Transfer Pricing In Vertically Integrated Industries*

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Abstract: Tax officials judge whether a multinational's transfer price is consistent with the arm's-length standard, the price at which two independent firms would carry out a similar transaction, by using data from comparable but independent transactions. In vertically integrated industries, the only source of comparable data may be from controlled (non-independent) transactions. Conventional wisdom asserts that standard arm's-length methods cannot perform well in such markets because the comparability rules encourage the integrated firms to collude tacitly on transfer prices in a way that amplifies tax-differential incentives. In this paper, we show that strategic linkages between vertically integrated firms operating in the same final good market moderate, and can possibly reverse, tax-differential incentives if the correct comparison method is used. The Cost-Plus method turns out to be the most effective in limiting the equilibrium amount of profit-shifting out of the high-tax country and it yields the highest tax revenues for the high-tax country. These benefits are shown to strengthen when the firms have private cost information.

Keywords: transfer pricing, vertical integration, incentive comparability

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1. Introduction

A transfer price is used to value the exchange of an asset or service between two subsidiaries of the same multinational. When the subsidiaries are incorporated in countries with different tax rates, the multinational has an incentive to set the transfer price to shift profits into the low-tax country. To mitigate this incentive, OECD countries audit multinational transfer prices to determine whether each transfer price meets the arm's-length standard: the transfer price equals the price two arm's-length or independent firms, firms not controlled by the same multinational, would trade. When an audit determines that a transfer price does not meet the arm's-length standard, the tax authority imputes an appropriate transfer price, calculates a new tax liability, and assesses penalties.

To detect deviations from arm's-length pricing in practice, tax authorities test whether a transfer price meets the arm's-length standard by comparing data from the audited firm to data from comparable transactions between independent buying and selling firms. The general idea is the data from the independent firms will approximate a competitive price. Reliance on transactions between independent firms seems obvious. Suppose instead the comparable data comes from controlled transactions, transactions where every buyer and seller pair is commonly owned by some other multinational. Each multinational active in the same product markets now has an expectation of being part of its competitors' comparability group. This expectation amplifies each multinational's profit-shifting incentives by creating an incentive for multinational firms operating in the same markets to tacitly collude in setting their transfer prices. How then should tax officials audit transfer prices in vertically integrated industries for which no independent transactions exist?[3]

[3]Issue COMP 2 from the OECD (2003) invitation to comment on comparability issues asks, "Should information on third party enterprises that are part of another MNE group and as such engaged in controlled transactions with associated enterprises be systematically rejected, or should such information be regarded as providing useful information...?" Although the OECD does not have legislative power, it does promote legal conventions that can and have been adopted by many countries. For example, the

In the United States, §482 of the Internal Revenue Code (1994) repeatedly mandates the need to "locate two unrelated parties that are each comparable to one of the controlled taxpayers" and as such offers no guidance for dealing with industries in which no comparable uncontrolled transactions exist.[4] OECD guidelines on the other hand acknowledge that "evidence from enterprises engaged in controlled transactions with associated enterprises may be useful" but do not provide specific guidance on how best to use such information. (OECD 2001, paragraph 1.70). The purpose of this paper is twofold. First, we seek to challenge the conventional wisdom discouraging/prohibiting the use of data from controlled transactions to audit a multinational's transfer prices. Second, we will identify the best procedure for analyzing controlled data in order to minimize harmful profit-shifting incentives.

The following example from the oil and natural gas industry in Norway illustrates the relevance of our focus on vertically integrated industries. Because of technical demands, drilling in the North Sea is limited to a small number of large multinational companies. The special nature of deep-sea drilling means the market for property and liability insurance is quite thin and risk characteristics can vary from one platform to another. Like most large corporations, oil and gas companies self-insure their risk in a (captive) insurance subsidiary and then reinsure a portion of their risk with independent insurers. The difficult task for revenue authorities is finding comparable insurance rates on the retained risks.[5] In addition, differences in the tax treatment of insurance profits and oil and gas profits create an incentive to

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OECD (1995) transfer price guidelines describe procedures common to many OECD countries.

[4]For example, see §1.482.5(a). (U.S. Treasury 1994, p. 34974).

[5]While U. S. laws governing the tax deductibility of premiums paid to a captive insurance subsidiary are quite strong, European laws are more permissive. For most European countries the sole focus is on the transfer payments (i.e. premiums) to adequately capitalized captive insurance companies. So while this specific example is not likely to be relevant from a United States perspective, it is quite relevant from a European perspective (see also note 2) and the more general problem of intermediate inputs being provided exclusively via vertically integrated multinationals is not unique to Europe.

shift profits into the insurance subsidiaries.[6] Because of the thin market and idiosyncratic risk features, similar yet independent data for comparability calculations is not available. On the other hand, the presence of several multinational companies in the North Sea generates good, albeit controlled, comparable data.

To better understand why it is desirable to use transfer price data from independent parties, consider how revenue authorities conduct transfer price audits. Data from transactions between independent parties is collected and compared to the audited party's data. Regulations from OECD countries specify several different methods for analyzing this data. The Comparable Uncontrolled Price method compares the audited transfer price to the comparable transaction prices, the Cost-plus method compares cost mark-ups of the selling firms, and the Comparable Profit method allows tax officials to compare profitability ratios (e.g. operating profit/sales and gross profit/operating expenses). The Resale Price method compares the mark-up over the transfer price reflected in the final good prices of the buying firms. Finally, the Profit-Split method compares profit shares between the buying and selling parties. If the value of the audited party's data implied by one of these methods falls within the 25th and the 75th-percentile of the data from the independent transactions, the transfer price is deemed acceptable. When the tested transfer price falls outside this range, the tested party's price is adjusted, a new tax liability is calculated, and penalties are imposed.[7]

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[6]For example, in Norway the marginal corporate tax rate for profits earned from North Sea operations is 78% while profit tax rates from "mainland" activities can range from 28% down to 0%. In the United Kingdom, captive insurance companies also enjoy favorable tax treatment relative to self-insurance.

[7]The study of transfer pricing behavior in economics dates back at least to Horst (1971) and Copithorne (1971). Since then the influence of tax-induced transfer pricing has been considered on the effect of transfer pricing on a government's choice of a tax base (Haufler and Schjelderup (2000) and a double tax rule (Weichenrieder (1996)), on the interaction between trade policies and corporate tax

As long as the audited firm cannot influence the comparable transactions, the data from all five methods should approximate a competitive price. One key way the audited firm can influence the independent transaction prices is if the audited multinational and the independent firms operate in similar markets and the multinational has some market power. Samuelson (1982) was the first to argue that equilibrium market prices are partially determined by the tax motives of an integrated firm. Halperin and Srinidhi (1996) extended this analysis by explicitly incorporating features of the comparability methods codified in the 1994 regulations. Both articles ensured the existence of independent transactions by assuming only one active multinational in the intermediate good market. Nonetheless, the use of comparable data was shown to result in equilibrium transfer prices that are distorted from efficient, competitive levels. With no independent transactions, one might expect these distortions to be magnified due to the coordinating effect created when each multinational firm recognizes that its transfer price helps establish a comparison basis for auditing the transfer prices of the multinational firms with which it competes. While this expectation is correct for some of the methods outlined above, it is not true for all of the methods. We show in this paper the correct method can actually induce tacit coordination consistent with a tax authority's interests.

Regardless of whether final good production or intermediate good production is located in a high-tax country, the Cost-Plus method shifts the least profit out of the high-tax country and may result in profits being shifted into the high-tax country. The Cost-Plus method also generates the most tax revenue for the high-tax country. The advantage associated with the Cost-Plus method comes from the economic linkage between a firm's transfer price on intermediate goods and its final good production. For the

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policies (e.g. Bond (1980), Levinsohn and Slemrod (1993), and Schjelderup and Weichenreider (1999)), and on the efficacy of transfer price regulation when firms have private cost information (Donnenfeld and Prusa (1993), Gresik and Nelson (1994), Bond and Gresik (1996), Elitzur and Mintz (1996), and Calzolari (2004)). However, all of these rules abstract away from the specific comparability structure of existing regulations, and thus fail to capture any of the associated strategic effects.

purposes of this introduction, we explain this linkage for the case in which final good production is located in the high-tax country. Firms have two options for increasing the profit it shifts out of the high-tax country: increase its transfer price and increase its final good output (which requires more of the intermediate good). As a firm increases its transfer price, its profit margin on final good production increases which means each firm has an incentive to increase its transfer price and its output together. If final good production among the firms are strategic substitutes, then a combined increase in transfer prices and output will not only result in an increase in profits shifted out of the high-tax country it can also result in a decrease in final good profits via a decrease in the final good price. The Cost-Plus method creates the strongest trade-off between final good profits and transfer price profits, and thus results in the lowest level of profit-shifting out of the high-tax country and the largest tax revenues for the high-tax country. With enough firms in the final good market, the negative price effect will actually swamp the positive profit effect from high transfer prices and create coordination incentives that result in lower transfer prices than one would expect based solely on tax differentials.

In section 2, we present a complete information model of transfer pricing with oligopolistic multinationals that can accommodate all standard transfer price methods. Each multinational produces an intermediate good in one country and a final good in another country. In section 3, we confirm the extant intuition that tacit coordination of transfer prices arises under any of these methods. Moreover, the comparability structure of arm's-length methods creates multiple equilibria that can be indexed by the transfer price on which the firms coordinate. As in Alles and Datar (1998) and Narayanan and Smith (2000, p. 501), scope for tacit coordination arises because transfer prices and tax differentials help shape product-market competition. In both papers, transfer prices serve an internal managerial commitment function via sales division incentives as in Fershtman and Judd (1987). Since neither paper models the regulation of transfer prices, the question of how the specific regulatory method contributes to the product-market competition is not addressed. As in Halperin and Srinidhi (1996), we abstract away from the internal managerial effects of transfer prices and focus on the impact of the choice of regulation. Our paper differs from Halperin and Srinihdi (1996) in that we allow for competition between multinationals whereas they only allow competition between a multinational's subsidiary and a local, unintegrated, domestic competitor. This competition between multinationals generates tacit coordination not seen in Halperin and Srinihdi (1996) because of the comparability structure of transfer price regulations.

Our main results regarding the advantages of the Cost-Plus method are developed in section 3. In section 4, we introduce cost heterogeneity among the upstream subsidiaries to assess the extent to which our results depend on the base model's symmetry. Rather than increasing the set of equilibria, we identify an incentive condition we refer to as "incentive comparability" that limits the ability of multinationals in a vertically integrated industry to take advantage of profit-shifting incentives when they have private cost information. Section 5 offers concluding comments.

2. A Model of Transfer Price Regulation.

N multinationals, indexed by i, produce identical goods. Final good production for firm i, q_i, requires an intermediate good in a 1-1 ratio. Each multinational has an upstream subsidiary (u) responsible for intermediate good production and a downstream subsidiary (d) responsible for final good production. The u and d subsidiaries operate in different tax jurisdictions which for simplicity we refer to as countries. The country of incorporation and operation for all the final-good subsidiaries is D and the country of incorporation and operation for all the intermediate-good subsidiaries is U. Segmenting the upstream and downstream divisions in this way makes it easier for tacit coordination consistent with the conventional wisdom described in the introduction to arise. If some of the upstream subsidiaries were located in country U and some were located in country D, the profit-shifting interests of the multinationals would not coincide. Some multinationals would use a large transfer price to shift profits out of the high-tax country and some would use a small transfer price. Thus, our polar assumption aligns the interest of the multinationals in a way that biases our results in support of the conventional wisdom.

The multinationals compete in a Cournot game. Each downstream subsidiary earns operating profit excluding intermediate good costs of $\pi_i(q_i, Q_{-i})$ where $Q_{-i} = \Sigma_{j \neq i} q_j$. π_i is concave in q_i and decreasing in Q_{-i}. In the final product market, the goods are assumed to be classical and strategic substitutes, i.e. $\partial\pi_i/\partial Q_{-i} < 0$ and $\partial^2\pi_i/\partial Q_{-i}\partial q_i < 0$. Furthermore, operating profits are symmetric in the sense that if $q_i = q_j$, then $\partial\pi_i/\partial Q_{-i} = \partial\pi_j/\partial Q_{-j}$. Each downstream producer purchases the intermediate good from its parent's upstream subsidiary at the transfer price, ρ_i. Thus, the downstream producer's total operating profit equals $\pi_i(q_i, Q_{-i}) - \rho_i q_i$. These profits are taxed at an effective rate, t. In some cases it will be convenient to write $\pi_i(q_i, Q_{-i}) = R_i(q_i, Q_{-i}) - C(q_i)$ where, for the demand curve $P(q_i + Q_{-i})$, $R_i(q_i, Q_{-i}) = P(q_i + Q_{-i})q_i$ denotes firm revenue and $C(q_i)$ denotes the downstream producer's local production costs. We assume firm revenue is strictly concave, downstream costs are weakly convex, and $C(0) = 0$.

Each u subsidiary has convex production costs $K(q_i)$ with $K(0) = 0$. Thus u profit equals $\rho_i q_i - K(q_i)$. These profits are taxed at an effective rate, t^*. We do not consider the role of withholding taxes or double taxation issues for repatriated profits. In some cases, these effects can be captured by appropriate adjustments to t and t^*.

If $t > t^*$, the multinational has a tax incentive to set a high transfer price to shift profits into the low-tax country. This is consistent with the Norwegian example as revenues from oil sales are treated as offshore profits while the captive insurance subsidiaries who provide insurance for the offshore platforms are located onshore. In fact, it is common for multinationals in many industries to locate their captive insurance and other service-related subsidiaries in tax haven countries because of this tax incentive. If $t < t^*$, the multinational has a tax incentive to set a low transfer price, again to shift profits into the low-tax country.

The tax incentive gives the high-tax country an incentive to audit transfer prices and penalize firms when it finds evidence of profit-shifting. The methods used by OECD countries to audit transfer prices have several common features. Our model reflects the main features of these procedures. First, the tax authority collects data from other firms engaged in similar transactions to form a comparison cohort. Large multinationals are routinely audited. So to study the equilibrium effects of standard regulations when no independent comparables are available, the cohort for each multinational will be the other N-1 firms.[8]

Second, the tax authority calculates a financial statistic for the audited firm and each firm in the comparison cohort. Each method permitted under the national laws of OECD countries uses a specific financial statistic, F, which is a function of a firm's transfer price and some or all N quantities. For a given statistic, F, denote the value of the audited firm's statistic by $F_i = F(q_i, q_{-i}, \rho_i)$ where $q_{-i} = (q_1,...,q_{i-1},q_{i+1},...,q_N)$ and denote the vector of statistics for the other N–1 firms by $F_{-i}=(F_1,...,F_{i-1},F_{i+1},...,F_N)$. Since F_{-i} is a function of $q=(q_i,q_{-i})$ and ρ_{-i} where $\rho_{-i} = (\rho_1,...,\rho_{i-1},\rho_{i+1},...,\rho_N)$, we can denote the entire vector of cohort statistic values by $F_{-i}(q,\rho_{-i})$. Tax laws in many OECD countries, including the U.S., explicitly define five different financial statistics that can be used to audit a firm's transfer price.[9]

Comparable Uncontrolled Price (CUP) - Compares actual transfer prices: $F_i = \rho_i$.[10]

Cost-Plus (CP) - Compares cost mark-ups in the upstream subsidiary: $F_i = (\rho_i q_i - K(q_i))/K(q_i)$.

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[8]Harris and Sansing (1998) and Sansing (1999) identify how transfer price regulations distort product pricing and capital investment decisions when the tested party is a vertically integrated firm (as in our model) and the comparable firms are not integrated. Because all the firms in our model are vertically integrated, differences in corporate structure cannot explain the strategic effects we identify.

[9]The newest method based on a Profit-Split statistic is not listed here because Nash equilibria need not exist due to the large profit-shifting incentives created in a vertically integrated industry. Also, contrary to to Figure 1 in Halperin and Srinidhi (1996), IRC §1.428-8 does not recommend the Profit-Split rule for the case we consider here of no independent comparables. In fact, this section of the U.S. tax code specifically proscribes the use of the Profit-Split method with no independent comparables.

[10]The formula for each method is based on the definitions in U.S. Treasury (1994). OECD names and formulas for each method are similar.

Comparable Profit Method (CPM) - Compares a variety of d financial ratios. Common ratios include:

i) Sales/Operating Expenses†. $F_i = (C(q_i) + \rho_i q_i)/(P(q_i+Q_{-i})q_i)$.

ii) Average Gross Profit†.[11] $F_i = q_i/(\pi_i(q_i,Q_{-i}) - \rho_i q_i)$.

Resale Price (RP) - Compares price markups over cost in the downstream subsidiary.†

$F_i = P(q_i + Q_{-i})q_i/(P(q_i + Q_{-i})q_i - C(q_i) - \rho_i q_i)$.

For example, if a tax authority chose to use the CP statistic, it would calculate the mark-up over upstream production costs implied by a firm's transfer price and intermediate good production and compare the markups from all N firms. Alternatively with the CUP statistic, a tax authority would simply compare the transfer prices set by all N firms without regard for possible differences in production levels.

Third, to complete the description of a method, we need to describe how these statistics are used. The audited firm's transfer price is judged to be the result of profit-shifting, either too high or too low, if its financial statistic falls outside the interquartile range of the cohort's statistics. OECD regulations define a range of acceptable transfer prices (i.e. the interquartile range) because the exact way in which an independent buyer and seller would bargain to divide the surplus, $\pi_i(q_i,Q_{-i}) - K(q_i)$, will depend on each party's bargaining power as well as the number of other traders. Thus, in practice, tax authorities allow for some variation in financial statistics. (In the next section, we will show that for identical firms, there can be no variation in equilibrium.) When a firm's financial statistics falls outside the interquartile range, the tax authority rules that the firm's transfer price is the result of illegal profit-shifting, calculates a new tax liability based on the median cohort statistic, and adds a penalty for profit-shifting. Denote the adjustment and penalties by $\Phi(\rho,q)$. Given $\Phi(\rho,q)$, the multinational's total post-tax profit equals

[11] Statistics marked with an "†" are expressed as the inverse of the common definition of the ratio so that higher transfer prices yield higher values of the statistic. This is a normalization that has no effect on the analysis. We explain why this is true at the end of this section after the full structure of the audit procedure is developed.

$$\Pi_i(q,\rho) = (1 - t)(\pi_i(q_i,Q_{-i}) - \rho_i q_i) + (1 - t^*)(\rho_i q_i - K(q_i)) - \Phi(\rho,q). \quad (1)$$

The exact form of $\Phi(\cdot,\cdot)$ depends on the choice of F. To simplify the formal description of $\Phi(\cdot,\cdot)$, we focus primarily on the case in which $t > t^*$. Penalties and adjustments for the case in which $t < t^*$ are defined in an analogous way. We also assume initially that only the high-tax country has an incentive to audit for evidence of profit-shifting. This assumption is made to test the conventional wisdom about comparing financial data in a vertically integrated industry. If it is correct, then all a low-tax country could accomplish by auditing is a decrease in tax revenues.[12] We will discuss strategies for the low-tax country at the end of section 3.

When $t > t^*$, country D has an incentive to audit for upward transfer price distortions. Let $\lfloor x \rfloor$ be the largest integer less than or equal to x and let $F_i^*(q,\rho_{-i})$ equal the $\lfloor 3N/4 \rfloor^{th}$ largest value from $F_{-i}(q,\rho_{-i})$. Consistent with the common regulatory practice penalizing a firm whose statistic value falls outside the interquartile range, F_i^* is the highest value of i's statistic that will not trigger penalties given (q_{-i},ρ_{-i}).[13] Firm i will be penalized if, and only if, $F_i > F_i^*$. (When $t < t^*$, the U government will have an incentive to audit the multinationals for transfer prices that are set too low. In this case, an audited firm's statistic would be compared to the lowest statistic value that would not trigger a penalty.)

To define firm i's penalty, let $\bar{F}_i(q,\rho_{-i})$ denote the median value among i's comparison cohort, F_{-i}. As long as F_i is increasing in ρ_i, it is possible to define $\bar{\rho}$ as the solution to $F_i(q,\bar{\rho}) = \bar{F}_i(q,\rho_{-i})$ or

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[12]In the Norwegian example, the same government oversees tax policy in D and U. Thus the standard common agency criticism, that the U government might establish countervailing regulations, does not apply. Moreover, most captive insurance companies are located in tax havens where the marginal tax rate is effectively zero. Efforts by D to limit tax-shifting transfer pricing into these tax havens will not reduce the tax revenues collected by these countries and hence will also be unlikely to create countervailing regulations.

[13]There is nothing special about using the 75th percentile. Any threshold above the median that is an increasing function of the components of F_{-i} will generate the same strategic effects.

$\bar{\rho} = F_i^{-1}(\bar{F}_i;q)$ where the inverse of F_i is taken with respect to ρ_i. (Note that $\bar{F}_i(q,\rho_{-i}) \le F_i^*(q,\rho_{-i})$.) The tax office uses $\bar{\rho}$ to calculate a new tax liability when firm i fails the above comparison test. Since $\bar{\rho}$ implies firm i initially underpaid taxes to country D by $t(\rho_i - \bar{\rho})q_i$, a penalty of $\phi > 0$ is imposed resulting in an additional tax payment by firm i to country D equal to $(1+\phi)t(\rho_i - \bar{\rho})q_i$. Thus, for any monotonic statistic, $t > t^*$ implies

$$\Phi(\rho,q) = \begin{cases} 0 & \text{if } F_i(q,\rho_i) \le F_i^*(q,\rho_{-i}) \\ (1+\phi)t(\rho_i - \bar{\rho})q_i & \text{if } F_i(q,\rho_i) > F_i^*(q,\rho_{-i}). \end{cases} \tag{2}$$

If firm i is also required to restate its taxable income in country U, it would receive a refund equal to $t^*(\rho_i - \bar{\rho})q_i$. Since country U's policy on restating taxable income does not affect our analysis, we assume no restatement is required.

The choice of a statistic and the penalty rule (2) define a transfer price method. We will call a method *monotonic* as long as it uses a statistic for which $\partial F_i/\partial\rho_i > 0$. There is no loss of generality in defining the statistic so that $\partial F_i/\partial\rho_i$ is positive. Doing so is merely a normalization and has no effect on our results as the following example illustrates. Consider the Average Gross Profit statistic which is normally defined as $F_i = (\pi_i(q_i,Q_{-i}) - \rho_i q_i)/q_i$. When country D is the high-tax country, each multinational has an incentive to set ρ_i high which in turn generates a small average gross profit. In this case, country D would penalize a multinational whose average gross profit falls below the 25th percentile of cohort values. Note that this is equivalent to defining the statistic as the inverse of average gross profit and penalizing any firm whose inverse average gross profit is above the 75th percentile of cohort values (which we do above). For notation purposes, it is simpler to define statistics so that they all generate a penalty when a firm's statistic is too large (given $t > t^*$).

3. Market Equilibria with Standard Transfer Price Regulations

For each transfer price method, we will derive the Nash equilibria of a complete information game in which the multinationals simultaneously choose output and transfer prices. Once the multinationals have chosen their quantities and transfer prices, the *D* government automatically tests all *N* transfer prices using the pre-specified method and assesses any relevant penalties. We assume in this section that $\Phi(\cdot,\cdot)$, $K(\cdot)$, and the $\pi_i(\cdot,\cdot)$ are common knowledge among all *N* firms.

For each firm, the transfer prices of the other *N*-1 firms constitutes its comparable data. Only monotonic arm's-length methods are admissible.[14] This is done is Section 3.1. Since the comparison structure of the transfer price methods supports multiple equilibria, Section 3.2 compares the maximal-profit equilibria from each method. In Section 3.3, we then ask which method the tax authority in the high-tax country would prefer based on two different welfare measures: compliance with the arm's-length standard and tax revenue maximization.

3.1 Equilibria

Proposition 1. *For any monotonic arm's-length method, F, if t > t*, then it is a best-response for firm i to choose the highest transfer price that does not trigger a penalty, i.e.,* $F_i(q,\rho_i) = F_i^*(q,\rho_{-i})$.

Proof. From (1) and (2),

$$\frac{\partial \Pi_i}{\partial \rho_i} = \begin{cases} (t - t^*)q_i & \text{if } F_i \le F_i^* \\ -(\phi t + t^*)q_i & \text{if } F_i > F_i^*. \end{cases}$$

$\partial\Pi_i/\partial\rho_i$ is positive when $F_i \le F_i^*$ and negative when $F_i > F_i^*$. Thus, Π_i is maximized when $F_i = F_i^*$ which implies $\rho_i = F_i^{-1}(F_i^*(q,\rho_{-i});q)$. ***Q.E.D.***

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[14]One might object to using the term "arm's-length" to describe the methods we consider since no data on independent transactions exist. In this paper, we use this term to denote the fact that the methods are those defined by regulations that seek to identify arm's-length prices under normal information circumstances.

Proposition 1 supports the intuition that using data from controlled transactions can lead to *tacit* coordination of the financial statistics. In any equilibrium, F_i must equal F_j for all i and j. Having a higher statistic value generates a penalty while having a lower statistic value implies the firm could have engaged in more profit-shifting without penalty. Analogously, when $t < t^*$, having a too low a statistic value generates a penalty while having too high a statistic value implies the firm could have shifted more profit into the low-tax country without penalty. Proposition 1 also implies that there exist multiple equilibria indexed by the common value of the financial statistic the firms choose. Suppose all firms $j \neq i$ choose their transfer prices and quantities so that $F_j = \mu$. Rather than solving two first-order equations, firm i will maximize its profit by choosing the unique value of ρ_i that for each q_i implies $F_i = \mu$ and then solving one first-order equation in q_i. Thus, each value of μ corresponds to a different equilibrium. Among all equilibria, we wish to focus on the equilibrium that maximizes industry (and hence firm) profit and then determine how the high-tax country's choice of a specific transfer price method influences the transfer price and quantities in this maximal-profit equilibrium.

Using Proposition 1, define $\tau_i(q_i,q_{-i},\rho_{-i}) \equiv F_i^{-1}(F_i^*(q,\rho_{-i});q)q_i$ to be firm i's indirect transfer payments, $\rho_i q_i$, when it sets ρ_i equal to its optimal value for each q_i taking as given the transfer price and quantity choices of the other multinationals. This optimal transfer price implies no penalties in equilibrium and gives us the "indirect" profit function

$$\Pi_i^*(q_i,q_{-i},\rho_{-i}) = (1-t)\pi_i(q_i,Q_{-i}) - (1-t^*)K(q_i) + (t - t^*)\tau_i(q_i,q_{-i},\rho_{-i}). \tag{3}$$

Firm i's problem of choosing ρ_i and q_i to maximize Π_i subject to $F_i = F_j$ is equivalent to choosing q_i to maximize Π_i^*. Table 1 reports the τ_i function for each method when every other firm produces q^* and chooses the transfer price ρ^*. For instance, if the CP method is used and the firms in i's cohort all have cost markups equal to μ, then Proposition 1 implies that for any q_i the maximum profit firm i can earn will imply $\rho_i q_i$ equals $(1+\mu)K(q_i)$ where $1+\mu = \rho^* q^*/K(q^*)$. However, if the CUP method is used and all the firms in i's cohort have transfer prices equal to ρ^*, then the maximum profit implied by Proposition 1 will equal $\rho^* q_i$.

Table 1 reveals three interesting properties of the τ_i functions. First, the Resale Price method and all variants of the CPM method induce the same indirect transfer payment function. This is due to the assumption that the firms are producing identical goods. (For convenience, we will refer to all of these methods as RP.) Second, if firm costs ($C(\cdot)$ and $K(\cdot)$) are linear then all the methods generate identical strategic effects as $\tau_i = \rho^* q_i$. Third, τ_i is linear in ρ^* because the standard statistics are all linear in the transfer price payments of each cohort firm ($\rho^* q^*$).

Method	$\tau_i(q_i, q^*_{-i}, \rho^*_{-i})$	$\partial\tau_i(q_i, q^*_{-i}, \rho^*_{-i})/\partial q_i$
CUP	$\rho^* q_i$	ρ^*
CP	$\rho^* q^* K(q_i)/K(q^*)$	$\rho^* q^* K'(q_i)/K(q^*)$
CPM and RP	$\rho^* q_i - C(q_i) + C(q^*) q_i/q^*$	$\rho^* - C'(q_i) + C(q^*)/q^*$

Table 1: Indirect transfer payment functions

Given the symmetry assumptions in this model, we will focus on symmetric equilibria described by an equilibrium firm quantity, q^*, and an equilibrium transfer price, ρ^*.[15] From (3), q^* and ρ^* will be a symmetric equilibrium quantity and transfer price only if

$$\partial\Pi^*_i(q^*, q^*_{-i}, \rho^*_{-i})/\partial q_i = (1-t)\partial\pi_i(q^*, Q^*_{-i})/\partial q_i - (1-t^*)K'(q^*) + (t - t^*)\partial\tau_i(q^*, q^*_{-i}, \rho^*_{-i})/\partial q_i = 0 \qquad (4)$$

and $\partial^2\Pi^*_i/\partial q_i^2 \le 0$ where q^*_{-i} is the N-1 dimensional vector (q^*,...,q^*), $Q^*_{-i} = (N-1)q^*$, and ρ^*_{-i} is the N-1 dimensional vector (ρ^*,...,ρ^*). Given ρ^*, q^* is the symmetric equilibrium quantity for which each firm's after-D-tax marginal profit plus its after-tax marginal transfer price payments equals its after-U-tax marginal costs. For each q^*, the linear structure of τ_i referenced above implies there is a unique ρ^* that solves (4) when $q_i = q^*$. Denote this transfer price by $\rho^*(q^*)$. (q^*,$\rho^*(q^*)$) is a *symmetric equilibrium* if, and only if, $P(q^*, Q^*_{-i}) > 0$, $\rho^*(q^*) \ge 0$, and $\Pi^*_i(q_i, q^*_{-i}, \rho^*_{-i})$ is maximized at $q_i = q^*$.

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[15]Under mild conditions on demand (such as weakly concave inverse demand), only symmetric equilibria will exist.

Eq. (4) implies a negative transfer price for all three methods when $t > t^*$ if q^* is too small. For CUP and CP, $\rho^*(q^*)$ will be positive as long as $q^* > \breve{q}$ where $\breve{q}$ solves $(1-t)\partial\pi_i(\breve{q},\breve{q}_{-i})/\partial q_i = (1-t^*)K'(\breve{q})$. For RP, $\rho^*(q^*)$ will be positive for all $q^* > \breve{q}'$ where $\breve{q}' > \breve{q}$. When $t < t^*$, (4) implies a negative transfer price if q^* is too large. $\rho^*(q^*)$ will be positive with CUP and CP if $q^* < \breve{q}$ and with RP if $q^* < \breve{q}'$. Thus, one key difference between the $t > t^*$ case and the $t < t^*$ case is that the latter case provides a natural limit on the profits a multinational can shift into the low-tax country.

A second difference between the two cases is that Π_i^* is globally concave in q_i with CUP and RP when $t > t^*$ while Π_i^* is globally concave in q_i with all 3 methods when $t < t^*$. In the appendix we show that a sufficient condition to guarantee that Π_i^* is globally concave in q_i with CP when $t > t^*$ is for q^* to be less than $\bar{q}$ where $\partial\pi_i(\bar{q},\bar{q}_{-i})/\partial q_i = 0$. (Note that $\bar{q}$ is always greater than $\breve{q}$.) In the absence of profit-shifting, no firm would choose to produce above $\bar{q}$. Just as with a negative transfer price, a country can rule out transfer prices above $\rho^*(\bar{q})$ on a priori grounds since it is incompatible with any deal between an independent buyer and an independent seller. Moreover, for a given tax rate t, country D's tax revenues at any equilibrium which equal $t(\pi_i(q^*,q_i^*) - \tau_i(q^*,q_{-i}^*,\rho^*(q^*)_{-i}))$ are decreasing for all $q^* \geq \bar{q}$. Following this discussion, we define a *regular symmetric equilibrium.*

Definition. *a) When $t > t^*$,* $(q^*,\rho^*(q^*))$ is a *regular symmetric equilibrium* if, and only if, $P(q^*,q_{-i}^*) > 0$, $\rho^*(q^*) \geq 0$, and $q^* \leq \bar{q}$.

b) When $t < t^*$,$(q^*,\rho^*(q^*))$ is a *regular symmetric equilibrium* if, and only if, $P(q^*,q_{-i}^*) > 0$ and $\rho^*(q^*) \geq 0$.

Eqs. (3) and (4) show any strategic differences generated by the different transfer price methods will be due to differences in the indirect transfer payments. Notice using Table 1 that τ_i is linear in q_i with CUP, convex in q_i with CP, and concave in q_i with RP. These differences will generate different marginal transfer payments which in turn will create different strategic effects. We report these marginal transfer payments in the last column of Table 1. To understand the role of the marginal transfer payments, consider a given equilibrium quantity, q^*. Based on (4), the only way q^* can be an equilibrium quantity for each of the three methods, is if the $\partial\tau_i/\partial q_i$ term has the same value. For a fixed value of ρ^*, the concavity of τ_i under RP means $\partial\tau_i/\partial q_i$ will be smaller than $\partial\tau_i/\partial q_i$ under either CUP or CP. The only way to have $\partial\tau_i/\partial q_i$ under RP equal $\partial\tau_i/\partial q_i$ under CUP or CP at q^* is for RP to have a larger equilibrium value of $\rho^*(q^*)$. A similar argument applies to a comparison of CUP and CP.

Proposition 2. *Assume K(·) and C(·) are strictly convex. For all regular symmetric equilibrium values of* q^*, $\rho^*_{CP}(q^*) \leq \rho^*_{CUP}(q^*) < \rho^*_{RP}(q^*)$.

Proof. See appendix.

Proposition 2 shows that the CP method uses the convexity of upstream production to create the incentive for the smallest transfer price while the RP methods use the convexity of downstream production to create the incentive for the largest transfer price. (For $q^* \neq \breve{q}$, ρ^*_{CP} will be strictly less than ρ^*_{CUP}.) The differences between CP and RP arise because the two involve different mark-up bases. CP is based on the mark-up of transfer price payments over upstream costs while RP is based on downstream revenues relative to downstream profit net of transfer price payments. With CP, higher upstream costs imply a lower mark-up for a fixed transfer price whereas with RP higher downstream costs imply a larger mark-up. Different statistics create different strategic effects.

When country *D* is the high-tax country, profit-shifting is associated with high transfer prices (holding output fixed) so CP induces the least profit-shifting. When country *U* is the high-tax country, profit-shifting is associated with low transfer prices (holding output fixed) so RP induces the least profit-shifting. Moreover, this ranking arises for very general reasons that could be applied, not just to the standard methods, but to many alternative methods that might be proposed. In any regular symmetric equilibrium, $\tau_i(q^*, q^*_{-i}, \rho^*_{-i}) = \rho^*(q^*)q^*$ where $\rho^*(q^*)$ is defined by (4). Eq. (4) implies that all standard transfer price methods have indirect transfer payments of the form

$$\tau_i(q_i, q^*, \rho^*_{-i}) = \rho^* q_i Y(q_i, q^*) + X(q_i, q^*) \tag{5}$$

where $Y(q^*, q^*) = 1$ and $X(q^*, q^*) = 0$. For a regular equilibrium quantity, q^*, two different methods will generate different equilibrium transfer prices only if they generate different expressions for $\partial\tau_i/\partial q_i$. Specifically, an increase in the convexity of Y and/or an increase in the concavity of X implies a larger marginal transfer payment and hence a smaller equilibrium transfer price.

3.2 Maximal-profit equilibria

Since all the transfer price methods support a continuum of regular symmetric equilibria, for each method we will focus on the equilibrium that maximizes firm (and hence industry) profit. Not only do all of the multinationals agree that this maximal-profit equilibrium is the best for each of them individually, it is also the equilibrium most consistent with the conventional idea of the comparability methods inducing tacit coordination among the multinationals. Thus, for each method chosen by the high-tax country, we need to find the value of q^* for which $\Pi_i^{**}(q^*) \equiv \Pi_i^*(q^*, q^*_{-i}, \rho^*(q^*)_{-i})$ is maximized. We denote these quantities by q^*_{CUP}, q^*_{CP}, and q^*_{RP}. Given (3),

$$\Pi_i^{**}(q^*) = (1-t)\pi_i(q^*, Q^*_{-i}) - (1-t^*)K(q^*) + (t - t^*)\tau(q^*) \tag{6}$$

where $\tau(q^*) \equiv \tau_i(q^*, q^*_{-i}, \rho^*(q^*)_{-i}) = \rho^*(q^*)q^*$.

Using (4) to solve for $\rho^*(q^*)$ and multiplying by q^* implies

$$(t - t^*)\tau(q^*) = \begin{cases} \Gamma(q^*)q^* & \text{if CUP} \\ \Gamma(q^*)q^* + (t - t^*)(C'(q^*)q^* - C(q^*)) & \text{if RP} \\ K(q^*)\Gamma(q^*)/K'(q^*) & \text{if CP} \end{cases} \tag{7}$$

where $\Gamma(q^*) = (1-t^*)K'(q^*) - (1-t)\partial\pi_i(q^*, Q^*_{-i})/\partial q_i$. For instance, with CUP, (4) implies that $-\Gamma(q^*) + (t - t^*)\rho^*(q^*) = 0$ so $(t - t^*)\rho^*(q^*)q^* = \Gamma(q^*)q^*$. A positive transfer price implies that $\Gamma(\cdot)$ is positive when $t > t^*$ and negative when $t < t^*$. Differences in $(t - t^*)\tau'(\cdot)$ will create the potential for each method to induce different maximal-profit equilibrium quantities.

Proposition 3. *Assume $C(\cdot)$ and $K(\cdot)$ are strictly convex. If $t > t^*$, then* $\tau_{CP}(q^*_{CP}) \leq \tau_{CUP}(q^*_{CUP}) < \tau_{RP}(q^*_{RP})$.

If $t < t^*$ and if downstream industry revenue, $P(Nq^*)Nq^*$, is concave in q^*, then

$\tau_{CP}(q^*_{CP}) \geq \tau_{CUP}(q^*_{CUP}) = \tau_{RP}(q^*_{RP}) = 0$.

Proof. See appendix.

When country D is the high-tax country, the CP method produces the smallest amount of profit-shifting (lowest value of τ). We know from Proposition 2 that CP induces the smallest transfer price holding output constant when costs are strictly convex. CP also induces the smallest maximal-profit equilibrium quantity because the convexity of production costs also causes $\rho^*(\cdot)$ to increase at a slower rate with respect to q^* than the other methods. (The reason for the weak inequality is that with enough firms, the maximal-profit equilibrium quantity will imply a zero transfer price for CP and CUP.) Another way to identify the source of CP's advantage begins by applying the Envelope Theorem to (6) which yields

$$\frac{d\Pi_i^{**}(q^*)}{dq^*} = (1-t)(N-1)\underbrace{\frac{\partial \pi_i(q^*, Q^*_{-i})}{\partial Q_{-i}}}_{\substack{\uparrow \\ \textit{Profit} \\ \textit{Effect}}} + \underbrace{(t-t^*)\frac{\partial \tau_i}{\partial \rho^*}\frac{d\rho^*}{dq^*}}_{\substack{\uparrow \\ \textit{Transfer Price} \\ \textit{Effect}}} + \underbrace{(t-t^*)\frac{\partial \tau_i(q^*, q^*_{-i}, \rho^*(q^*)_{-i})}{\partial q^*_{-i}}}_{\substack{\uparrow \\ \textit{Comparability} \\ \textit{Effect}}}. \tag{8}$$

Tacitly coordinating on an equilibrium with a higher quantity affects firm profits in three ways. The first is the profit effect. When all firms increase production, every firm's equilibrium operating profit falls due to our strategic substitutes assumption. The second effect is the transfer price effect. When higher values of q^* increase ρ^*, they also increase the profit shifted out of country D. The main difference between the methods is due to the third effect which we refer to as the comparability effect. Notice in Table 1 that τ_i does not depend on the output of other firms with CUP. With CP, increasing q^* while holding ρ^* fixed raises average upstream costs, implies a smaller mark-up, and lowers τ_i. With RP, increasing q^* while holding ρ^* fixed raises average downstream costs, implies a larger mark-up, and increases τ_i. Only CP generates a negative comparability effect which works to discourage tacit coordination on a high-output/high-transfer-price equilibrium relative to CUP and RP.[16]

When country *U* is the high-tax country, the effect of each method on the maximal-profit equilibrium quantity conflicts with the Proposition 2 effect on the transfer price. With regard to RP and CUP, while RP supports a higher transfer price than CUP (less profit-shifting) for the same output, it also supports a larger maximal-profit equilibrium quantity (more profit-shifting). In the proof of Proposition 3, we show that the quantity effect is so strong under CUP and RP that Π_i^{**} will be strictly increasing in q^* unless downstream industry revenue is sufficiently convex in q^*. Our assumption of concave industry revenue is thus only a sufficient condition for CP to shift fewer profits out of country *U* and is guaranteed as long as demand is not too convex. Under this mild restriction on demand, CUP and RP will result in a zero transfer price so only CP has the potential to avoid the conventional concern of extreme income-shifting. The following example shows that the effect of CP is sufficient to yield non-extreme transfer pricing.

Example 1. Assume $\pi_i(q_i, Q_{-i}^*) = (a - q_i - (n-1)q^*)q_i$ and $K(q) = \kappa q_i^2$. Π_i^{**} is strictly concave in q^* for $N > 1$ so $d\Pi_i^{**}/dq^* = 0$ implies $q_{CP}^* = a/(2(N-1))$. For any t and t^*, a zero transfer price corresponds to $q^* = \breve{q} = (1-t)a/((1-t)(N+1)+2(1-t^*)k)$ while, for $t > t^*$, the maximum regular equilibrium transfer price occurs at $q^* = \bar{q} = a/(N+1)$. q_{CP}^* is strictly less than $\bar{q}$ when $N > 3$ and q_{CP}^* is strictly less than $\breve{q}$ when $N > 3 + 2(1-t^*)k/(1-t)$. For $t > t^*$, a few firms can be sufficient to avoid extreme profit-shifting out of country *D*. In fact, if *N* is large enough, the CP method can actually reverse the tax differential incentive to shift profit out of country *D* and result in a net incentive for the firms to shift profit into country *D*. For $t < t^*$, the CP method will also discourage extreme profit-shifting out of the high-tax country (now

[16]The alternative assumption of strategic complements in the final good market will not change the differences in the comparability effect. The profit effect will become positive which will make a corner solution at $\bar{q}$ more likely.

country U) with at least $3 + 2(1-t^*)k/(1-t)$ firms.

3.3. Arm's-Length Pricing and Tax Revenues.

Propositions 2 and 3 point to the CP method as the preferred method for minimizing the amount of profit-shifting out of the high-tax country. In this section, we ask whether the amount of profit-shifting under CP is consistent with the stated objective of transfer price regulations: arm's-length pricing. We then examine the effect of using the CP method on each country's tax revenues.

3.3.1 Arm's-Length Pricing

In the United States, IRS transfer price regulations begin with "The purpose of section 482 is to ensure that taxpayers clearly reflect income attributable to controlled transactions, and to prevent the avoidance of taxes with respect to such transactions." (U.S. Department of Treasury (1994), §1.482-1(a), pp. 34990) "In determining the true taxable income of a controlled taxpayer, the standard to be applied in every case is that of a taxpayer dealing at arm's length with an uncontrolled taxpayer." (§1.482-1(b)). Article 9 of the OECD Model Tax Convention expresses a similar intent (OECD (2001), p. G-1). The concept of an arm's-length price has been interpreted as representing the price that would arise in a competitive market for the intermediate good. In a symmetric, competitive equilibrium, the downstream firms would choose their quantities taking the intermediate good price, ρ, as given by setting $\partial\pi_i/\partial q_i = \rho$ and upstream firms would choose their quantities taking the intermediate good price as given by setting $K'(q_i) = \rho$. In equilibrium, $\partial\pi_i/\partial q_i$ equals $K'(q_i)$. Let q^e denote this equilibrium quantity. It is also the equilibrium quantity that would arise if t was equal to t*. The equilibrium transfer price would then equal $\rho^e = K'(q^e)$.

In the absence of tax-motivated profit-shifting, each firm should be expected to declare transfer price payments of $\tau^e = K'(q^e)q^e$. In general, CP will not result in an arm's-length price but it will come closer to an arm's-length price than CUP and RP. When $t > t^*$, $\tau_{CP}(\bar{q}) = (1-t^*)K(\bar{q})/(t-t^*)$ is greater than $K(q^e)$. So for N sufficiently small, $\tau_{CP}(q^*_{CP})$ will be greater than τ^e indicating tax-motivated profit-shifting out of country D. This is illustrated in Figure 1 which plots $\tau_{CP}(q^*_{CP})$ (solid line), τ^e (dot-dashed line), and $\tau_{CP}(\bar{q})$ (dashed line) for example 1 when t=.5 and t*=.2. Figure 1 also shows that with enough firms $\tau_{CP}(q^*_{CP})$ can be less than τ^e. For $N > 8$, the maximal-profit equilibrium transfer price will be zero. Thus, with enough firms, the CP method can result in profit-shifting into country D even though it is the high-tax country. Figure 2 illustrates the relationship between $\tau_{CP}(q^*_{CP})$ (solid line) and τ^e (dot-dashed line) for example 1 when t = .2 and t* = .5. ($\bar{q}$ is not relevant when $t < t$*). CP will induce a positive

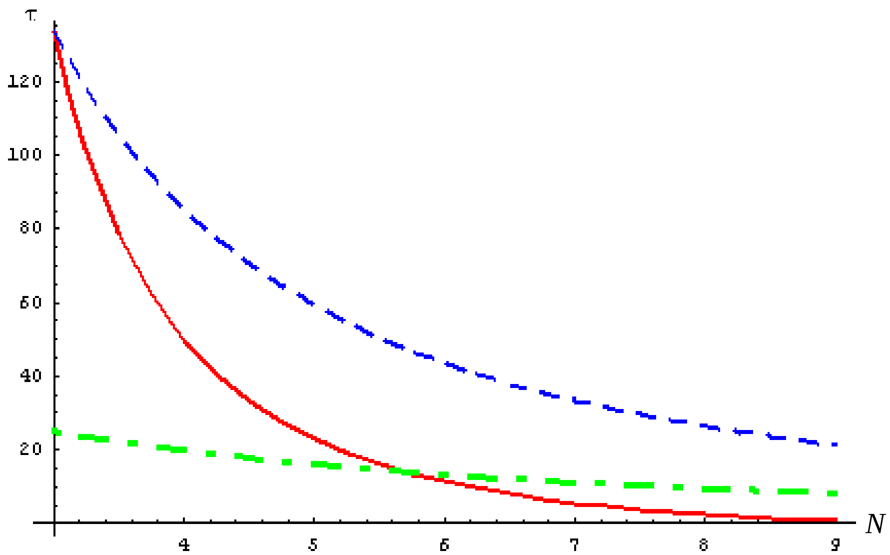

Figure 1

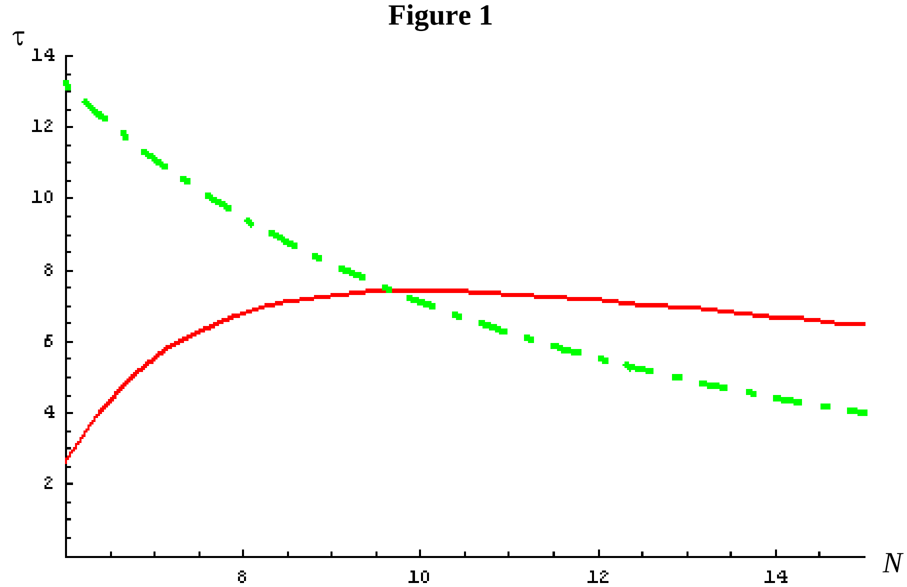

Figure 2

profit-maximal equilibrium transfer price for $N > 5$. For N a little greater than 5, $\tau_{CP}(q^*_{CP})$ will be less than τ^e indicating profit-shifting into low-tax country *D*. However, with 10 or more firms, $\tau_{CP}(q^*_{CP})$ will be greater than τ^e indicating profit-shifting into country *U*. So again we see that the CP method can induce profit-shifting into the high-tax country.

3.3.2 Tax Revenues

Tax revenues for country *D* equal $t(\pi_i(q_i^*, q_{-i}^*) - \tau_{CP}(q^*))$ and tax revenues for country *U* equal $t^*(\tau_{CP}(q^*) - K(q^*))$. Since the tax offices responsible for setting transfer price regulations in OECD countries do not set the country's tax rate, we fix *t* and *t** and ask whether using the CP method increases the high-tax country's tax revenues relative to the extreme transfer pricing with CUP or RP.

Proposition 4. *If equilibrium downstream industry revenue, P(Nq*)Nq*, is concave in q*, then when t > t*, country D collects strictly higher tax revenues under CP than under CUP or RP and when t < t*, country U collects strictly higher tax revenues under CP than under CUP or RP as long as* $\rho^*_{CP}(q^*_{CP}) > 0$.

Proof. See appendix.

The key to proving Proposition 4 is showing that the regular equilibrium that maximizes the high-tax country's tax revenues generates a quantity less than q^*_{CP}. This is always true because at q^*_{CP} downstream profit, $\pi_i(q^*, Q^*_{-i})$, is strictly decreasing in *q**. Since $d\pi_i(q^*, Q^*_{-i})/dq^* = \partial\pi_i/\partial q_i + (N-1)\partial\pi_i/\partial Q_{-i}$, the only way $\pi_i(q^*, Q^*_{-i})$ can be decreasing in *q** is if the negative effect on downstream profit due to strategic substitution dominates the direct effect a firm gets from moving to a lower-quantity equilibrium. Since a lower equilibrium quantity implies a reduction in profits shifted out of the high-tax country, it can only increase global after-tax firm profit if the net effect of all firms producing less in the downstream market is an increase in downstream profit.

3.4 The response of the low-tax country.

The conventional concern by a high-tax country of using a standard comparison method to audit for tax-induced profit-shifting in a vertically integrated industry is that it will result in extreme profit-shifting out of the high-tax country. Propositions 2-4 show that this concern need not be correct if CP is used. However, example 1 also shows that with enough firms in the industry CP can actually result in profit-shifting into the high-tax country. One would expect a response by the low-tax country to counter such an incentive. It turns out that options available to the low-tax country are limited given the direct incentive firms face to shift profits out of the high-tax country.

For any regular equilibrium, every firm will meet the arm's-length standard and avoid penalties and adjustments. No matter what financial statistic a low-tax country uses, the firms will report identical values in equilibrium. Thus, in order to prevent profit-shifting out of the low-tax country, the policies of the low-tax country must change the maximal-profit equilibrium. Clearly, this will not happen if the low-tax country uses the same method as the high-tax country. Pairing the CUP or RP method with the CP method will not help either.

Figure 3 illustrates the incentives created when the low-tax country uses the CUP or RP method and the high-tax country uses the CP method. For a given ρ* and q^*, each line represents the combination of ρ_i and q_i for which $F_i = F_i^*$ for each method. The lines intersect at (ρ^*,q^*). If $t > t^*$, country D will penalize any firm that chooses (ρ_i,q_i) above the CP line and country U will penalize any

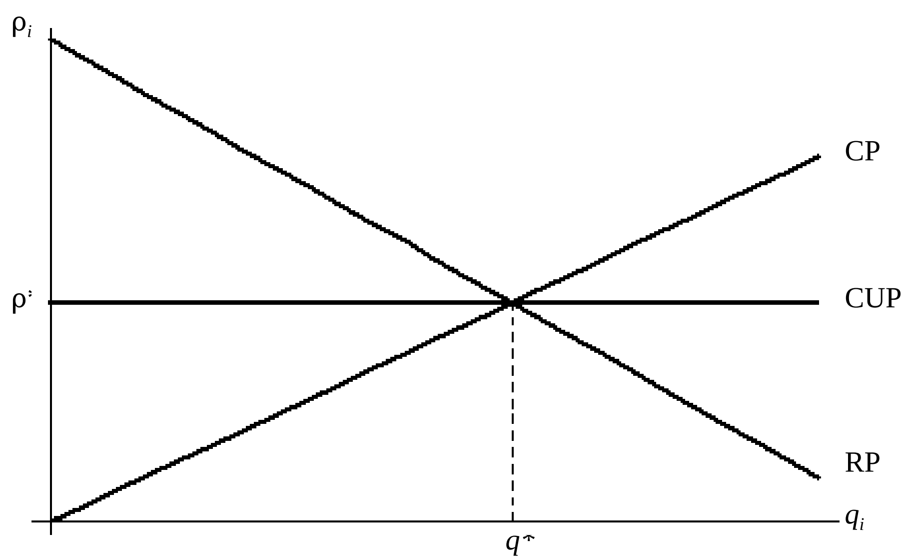

Figure 3

firm that chooses (ρ_i,q_i) above either the CUP line or the RP line. For $q_i < q^*$, the profit-maximizing value of ρ_i for firm *i* will be above the CP line only if country *U* 's penalty (ϕ_U) is big enough relative to country *D*'s penalty (ϕ_D). Following the analysis of Proposition 1, increasing ρ_i above the CP line causes firm *i* to incur a per unit penalty from country *D* equal to $\phi_D\, t + t^*$ but it also decreases the per unit penalty paid to country *U* by $\phi_U\, t^* + t$. Firm *i* will be willing to incur the country *D* penalty in order to avoid the country *U* penalty as long as ϕ_U is sufficiently large relative to ϕ_D. For $q_i > q^*$, the firm can and will avoid penalties from both countries by choosing ρ_i on the CP line. For (ρ^*,q^*) to be an equilibrium, we now require $\partial\Pi_i^*/\partial q_i \leq 0$ based on CP for $q_i > q^*$ and $\partial\Pi_i^*/\partial q_i \geq 0$ based on CUP or RP for $q_i < q^*$. For any q^*, the inequalities in the first-order conditions imply there will now exist a range of equilibrium values of ρ^*. The maximal-profit equilibrium within this range will correspond to the largest value of ρ^* which is the same value used in the prior CP analysis. If $t < t^*$, the use of CUP or RP by the low-tax country will now affect the Proposition 1 analysis for $q_i > q^*$ and will again imply a range of transfer prices consistent with each equilibrium quantity. Once again, the maximal-profit equilibrium will coincide with that from the prior CP analysis. To affect the maximal-profit equilibrium when the high-tax country uses CP, the low-tax country would need a statistic that generates a steeper iso-value line than CP. None of the standard methods do.

The only provision for limiting the extent to which profits would be shifted out of the low-tax country would be to invoke what Ault and Bradford (1990) refer to as a "commensurate with income" standard. This would allow the low-tax country to impose penalties a priori on transfer prices that imply negative pre-tax profit.

4. Private cost information

Up to this point, the analysis has relied extensively on the fact that all the firms were identical. In this section, we introduce private and heterogeneous costs and show that doing so strengthens the performance of the CP method. Private information has the potential to greatly increase the number of equilibria. We will show that this does not happen with the CP method due to an equilibrium condition we call "incentive comparability."

Let upstream costs now equal $K(q_i,\theta_i)$ where θ_i is a firm-specific parameter known only to firm i such that $K_{\theta_i} > 0$ and $K_{q\theta_i} > 0$. We assume $\theta_i \in \{L,H\}$ with $L < H$ and $\theta_i = L$ with probability λ. Since the incentive for CP to shift the most profit out of the high-tax country occurs when there are two firms, we assume that $N = 2$. Then for the CP method,

$$\tau_i(q_i,q^*(\theta_j),\rho^*(\theta_j),\theta_i) = \frac{\rho^*(\theta_j)q^*(\theta_j)}{K(q^*(\theta_j),\theta_j)} \cdot K(q_i,\theta_i)$$

where the scalar functions $\rho^*(\theta_i)$ and $q^*(\theta_i)$ denote a proposed transfer price/output strategy.

For brevity we will present the analysis when $t > t^*$. Given $(\rho^*(\theta_j),q^*(\theta_j))$, firm i's seeks to maximize its expected profit,

$$E_{\theta_j}\Pi_i(\rho_i,q_i,\theta_i,\theta_j) = (1-t)E_{\theta_j}\pi_i(q_i,q^*(\theta_j)) - (1-t^*)K(q_i,\theta_i) + (t-t^*)\rho_i q_i - E_{\theta_j}Penalty, \tag{9}$$

where

$$E_{\theta_j}Penalty = (1+\phi)tq_i[\lambda \max\left\{0,\rho_i - \frac{\rho^*(L)q^*(L)}{K(q^*(L),L)} \cdot \frac{K(q_i,\theta_i)}{q_i}\right\} + (1-\lambda)\max\left\{0,\rho_i - \frac{\rho^*(H)q^*(H)}{K(q^*(H),H)} \cdot \frac{K(q_i,\theta_i)}{q_i}\right\}]. \tag{10}$$

[17]

With probability λ, firm i faces a low-cost competitor and with probability 1-λ, it faces a high-cost competitor. In (10), a penalty is imposed on firm i when it has a larger cost mark-up than firm j.

Firm i's best response depends on the order of its competitor's low-type and high-type mark-ups. Three cases are possible:

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[17]Because there are only 2 firms, the value of each firm's statistic defines both the statistic value above which a penalty is imposed, F^*, and the transfer price adjustment, $\bar{F}$. That is, firm 1 will be penalized if $F_1 > F_2$, and in this case, firm 1's transfer price is adjusted so that $F_1 = F_2$.

i) $\rho^*(L)q^*(L)/K(q^*(L),L) > \rho^*(H)q^*(H)/K(q^*(H),H)$,

ii) $\rho^*(L)q^*(L)/K(q^*(L),L) < \rho^*(H)q^*(H)/K(q^*(H),H)$, and

iii) $\rho^*(L)q^*(L)/K(q^*(L),L) = \rho^*(H)q^*(H)/K(q^*(H),H)$.

If $(\rho^*(\theta_j),q^*(\theta_j))$ implies case (i), then

$$\frac{\partial E_{\theta_j}\Pi_i}{\partial\rho_i} = \begin{cases} (t - t^*)q_i & \text{if } \dfrac{\rho_i q_i}{K(q_i,\theta_i)} \leq \dfrac{\rho^*(H)q^*(H)}{K(q^*(H),H)} \\ [1 - (1-\lambda)(1+\phi)]t - t^* & \text{if } \dfrac{\rho^*(H)q^*(H)}{K(q^*(H),H)} < \dfrac{\rho_i q_i}{K(q_i,\theta_i)} \leq \dfrac{\rho^*(L)q^*(L)}{K(q^*(L),L)} \\ -(\phi t + t^*)q_i & \text{if } \dfrac{\rho_i q_i}{K(q_i,\theta_i)} > \dfrac{\rho^*(L)q^*(L)}{K(q^*(L),L)}. \end{cases} \tag{11}$$

Which mark-up, firm i will seek to match depends on the probability of facing a low-cost competitor. If $[1-(1-\lambda)(1+\phi)]t - t^* > 0$, firm i's optimal choice of ρ_i implies $\rho_i q_i/K(q_i,\theta_i) = \rho^*(L)q^*(L)/K(q^*(L),L)$ which in equilibrium implies case (iii) for $\theta_i = H$. On the other hand, if $[1-(1-\lambda)(1+\phi)]t - t^* < 0$, then $\rho_i q_i/K(q_i,\theta_i) = \rho^*(H)q^*(H)/K(q^*(H),H)$ which in equilibrium also implies case (iii) for $\theta_i = L$. Together these two observations imply that case (i) cannot arise in equilibrium. Similar arguments show that case (ii) cannot arise in equilibrium. They also imply that case (iii) will be an equilibrium configuration.

Proposition 5. *In any symmetric equilibrium under CP*

$$\rho^*(L)q^*(L)/K(q^*(L),L) = \rho^*(H)q^*(H)/K(q^*(H),H). \tag{12}$$

We call (12) an "incentive comparability" constraint. For CUP, the analog to (12) is $\rho^*(L) = \rho^*(H)$; for RP no equivalent equilibrium condition need arise. Thus, introducing private information with RP will increase the dimensionality of the set of equilibria. With CP and CUP, the set of equilibria is still indexed by single parameter, just as in the complete information case.

For each θ_i, let $\rho_i(\theta_i)$ denote firm i's best-response transfer price to $(\rho^*(\theta_j),q^*(\theta_j))$. Using (12), let the common value of firm j's statistic be denoted by $\mathscr{F}$. Then for $\theta_i \in \{L,H\}$, $\rho_i(\theta_i) = \mathscr{F}K(q_i(\theta_i),\theta_i)/q_i(\theta_i)$, $\tau_i(q_i,\theta_i,\mathscr{F}) = \mathscr{F}K(q_i,\theta_i)$, and regular equilibria are defined by

$$\frac{\partial E_{\theta_j}\Pi_i^*}{\partial q_i} = (1-t)[\lambda\partial\pi_i(q^*(\theta_i),q^*(L))/\partial q_i + (1-\lambda)\partial\pi_i(q^*(\theta_i),q^*(H))/\partial q_i] \\ - [1-t^* + \mathscr{F}(t^* - t)]\partial K(q^*(\theta_i),\theta_i)/\partial q_i = 0. \tag{13}$$

Repeating the analysis of section 3 would involve maximizing expected equilibrium firm profit with respect to $\mathscr{F}$ (with the expectation taken over θ_i and θ_j) and will yield qualitatively similar results. Instead, we wish to highlight an aspect of (12) that enhances the control of the tax authority in the high-tax country. In optimal regulation papers such as Baron and Myerson (1982), the regulator is presumed to have sufficient information to specify the profit of the firm with the highest possible cost after which incentive compatibility determines the profits of any lower cost firm. In the same spirit, (12) shows that if the high-tax country can specify the maximum acceptable statistic value for a firm claiming to have the highest possible costs, in equilibrium the high-cost and low-cost firms will produce the same statistic value. Now instead of resolving the multiple equilibrium problem by focusing on the firms' preferred equilibrium, the choice of a statistic value for the high-cost firm by the high-tax country allows the high-tax country to implement its preferred equilibrium. In fact, choosing $\mathscr{F}$ close to one will result in equilibrium transfer prices below their arm's-length levels. So even with just two firms, the CP method can lead to profit-shifting into the high-tax country.

5. Concluding Remarks

United States transfer price regulations specifically proscribe the use of data from competing multinationals to audit each other's transfer prices for fear of inducing tacit coordination that facilitates extreme tax-induced profit-shifting. OECD guidelines acknowledge that there may be some benefit to using data from such "controlled" transactions, but do not offer any guidance on how best to use this information. Both positions leave tax authorities with no systematic process for auditing transfer prices in vertically integrated industries. Our paper studies the equilibrium incentives created by standard transfer price methods in a vertically integrated industry to determine which encourage aggressive tax-induced transfer pricing and which discourage such behavior. While the tacit coordination incentives of concern are present in our model, we show each of the standard methods produces different coordination incentives based on the economic relationship between the tax savings a multinational earns via profit-shifting and the operating profit it earns in its product market. On the margin, a multinational will produce the quantity for which its after-tax downstream operating profit plus its marginal tax savings from shifting profit out of the high-tax country equals its after-tax upstream production costs. A higher transfer price increases the marginal tax savings and encourages more final good production. When each multinational operating in the final good market behaves the same way, each firm's increased output imposes a revenue destruction externality on its competitors. If the revenue destruction effect is large enough, in equilibrium it will offset any tax savings from a higher transfer price.

Our analysis reveals that the Cost-Plus method generates the strongest revenue destruction incentives and thus discourages the multinationals from coordinating on transfer prices that shift a lot of profit out of the high-tax jurisdiction. Surprisingly, this is true regardless of whether the upstream divisions or the downstream divisions face the higher tax rate. Moreover, our analysis reveals that tax revenues are the largest when the high-tax country uses the Cost-Plus method. We conclude that the conventional intuition regarding the use of standard transfer price regulations in the absence of independent comparable data is incomplete because it ignores the strategic externalities that can exist between an integrated firm's upstream and downstream markets.

Allowing for private cost information only serves to reinforce the attractiveness of the Cost-Plus method. Our analysis identifies an equilibrium property of comparability methods applied to vertically integrated competitors which we refer to as "incentive comparability." This equilibrium condition strengthens the strategic effects of the Cost-Plus method beyond those identified in our complete information analysis by linking the mark-up values of low-cost firms to the mark-up values of high-cost firms. With two multinationals and two cost types, the low-cost and the high-cost mark-ups will be equal. With two multinationals and more than two cost types, the low-cost and high-cot mark-ups will satisfy an inequality condition. In either case, the private information incentive properties of the Cost-Plus method strengthen a high-tax country's ability to limit outbound profit-shifting. We leave the case of three or more firms for future research.

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Appendix

Proof of concavity of Π_i^* *with respect to* q_i.

CUP: Since τ_i is linear in q_i, the strict concavity of π_i and the convexity of $K(\cdot)$ imply that $\Pi_i^*(q_i, q_{-i}^*, \rho^*(q^*)_{-i})$ is strictly concave in q_i for all q^*.

CP: From Table 1,

$$\Pi_i^* = (1-t)\pi_i(q_i, Q_{-i}^*) - (1-t^*)K(q_i) + (t-t^*)\rho^* q^* K(q_i)/K(q^*) \tag{A.1}$$

and from (4),

$$(t-t^*)\rho^*(q^*)q^* = [(1-t^*)K'(q^*) - (1-t)\partial\pi_i(q^*, Q_{-i}^*)/\partial q_i]K(q^*)/K'(q^*). \tag{A.2}$$

Together (A.1) and (A.2) imply

$$\Pi_i^* = (1-t)\pi_i(q_i, Q_{-i}^*) - (1-t)\partial\pi_i(q^*, Q_{-i}^*)/\partial q_i \cdot K(q_i)/K'(q^*). \tag{A.3}$$

When $t < t^*$, $\rho^*(q^*)$ will be positive if, and only if, the bracketed term in (A.2) is negative. This implies $\partial\pi_i(q^*, Q_{-i}^*)/\partial q_i$ must be positive. Hence, (A.3) must be strictly concave in q_i for all q^*. When $t > t^*$, the bracketed term must be positive. Thus, a sufficient condition for (A.3) to be globally concave in q_i is $\partial\pi_i(q^*, Q_{-i}^*)/\partial q_i \geq 0$.

RP: From Table 1,

$$\Pi_i^* = (1-t)(P(q_i+Q_{-i}^*)q_i - C(q_i)) - (1-t^*)K(q_i) + (t-t^*)(\rho^* q_i - C(q_i) + C(q^*)q_i/q^*)$$

or

$$\Pi_i^* = (1-t)P(q_i+Q_{-i}^*)q_i - (1-t^*)C(q_i) - (1-t^*)K(q_i) + (t-t^*)(\rho^* q_i + C(q^*)q_i/q^*). \tag{A.4}$$

The concavity of downstream revenues and the convexity of $C(\cdot)$ and $K(\cdot)$ imply that (A.4) is globally concave in q_i for all q^*.

Proof of Proposition 2. For any method, $\tau_i(q^*, q_{-i}^*, \rho_{-i}^*) = \rho^*(q^*)q^*$ in a regular symmetric equilibrium and $\rho^*(q^*)$ is defined by (4). Let $\gamma(q^*) = [(1-t^*)K'(q^*) - (1-t)\partial\pi_i(q_i^*, Q_{-i}^*)/\partial q_i]/(t-t^*)$ and let $\delta(q^*) = C'(q^*)q^* - C(q^*)$. $\rho^*(q^*) \geq 0$ implies $\gamma(q^*) \geq 0$ and the strict convexity of $C(\cdot)$ implies $\delta(q^*) > 0$. Using (4), $\rho_{CUP}^* - \rho_{CP}^* = \gamma(q^*)(1 - K(q^*)/(q^* K'(q^*))) \geq 0$ implies $\rho_{CUP}^* \geq \rho_{CP}^*$ and

$\rho^*_{CUP} - \rho^*_{RP} = -\delta(q^*) < 0$ implies $\rho^*_{CUP} < \rho^*_{RP}$. ***Q.E.D.***

Proof of Proposition 3.

*CUP vs. CP when t > t**. First note that (7) implies

$$(t - t^*)(\tau'(q^*)|_{\mathrm{CUP}} - \tau'(q^*)|_{CP}) = \Gamma'(q^*)(q^* - K(q^*)/K'(q^*)) + \Gamma(q^*)K(q^*)K''(q^*)/(K'(q^*)^2). \tag{A.5}$$

Since the firms' final goods are strategic substitutes, $\Gamma'(\cdot)$ is positive and since $t > t^*$, $\Gamma(\cdot)$ is positive. Thus, (A.5) is positive as long as $K(\cdot)$ is strictly convex. Given (6), this means $q^*_{CP} \le q^*_{CUP}$. Next, differentiating (4) with respect to q^* implies

$$\rho^{*\prime}(q^*) = \frac{\partial^2\Pi_i/\partial q_i^2 + (1-t)(N-1)\partial^2\pi_i/\partial q_j\partial q_i + (t-t^*)\partial^2\tau_i/\partial q^*_{-i}\partial q_i}{-(t-t^*)\partial^2\tau_i/\partial\rho^*\partial q_i}. \tag{A.6}$$

The denominator of (A.6) is strictly negative. Since $\partial^2\tau_i/\partial q^*_{-i}\partial q_i$ is zero under CUP and negative under CP, the numerator is also strictly negative. Thus $\rho^{*\prime}(q^*)$ is strictly positive for both CUP and CP. Therefore, $\tau_{CP}(q^*_{CP}) = \rho^*_{CP}(q^*_{CP})q^*_{CP} \le \rho^*_{CP}(q^*_{CUP})q^*_{CUP} \le \rho_{CUP}(q^*_{CUP})q^*_{CUP} = \tau_{CUP}(q^*_{CUP})$.

*CUP vs. RP when t > t**. First, (7) implies $(t - t^*)(\tau'_{CUP}(q^*) - \tau'_{RP}(q^*)) = -(t - t^*)C''(q^*)q^* < 0$ where the strict inequality is due to the strict convexity of $C(\cdot)$. Thus, $q^*_{CUP} \le q^*_{RP}$. Second,

$$\tau_{CUP}(q^*_{CUP}) - \tau_{RP}(q^*_{RP}) = \tau_{CUP}(q^*_{CUP}) - \tau_{CUP}(q^*_{RP}) + \tau_{CUP}(q^*_{RP}) - \tau_{RP}(q^*_{RP}). \tag{A.7}$$

The first difference on the right-hand side of (A.7) is non-positive because $\tau_{CUP}(q^*)$ is increasing in q^* and the second difference is negative by Proposition 2.

*t < t**. The above comparison between CUP and RP shows that with $t < t^*$, (7) still implies $q^*_{CUP} \le q^*_{RP}$. So consider Π_i^{**} under CUP. Explicitly accounting for downstream revenues and costs gives us

$$\Pi_i^{**}(q^*)|_{CUP} = (1-t)(R_i(q^*, q^*_{-i}) - C(q^*)) - (1-t^*)K(q^*)$$
$$+ (1-t^*)K'(q^*)q^* - (1-t)(\partial R_i(q^*, q^*_{-i})/\partial q_i - C'(q^*))q^*$$

or

$$\Pi_i^{**}(q^*)|_{CUP} = (1-t)(R_i(q^*,q_{-i}^*) - \partial R_i(q^*,q_{-i}^*)/\partial q_i \cdot q^*) \\ - (1-t^*)(K(q^*) - K'(q^*)q^*) + (1-t)(C'(q^*))q^* - C(q^*)). \tag{A.8}$$

Therefore,

$$d\Pi_i^{**}(q^*)/dq^*|_{CUP} = (1-t)(\partial R_i(q^*,q_{-i}^*)/\partial q_{-i}^* - (d/dq^*)(\partial R_i(q^*,q_{-i}^*)/\partial q_i) \cdot q^*) \\ + (1-t^*)K''(q^*)q^* + (1-t)C''(q^*))q^*. \tag{A.9}$$

Since $R_i(q_i,q_{-i}^*) = P(q_i + (N-1)q^*)q_i, \partial R_i(q^*,q_{-i}^*)/\partial q_{-i}^* = (N-1)P'(Nq^*)q^*$ and $\partial R_i(q^*,q_{-i}^*)/\partial q_i = P(Nq^*) + P'(Nq^*)q^*$. Concave industry revenue implies $NP''(Nq^*)q^* < -2P'(Nq^*)$ which in turn implies that the first line of (A.9) is positive. Thus, $d\Pi_i^{**}/dq^*|_{CUP}$ is strictly positive for all q*. Hence, q_{CUP}^* must equal $\breve{q}$. $\rho_{CUP}^*(\breve{q}) = 0$ so $\tau_{CUP}(q_{CUP}^*) = 0$. Turning to the RP method, it is straightforward to show that under the same assumptions on industry revenue, $d\Pi_i^{**}/dq^*|_{RP}$ is strictly positive for all q*. Thus, the profit-maximal equilibrium under RP will also imply $\rho_{RP}^*(q_{RP}^*) = 0$. Finally, Example 1 shows that it is possible for CP to generate a positive transfer price under the same circumstances that lead CUP and RP to generate a zero transfer price. ***Q.E.D.***

Proof of Proposition 4. For fixed tax rates $t > t$*, country *D*'s tax revenues are maximized when

$$\partial\pi_i/\partial q_i + (N-1)\partial\pi_i/\partial Q_{-i} - \tau_{CP}'(q^*) = 0. \tag{A.10}$$

Evaluating the left-hand side of (A.10) at q_{CP}^* yields marginal tax revenue for *D* of

$$(1-t^*)(\partial\pi_i/\partial q_i + (N-1)\partial\pi_i/\partial Q_{-i} - K')/(t-t^*). \tag{A.11}$$

Similarly, for fixed tax rates $t < t$*, country *U* 's tax revenues are maximized when

$$\tau_{CP}'(q^*) - K'(q^*) = 0. \tag{A.12}$$

Evaluating the left-hand side of (A.12) at q^*_{CP} yields marginal tax revenue for U of

$$(1-t)(K' - \partial\pi_i/\partial q_i - (N-1)\partial\pi_i/\partial Q_{-i})/(t-t^*). \tag{A.13}$$

The proof consists of evaluating (A.11) and (A.13) in three steps.

Step 1. In this step, we show tax revenues are at least as high under CP as under CUP and RP when q^*_{CP} is equal to the largest value consistent with a regular equilibrium. Suppose $t > t^*$. The left-hand side of (A.10) is strictly negative at $\bar{q}$. Thus, country D's tax revenues cannot be maximized at $\bar{q}$. If $q^*_{CP} = \bar{q}$, then by Proposition 2, CP must result in larger tax revenues for country D than CUP or RP. Now suppose $t < t^*$. The left-hand side of (A.12) is strictly negative at $\breve{q}$. Thus, country U's tax revenues cannot be maximized at $\breve{q}$. However, if $q^*_{CP} = \breve{q}$, then by Proposition 3, country U tax revenues will be the same with all three methods.

Step 2. We show in this step that $d\pi_i(q^*, Q^*_{-i})/dq^* = \partial\pi_i/\partial q_i + (N-1)\partial\pi_i/\partial Q_{-i}$ must be negative at q^*_{CP} whenever $q^*_{CP} < \bar{q}$ for $t > t^*$ and whenever $q^*_{CP} < \breve{q}$ for $t < t^*$. Suppose to the contrary that at $q^*_{CP} < \bar{q}$ when $t > t^*$ or at $q^*_{CP} < \breve{q}$ when $t < t^*$, $d\pi_i/dq^* \geq 0$. Then from (6), CP implies

$$d\Pi^{**}_i/dq^* = (1-t)(d\pi_i/dq^* - (K/K')(\partial^2\pi_i/\partial q_i^2 + (N-1)\partial^2\pi_i/\partial Q_{-i}\partial q_i) + (KK''/(K')^2)\partial\pi_i/\partial q_i). \tag{A.14}$$

For all regular equilibrium values of q^*, the first and third terms in (A.14) are non-negative and the second term is strictly positive. But this means q^*_{CP} could not have maximized Π^{**}_i. Because downstream industry revenue is concave in q^* so too is $\pi_i(q^*, Q^*_{-i})$. Hence, we must have that $d\pi_i/dq^* < 0$ for the values of q^*_{CP} considered in this step.

Step 3. By Step 2, (A.11) and (A.13) are both negative at q^*_{CP}. Thus, $q^{DTR} < q^*_{CP}$ and $q^{FTR} < q^*_{CP}$. Concave equilibrium downstream industry revenue then implies that the left-hand sides of (A.10) and (A.12) are negative for all $q^* \geq q^*_{CP}$. Thus, when country D is the high-tax country, its revenues are increasing as q^* falls from $\bar{q}$ to q^*_{CP} and when country U is the high-tax country, its revenues are increasing as q^* falls from $\breve{q}$ to q^*_{CP}. ***Q.E.D.***