

How to Maximize a Likelihood Function
ECON 60303

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Problem setup

$f(y_i | x_i, \hat{\beta})$ is a pdf that describes the data generating process

There are n observations in the sample

x_i is $(1 \times j)$

$\hat{\beta}$ is $(k \times 1)$

Log likelihood function $\ell = \sum_{i=1}^n \ln f(y_i | x_i, \hat{\beta})$

Gradient $(k \times 1)$: $g = \frac{\partial \ell}{\partial \hat{\beta}}$ $cc = g'(-H^{-1})g$

Hessian $(k \times k)$: $H = \frac{\partial^2 \ell}{\partial \hat{\beta} \partial \hat{\beta}'}$

$Cov(\hat{\beta}) = -H^{-1}$

Convergence criteria: $cc = g'(-H^{-1})g$

Quasi-Newton search method:

Let $\hat{\beta}_t$ be the t 'th iteration of search algorithm.

Corresponding values for the other parameters are $g_t, (-H_t)^{-1}$

Initially fix $\lambda=1$

$\hat{\beta}_{t+1} = \hat{\beta}_t + \lambda(-H_t)^{-1} g_t$

Basic Algorithm:

1. Obtain starting values $\hat{\beta}_0$
2. Calculate $g_0, (-H_0)^{-1}$
3. Update $\hat{\beta}_1 = \hat{\beta}_0 + \lambda(-H_0)^{-1} g_0$
4. Is $\ell(\hat{\beta}_1) > \ell(\hat{\beta}_0)$
5. If no, cut λ in half and go 3
6. If yes, calculate $g_1, (-H_1)^{-1}$
7. Has model converged?
8. If no, go to 3 and update beta
9. If yes, printout results

Flow Chart for MLE Search Routine
Quasi-Newton Method

