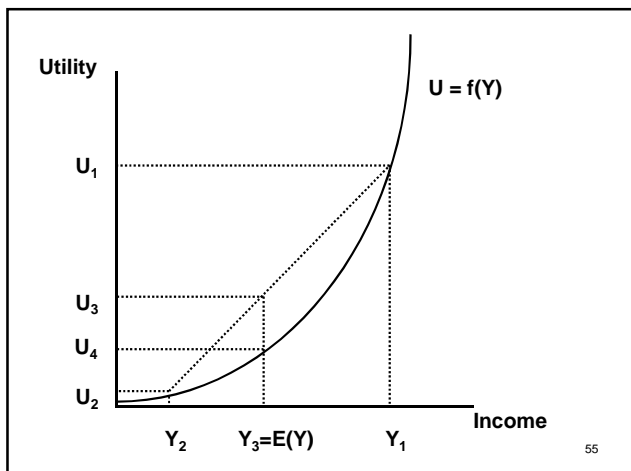
Risk Loving

- The desire to shed risk is due to the assumption of declining marginal utility of income
- Consider the next situation.
- The graph shows increasing marginal utility of income
- $U'(Y_1) > U'(Y_2)$ even though $Y_1 > Y_2$

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- What does this imply about tolerance for risk?
- Notice that at $E(Y) = Y_3$, expected utility is U_3 .
- Utility from a certain stream of income at Y_3 would generate U_4 . Note that $U_3 > U_4$
- This person prefers an uncertain stream of Y_3 instead of a certain stream of Y_3
- This person is 'risk loving'. Again, the result is driven by the assumption of $U'' > 0$

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Risk Neutral

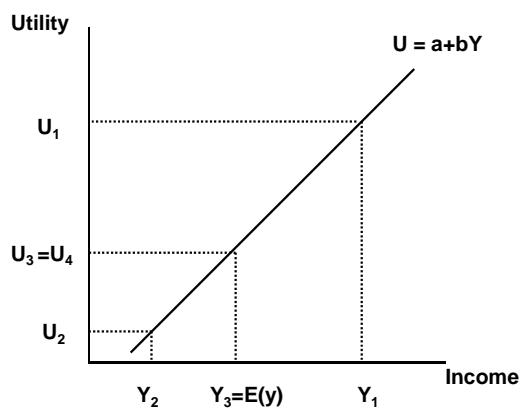
- If utility function is linear, the marginal utility of income is the same for all values of income
 - $U' > 0$
 - $U'' = 0$
- The uncertain income $E(Y)$ and the certain income Y_3 generate the same utility
- This person is considered risk neutral
- We usually make the assumption firms are risk neutral

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Example

- 25% chance of \$100
- 75% chance of \$1000
- $E[Y] = 0.25(100) + 0.75(1000) = \775
- $U = Y$
- Compare to certain stream of \$775

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Benefits of insurance

- Assume declining marginal utility
- Person dislikes risk
 - Is willing to receive lower certain income rather than higher expected income
- Firms can capitalize on the dislike for risk by helping people shed risk via insurance

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Simple insurance example

- Suppose income is known (Y_1) but random shocks can reduce income
 - House or car is damaged
 - Can pay \$ to repair, return you to a state of normal
- L is the loss if the bad event happens
- Probability of loss is P_1
- Expected utility without insurance is
- $E(U) = (1-P_1)U(Y_1) + P_1U(Y_1-L)$

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- Suppose you can buy insurance that costs you PREM. The insurance pay you to compensate for the loss L.
 - In good state, income is
 - $Y - \text{Prem}$
 - In bad state, paid PREM, lose L but receive PAYMENT, therefore, income is
 - $Y - \text{Prem} - L + \text{Payment}$
 - For now, let's assume $\text{PAYMENT} = L$, so
 - Income in the bad state is also
 - $Y - \text{Prem}$

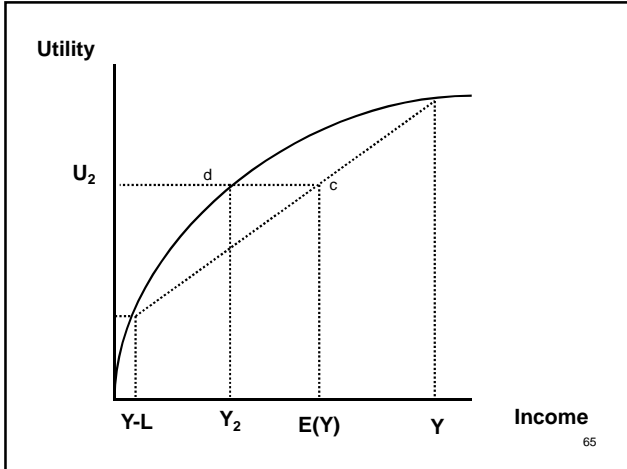
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- Notice that insurance has made income certain. You will always have income of $Y - \text{PREM}$
- What is the most this person will pay for insurance?
- The expected loss is p_1L
- Expected income is $E(Y)$
- The expected utility is U_2
- People would always be willing to pay a premium that equaled the expected loss

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- But they are also willing to pay a premium to shed risk (line cd)
- The maximum amount they are willing to pay is expected loss + risk premium

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- Suppose income is \$50K, and there is a 5% chance of having a car accident that will generate \$15,000 in loss
- Expected loss is $.05(15K) = \$750$
- $U = \ln(y)$
- Some properties of logs
 - $Y = \ln(x)$ then $e^y = \exp(y) = x$
 - $Y = \ln(x^a) = a \ln(x)$
 - $Y = \ln(xz) = \ln(x) + \ln(z)$

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- $E(U) = P \ln(Y-L) + (1-P)\ln(Y)$
- $E(U) = 0.05 \ln(35,000) + 0.95 \ln(50,000)$
- $E(U) = 10.8$

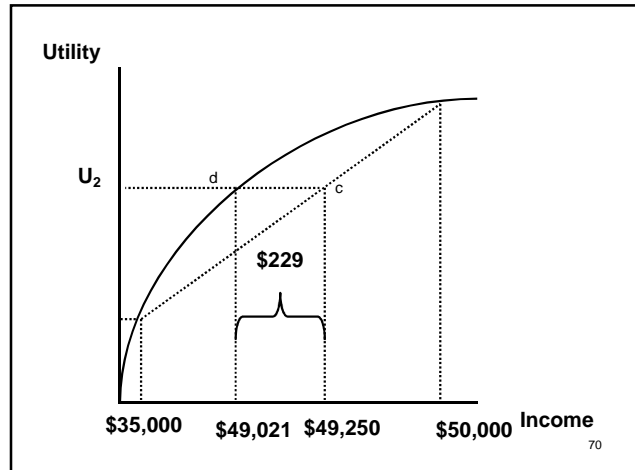
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- What is the most someone will pay for insurance?
- People would purchase insurance so long as utility with certainty is at least 10.8 (expected utility without insurance)
- $U_a = U(Y - \text{Prem}) \geq 10.8$
- $\ln(Y - \text{PREM}) \geq 10.8$
- $Y - \text{PREM} = \exp(10.8)$
- $\text{PREM} = Y - \exp(10.8) = 50,000 - 49,021 = 979$

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- Recall that the expected loss is \$750 but this person is willing to pay more than the expected loss to avoid the risk
- Pay \$750 (expected loss), plus the risk premium ($\$979 - \$750 = 229$)

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Supply of Insurance

- Suppose there are a lot of people with the same situation as in the previous slide
- Each of these people have a probability of loss P and when a loss occurs, they have L expenses
- A firm could collect money from as many people as possible in advance. If bad event happens, they pay back a specified amount.

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- Firms are risk neutral, so they are interested in expected profits
- Expected profits = revenues – costs
 - Revenues are known
 - Some of the costs are random (e.g., exactly how many claims you will pay)

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- Think of the profits made on sales to one person
- A person buys a policy that will pay them q dollars ($q \leq L$) back if the event occurs
- To buy this insurance, person will pay “ a ” dollars per dollar of coverage
- Cost per policy is fixed t

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- Revenues = aq
 - a is the price per dollar of coverage
- Costs = $pq + t$
 - For every dollar of coverage (q) expect to pay this p percent of time
- $E(\pi) = aq - pq - t$
- Let assume a perfectly competitive market, so in the long run $\pi = 0$
- What should the firm charge per dollar of coverage?
- $E(\pi) = aq - pq - t = 0$

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- $a = p + (t/q)$
- The cost per dollar of coverage is proportion to risk
- t/q is the loading factor. Portion of price to cover administrative costs
- Make it simple, suppose $t=0$.
 - $a = p$
 - If the probability of loss is 0.05, will change 5 cents per \$1.00 of coverage

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- In this situation, if a person buys a policy to insure L dollars, the ‘actuarially fair’ premium will be LP
- An actuarially fair premium is one where the premium equals the expected loss
- In the real world, no premiums are ‘actuarially fair’ because prices include administrative costs called ‘loading factors’

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How much insurance will people purchase?

- **With insurance**
 - Pay a premium that is subtracted from income
 - If bad state happens, lose L but get back the amount of insurance q
 - They pay $p+(t/q)$ per dollar of coverage. Have q dollars of coverage to pay a premium of $pq+t$ in total
- **Utility in good state**
 - $U = U[Y - pq - t]$

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- **Utility in bad state**
 - $U[Y - L + q - pq - t]$
- $E(u) = (1-p)U[Y - pq - t] + pU[Y-L+q-pq-t]$
- Simplify, let $t=0$ (no loading costs)
- $E(u) = (1-p)U[Y - pq] + pU[Y-L+q-pq]$
- Maximize utility by picking optimal q
- $dE(u)/dq = 0$

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- $E(u) = (1-p)U[Y - pq] + pU[Y-L+q-pq]$
- $dE(u)/dq = (1-p) U'(Y-pq)(-p)$
- $+ pU'(Y-L+q-pq)(1-p) = 0$
- $p(1-p)U'(Y-L+q-pq) = (1-p)pU'(Y-pq)$
- $(1-p)p$ cancel on each side

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- $U'(Y-L+q-pq) = U'(Y-pq)$
- Optimal insurance is one that sets marginal utilities in the bad and good states equal
- $Y-L+q-pq = Y-pq$
- Y 's cancel, pq 's cancel,
- $q=L$
- If people can buy insurance that is 'fair' they will fully insure losses.

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Insurance w/ loading costs

- Insurance is not actuarially fair and insurance does have loading costs
- Can show (but more difficult) that with loading costs, people will now under-insure, that is, will insure for less than the loss L
- Intuition? For every dollar of expected loss you cover, will cost more than a \$1
- Only get back \$1 in coverage if the bad state of the world happens

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- Recall:
 - q is the amount of insurance purchased
 - Without loading costs, cost per dollar of coverage is p
 - Now, for simplicity, assume that price per dollar of coverage is pK where $K > 1$ (loading costs)
- Buy q \$ worth of coverage
- Pay qpK in premiums

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- $E(u) = (1-p)U[Y - pqk] + pU[Y-L+q-pqk]$
- $dE(u)/dq = (1-p) U'(Y-pqk)(-pk)$
- $+ pU'(Y-L+q-pqk)(1-pk) = 0$
- $p(1-pk)U'(Y-L+q-pq) = (1-p)pkU'(Y-pq)$
- p cancel on each side

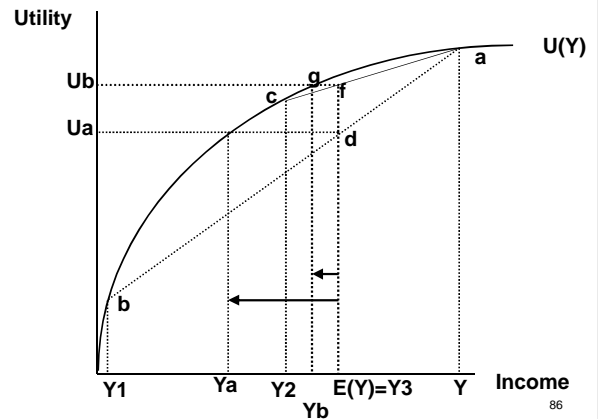
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- $(1-pk)U'(Y-L+q-pkq) = (1-p)kU'(Y-pkq)$
- $(a)(b) = (c)(d)$
- Since $k > 1$, can show that
- $(1-pk) < (1-p)k$
- Since $(a) < (c)$, must be the case that
- $(b) > (d)$
- $U'(Y-L+q-pkq) > U'(Y-pkq)$
- Since $U'(y_1) > U'(y_2)$, must be that $y_1 < y_2$

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- $(Y-L+q-pqk) < (Y-pqk)$
- Y and $-pqk$ cancel
- $-L + q < 0$
- Which means that $q < L$
- When price is not 'fair' will not fully insure

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Demand for Insurance

- Both people have income of Y
- Each person has a potential health shock
 - The shock will leave person 1 w/ expenses of E_1 and will leave income at $Y_1=Y-E_1$
 - The shock will leave person 2 w/ expenses of E_2 and will leave income at $Y_2=Y-E_2$
- Suppose that
 - $E_1 > E_2, Y_1 < Y_2$

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- Probabilities the health shock will occur are P_1 and P_2
- Expected Income of person 1
 - $E(Y)_1 = (1-P_1)Y + P_1(Y-E_1)$
 - $E(Y)_2 = (1-P_2)Y + P_2(Y-E_2)$
 - Suppose that $E(Y)_1 = E(Y)_2 = Y_3$

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- **In this case**

- Shock 1 is a low probability/high cost shock
- Shock 2 is a high probability/low cost shock

- **Example**

- $Y = \$60,000$
- Shock 1 is 1% probability of \$50,000 expense
- Shock 2 is a 50% chance of \$1000 expense
- $E(Y) = \$59500$

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- **Expected utility locus**

- Line ab for person 1
- Line ac for person 2

- **Expected utility is**

- U_a in case 1
- U_b in case 2

- **Certainty premium –**

- Line (de) for person 1, Difference $Y_3 - Y_a$
- Line (fg) for person 2, Difference $Y_3 - Y_b$

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Implications

- **Do not insure small risks/high probability events**

- If you know with certainty that a cost will happen, or, costs are low when a bad event occurs, then do not insure
- Example: teeth cleanings. You know they happen twice a year, why pay the loading cost on an event that will happen?

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- **Insure catastrophic events**

- Large but rare risks
- **As we will see, many of the insurance contracts we see do not fit these characteristics – they pay for small predictable expenses and leave exposed catastrophic events**

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Some adjustments to this model

- The model assumes that poor health has a monetary cost and that is all.
 - When experience a bad health shock, it costs you L to recover and you are returned to new
- Many situations where
 - health shocks generate large expenses
 - And the expenses may not return you to normal
 - AIDS, stroke, diabetes, etc.

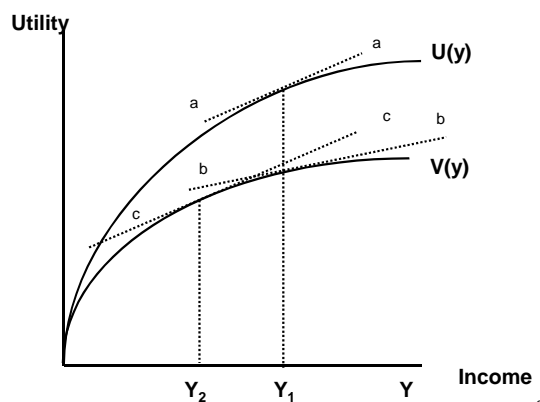
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- In these cases, the health shock has fundamentally changed life.
- We can deal with this situation in the expected utility model with adjustment in the utility function
- “State dependent” utility
 - $U(y)$ utility in healthy state
 - $V(y)$ utility in unhealthy state

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- Typical assumption
 - $U(Y) > V(Y)$
 - For any given income level, get higher utility in the healthy state
 - $U'(Y) > V'(Y)$
 - For any given income level, marginal utility of the next dollar is higher in the healthy state

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Note that:

- At Y_1 ,
 - $U(Y_1) > V(Y_1)$
 - $U'(Y_1) > V'(Y_1)$
 - Slope of line aa > slope of line bb
- Notice that slope line aa = slope of line cc
 - $U'(Y_1) = V'(Y_2)$

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What does this do to optimal insurance

- $E(u) = (1-p)U[Y - pq - t] + pV[Y-L+q-pq-t]$
- Again, lets set $t=0$ to make things easy
- $E(u) = (1-p)U[Y - pq] + pV[Y-L+q-pq]$

- $dE(u)/dq = (1-p)(-p)U'[Y-pq]$
 $+p(1-p)V'[Y-l+q+pq] = 0$
- $U'[Y-pq] = V'[Y-l+q-pq]$

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- Just like in previous case, we equalize marginal utility across the good and bad states of the world
- Recall that
 - $U'(y) > V'(y)$
 - $U'(y_1) = V'(y_2)$ if $y_1 > y_2$
- Since $U'[Y-pq] = V'[Y-l+q-pq]$
- In order to equalize marginal utilities of income, must be the case that $[Y-pq] > [Y-l+q-pq]$

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- Income in healthy state > income in unhealthy state
- Do not fully insure losses. Why?
 - With insurance, you take \$ from the good state of the world (where MU of income is high) and transfer \$ to the bad state of the world (where MU is low)
 - Do not want good money to chance bad

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