

Formula Apportionment vs. Separate Accounting:
A Private Information Perspective¹

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Abstract: In 2002, the European Commission recommended that member countries use formula apportionment procedures to tax multinational companies. This departure from the standard separate accounting (transfer pricing) approach is an attempt to reduce the costs and distortions associated with auditing transfer prices. Unfortunately, apportionment formulas create their own economic distortions and, contrary to popular belief, they do not eliminate distortions due to asymmetric information between the multinational and the national tax authorities. In this paper, I explicitly model the role of private information in two tax competition games: one in which tax liabilities are calculated under formula apportionment and one in which tax liabilities are calculated under separate accounting and transfer prices are audited. Switching to a formula apportionment system affects the after-tax profit of multinationals and the tax revenues paid by both domestic and foreign firms. The direction and magnitude of the changes depend on the accuracy of the auditing technology and non-monotonically on multinational costs. The switch will have different effects on the tax receipts from domestic and foreign firms.

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1. Introduction.

What is the best system for taxing multinational firms? Recent efforts to develop an integrated EU tax policy have increased interest in this longstanding question. The predominant system used worldwide, separate accounting (SA), calculates national tax liabilities of multinationals by treating subsidiaries located in different countries as distinct companies. Each subsidiary calculates its tax liability based on the laws of its host country and any transactions between subsidiaries of the same multinational are valued for tax purposes by transfer prices. In 2002, the European Commission (European Commission, 2002) proposed four alternatives to SA. Three of the proposals (HST, CCTB, and CHTB) adopt a formula apportionment (FA) approach while the fourth proposes an EU-wide tax.² Under FA, a multinational would report its EU-wide taxable income to every EU country in which it is active and this income would be allocated among each EU country for tax purposes based on a formula that could use a variety of relative cost and revenue ratios.³

An important feature of FA is a reduced reliance on transfer prices which has several effects. First, as Mintz (2004) argues FA creates lower compliance costs as transfer prices on intra-EU transactions no longer need to be audited. Moreover, increased EU integration is expected to make auditing less effective and more costly. Second, Hellerstein and McLure (2004) argue that FA creates its own valuation issues the shift away from SA was supposed to avoid. Third, numerous authors have pointed out that while SA distorts production and pricing decisions through income shifting, FA does so

²The Home State Taxation plan (HST) allows a company with headquarters in the EU to calculate its EU-wide income using its home country tax laws. The Common Consolidated Tax Base plan (CCTB) proposes an EU-wide set of tax base definitions that each company could instead use. The Compulsory Harmonized Tax Base (CHTB) would mandate a single tax base definition. All three plans would allocate a firm's total EU income across member countries by using an apportionment formula.

³For excellent descriptions and critiques of all four proposals, see Mintz and Martens-Weiner (2003), Zodrow (2004), Devereux (2004), Hellerstein and McLure (2004), Mintz (2004), and Sørensen (2004). In December 2005, the Commission approved a voluntary pilot program in which small to medium size companies could use the HST plan (European Commission (2005b)).

through revenue shifting and its effects on factor returns.⁴ Fourth, SA and FA create different tax competition incentives for each country. In a complete information, homogeneous firm model in which multinationals face a reduced form cost of income shifting, Nielsen, Raimondos-Møller, and Schjelderup (2009) show that a shift from SA to FA can raise or lower equilibrium tax rates and/or tax revenues.⁵ Sørensen (2003) imposes more structure on the production functions of the multinationals than Nielsen et al. (2009) but still finds that the net tax competition externalities and the equilibrium changes due to a shift from SA to FA can go either way. Introducing trade costs as in Kind, Midelfart, and Schjelderup (2005) also yields qualitatively similar results. Thus, except for lower compliance costs, these properties suggest that a shift from SA to FA may but need not improve welfare.

Missing from the above papers is the fact that taxing multinationals is inherently a problem of firm heterogeneity and private information, with SA and FA being two possible solutions. If host governments were fully informed about the economic structure of each multinational, tax codes could easily be developed to avoid strategic tax planning distortions. Because host governments could never hope to be that well informed, their tax policies must account for such distortions. While the above complete information models are important for identifying some of the output, factor, and tax competition distortions generated by SA and FA, they were not set up to capture the information rent distortions explicitly present with heterogeneous, privately informed firms under SA and but also present, perhaps less obviously, with FA. Thus, this paper expands on the above literature by identifying the economic differences between SA and FA that are directly attributable to private information.

Section 2 presents a model of tax competition between two countries over the profits of two multinationals. To introduce a role for transfer prices, each multinational will produce an intermediate good in its home country, the cost of which is the firm's private information. Under SA, the transfer prices used to value the intermediate goods produced for foreign production will be subject to auditing. Unlike the majority of auditing papers, the governments have an auditing technology which yields an unbiased but noisy signal of a firm's true cost and which creates incentive effects linked to a firm's compliance efforts under SA. As Baron and Besanko (1984) demonstrate, auditing rules create

⁴Gordon and Wilson (1986) were the first to identify and characterize the factor-return distortions created by apportionment formulas.

⁵Nielsen et al. (2003) analyze the case in which the multinationals have market power. Their results complement the conclusions of Hellerstein and McLure (2004) by showing that transfer prices can still provide a strategic income shifting role under FA by affecting the multinationals' pricing power.

differential information rent effects that can distort firm decisions and tax competition incentives. Nonetheless, the most common modeling approach in the tax competition literature assumes a reduced form cost to transfer price distortions that is not information based and cannot generate heterogeneous firm behavior, e.g., see Gérard (2005) and Nielsen et al. (2009). Under FA, auditing is no longer necessary but private information effects will still persist because of the heterogeneous response of firms to the apportionment formula. Because existing studies of tax competition with FA do not include any cost heterogeneity among multinationals, they are not able to identify how the information rent distortions arising from private, heterogeneous firms affect equilibrium behavior.⁶

Section 3 analyzes tax competition equilibria under a revenue-based FA rule while section 4 analyzes tax competition equilibria under SA with auditing. Section 5 compares the FA and the SA equilibria. The comparisons yield three main results. First, similar to Nielsen et al. (2003, 2009) and Kind et al. (2005), which system results in higher equilibrium tax rates or tax revenues, depends on the accuracy of the auditing technology available to the countries. If the auditing technology is very precise, equilibrium tax rates under SA will be higher than under FA. If the auditing technology is very noisy, FA will yield higher tax rates. Similar results are also obtained with respect to expected equilibrium tax revenues.

Second, if FA results in lower equilibrium tax rates, after-tax multinational profit will be higher under FA for all firm types. However, if FA results in higher equilibrium tax rates, only multinationals with costs in the middle of the type distribution will earn higher after-tax profits. This is due to differences in marginal information rents under SA and FA. Relative to FA, auditing under SA reduces a firm's marginal profit of foreign sales because the volume of income a firm can shift with its transfer prices is proportional to its foreign sales. With lower foreign sales (relative to FA), the firm's ability to

⁶Burbidge, Cuff, and Leach (2006) demonstrate the impact of firm heterogeneity on tax competition equilibria when comparing profit tax systems to personal and profit tax systems. However, they do not allow the tax policies to differ based upon firm differences. Cremer and Gahvari (2000) study a costly state falsification model in which countries can compete not only in tax rates but also in the degree of enforcement of income reporting. While transfer pricing is not one of the channels through which income can be concealed, they do find that differential enforcement has strategic value to competing governments. Cremer, Marchand, and Pestieau (1990) derive the optimal income tax in a private information model in which there is both evasion and auditing but do not consider any SA, FA, or tax competition issues.

earn information rents is also lower. Auditing also effects a firm's marginal information rents through the penalties arising from negative audits and this effect is strongest for firms in the middle of the cost distribution. The net effect is that firms earn higher rents under FA with average-type firms doing the best relative to SA. When FA results in higher tax rates than SA, the rent advantage average-type firms enjoy with FA still persists but now both firms with the highest and the lowest costs earn lower rents as the effect of a higher tax rate outweighs the small pre-tax rent advantage. Thus, a key result of this paper is that the shift to FA need not affect firm profits monotonically.

Third, the switch to FA affects tax revenues from domestic and foreign subsidiaries differently. For example, if FA results in higher equilibrium tax rates, each country will collect more tax revenues from the most efficient and the average to high-cost domestic firms. At the same time, tax revenues will increase from the average to lowest-cost foreign firms. That is, this switch to FA reduces tax payments from some but not all of above average efficiency domestic units and the below average efficiency foreign units. Thus, a non-monotonic relationship arises between tax revenues and firm costs.

A companion paper, Gresik (2008), derives and compares the optimal SA mechanism to the optimal FA mechanism when the countries operate as a union but do not coordinate tax rates (consistent with the first three EC proposals). Since similar non-monotonic effects persist when comparing optimal SA and FA mechanisms, it is unlikely that they are due to the specific rules used in this analysis. The non-monotonic effects reinforce the importance of evaluating multinational tax policies in the context of private information models as complete information, homogeneous firm models cannot generate such results. While we do not formally model any political economy effects, the second and third results suggest that political support for the choice of tax system will also vary non-monotonically with respect to firm costs. Section 6 offers concluding remarks.

2. The Basic Model.

There are two countries, A and B. Each is home to a multinational, a and b , respectively. Each multinational operates in both countries. None of the multinationals compete with each other. In order to compare our results with those in the extant literature, each of the four markets (a 's home market, a 's foreign market, b 's home market, and b 's foreign market) are assumed to be identical. Thus, if firm j sells q_j^i in country i , it will earn revenue of $R(q_j^i)$.⁷ I assume $R(\cdot)$ is strictly concave, $R(0)=0$, $R'(0)>0$, and $R'(\cdot)$ is concave. The first three assumptions are standard. The fourth is adopted for technical

⁷Superscripts will refer to countries and letter subscripts will refer to companies. Numerical subscripts will refer to partial derivatives.

convenience. All the results of the model will still be true as long as marginal revenue is not too convex.

Production in both countries requires an intermediate good produced only in the home country. Production cost for firm j 's intermediate good is $C(q_j^j, q_j^i, G(\theta_j)) = G(\theta_j)(q_j^j + q_j^i)$ where $i \neq j$. This cost is assumed to be tax deductible. Using a linear cost structure allows us to focus on the non-scale effects of each multinational's private information.

The parameter θ_j is firm j 's private information. It is independently drawn from $(-\infty, \infty)$ according to the distribution $G(\theta_j)$ with continuous density $g(\theta_j)$ and mean $\hat{\theta}$. This distribution information is common knowledge to both firms and both countries. This cost function satisfies the standard single-crossing properties. Namely, $C_3(\cdot, \cdot; G(\theta_j)) > 0$, $C_{13}(q_j^j, \cdot; G(\theta_j)) > 0$, and $C_{23}(\cdot, q_j^i; G(\theta_j)) > 0$. Higher values of θ_j correspond to higher production costs and higher marginal production costs. This formulation is equivalent to defining the firm's type as $s_j = G(\theta_j)$ and assuming that s_j is uniformly distributed on $[0, 1]$. The reason for representing a firm's type as θ_j distributed on $(-\infty, \infty)$ instead of s_j distributed on a closed, compact interval is that it avoids technical problems associated with auditing a firm whose type s_j is close to either of the endpoints of its support. I explain this choice in more detail when the auditing technology is introduced. Finally, I assume revenues are large enough so that every type wishes to produce a positive quantity.

There also exist economic costs that do not enter into the definition of taxable income. An example would be the opportunity cost of any production-related capital. $K(q_i^i, q_i^j) = k(q_i^i + q_i^j)$ denotes such costs and are associated with final good production. Without such costs, taxes under FA would be pure profit taxes and would imply equilibrium tax rates of 100%.

I consider two distinct tax competition games: one in which tax liabilities are defined by FA rules and one in which tax liabilities are defined by SA rules. This approach captures the current state of EU discussions in which the countries seek to adopt a common tax system while retaining the right to compete in tax rates under the chosen system.⁸

⁸Since costs and quantities are observable, the governments could infer each multinational's private information. This is not permitted for two reasons. First, the choice of SA or FA rules is an explicit choice by governments to restrict the amount of inference in which they engage. The analysis is meant to be consistent with this choice and consistent with how these two systems use firm observations. Second, one can think of a more complicated model in which multidimensional private information and/or unobservable actions limit the ability of the governments to perfectly infer firm types. The cost functions can then be interpreted as reduced form costs that reflect any inference the government have

3. The Formula Apportionment Game.

3.1 Game definition and preliminary equilibrium analysis.

The FA game is a two-stage game. In stage 1, the two countries simultaneously set their tax rates, t^A and t^B . In the second stage, the multinationals choose their output levels. Tax liabilities are defined by a revenue-based formula so that

$$\pi_j(q_j^j, q_j^i, \theta_j) = \frac{(1 - t^A)R(q_j^A) + (1 - t^B)R(q_j^B)}{R(q_j^A) + R(q_j^B)} [R(q_j^A) + R(q_j^B) - C(q_j^j, q_j^i, G(\theta_j))] - K(q_j^j, q_j^i). \quad (3.1)$$

Concavity of the revenue function implies that global pre-tax profit is globally concave in q_j^j and q_j^i but need not imply concavity of global after-tax profit. Therefore, I assume π_j is quasi-concave in q_j^j and q_j^i . This assumption will be satisfied if demand is linear (in which case π_j will be strictly concave).

The term in square brackets equals the multinational's global taxable income. The proportion of this taxable income country i uses as its tax base is defined by the proportion of revenues the multinational earns in country i . The revenue-based formula is used for two reasons. While historically the most common formula used by American states puts equal weight on revenue, labor costs, and capital values, there is a trend towards over-weighting revenue.⁹ For symmetric countries, Proposition 4 from Anand and Sansing (2000) implies that the revenue-based formula arises as an equilibrium rule when states compete via their formulas.¹⁰ Second, Hellerstein and McLure (2004) point out that capital investment and labor costs can be subject to valuation problems not unlike those associated with transfer pricing. By using a revenue formula, no new information problems are introduced.

To focus on the marginal effects of SA and FA systems, this section derives the symmetric subgame perfect Nash equilibrium of the FA game. This is consistent with most complete information analyses. If demand in each market is approximately linear, the symmetric equilibrium of this game will be the unique equilibrium.

already conducted. This process is modeled explicitly in Gresik (2008) which derives optimal SA and FA rules.

⁹For 2008, eleven states used 100% revenue (sales)-based formulas and 22 other states place a weight of 50% or more on sales (Multistate Tax Commission, 2008).

¹⁰Proposition 4 of Anand and Sansing (2000) implies that each country will choose a revenue rule if demand is equal in each country.

Starting with the second stage, define firm j 's output levels that maximize (3.1) given t^A and t^B by $q_j^j(t^j, t^i, \theta_j)$ and $q_j^i(t^j, t^i, \theta_j)$. With identical revenue functions in all four markets, (3.1) implies that firm j 's foreign production will equal firm i 's home production when firms i and j have identical cost parameters or that

$$q_j^i(t^j, t^i, \theta_j) = q_i^i(t^i, t^j, \theta_j). \quad (3.2)$$

In the first stage, each country i maximizes expected tax revenues defined as

$$ETR^i(t^i, t^j) \equiv \mathcal{E}_{\theta_i} TR_i^i(t^i, t^j, \theta_i) + \mathcal{E}_{\theta_j} TR_j^i(t^i, t^j, \theta_j) \quad (3.3)$$

where \mathcal{E} denotes the expectation operator and the tax revenues country i collects from firm j equal

$$TR_j^i(t^i, t^j, \theta_j) = \frac{t^i \cdot R(q_j^i(t^j, t^i, \theta_j))}{R(q_j^j(t^j, t^i, \theta_j)) + R(q_j^i(t^j, t^i, \theta_j))} \quad (3.4)$$

$$[R(q_j^j(t^j, t^i, \theta_j)) + R(q_j^i(t^j, t^i, \theta_j)) - C(q_j^j(t^j, t^i, \theta_j), q_j^i(t^j, t^i, \theta_j), G(\theta_j))].$$

For country i , (3.2) and (3.4) imply for all t^i and t^j that country i 's domestically-served market and its foreign-served market generate the same tax revenues as a function of the firm's type. Thus, $TR_i^i(t^i, t^j, \theta_i) = TR_j^i(t^i, t^j, \theta_j)$ or $ETR^i(t^i, t^j) = 2\mathcal{E}_{\theta_i} TR_i^i(t^i, t^j, \theta_i)$. Thus, a symmetric equilibrium is defined by a value of t for which

$$\mathcal{E}_{\theta_i} \partial TR_i^i(t^i, t, \theta_i) / \partial t^i |_{t^i=t} = 0. \quad (3.5)$$

3.2 FA Equilibrium Properties.

Let $\Pi_j(t^j, t^i, \theta_j)$ denote the indirect profit function of firm j given its optimal quantity choices for any pair of tax rates and let $\Pi_j(t, \theta_j) \equiv \Pi_j(t, t, \theta_j)$. Thus,

$$\Pi_j(t, \theta_j) = (1-t)[R(q_j^j(t, t, \theta_j)) + R(q_j^i(t, t, \theta_j))] - C(q_j^j(t, t, \theta_j), q_j^i(t, t, \theta_j), \theta_j) - K(q_j^j(t, t, \theta_j), q_j^i(t, t, \theta_j)).$$

Using the Envelope Theorem,

$$\partial \Pi_j(t^j, t^i, \theta_j) / \partial \theta_j = -\frac{(1-t^j)R(q_j^j) + (1-t^i)R(q_j^i)}{R(q_j^j) + R(q_j^i)} \cdot (q_j^j + q_j^i)g(\theta_j) < 0. \quad (3.6)$$

For any pair of tax rates, higher cost multinationals will earn lower after-tax profit.

In a symmetric equilibrium, $t^A = t^B \equiv t$, so define $q^*(t, \theta_j) = q_j^j(t, t, \theta_j) = q_j^i(t, t, \theta_j)$ where $q^*(t, \theta_j)$ satisfies

$$(1-t)(R'(q^*(t, \theta_j)) - G(\theta_j)) = k \quad (3.7)$$

and (3.6) implies

$$\partial \Pi_j(t, \theta_j) / \partial \theta_j = -2(1-t)q^*(t, \theta_j)g(\theta_j). \quad (3.8)$$

Eq. (3.8) describes a multinational's marginal information rent under FA with equal tax rates. Eqs. (3.7) and (3.8) imply that higher cost multinationals will produce less and earn less profit than lower cost multinationals.

Furthermore

$$\partial \Pi_j(t, \theta_j) / \partial t = -(2R(q^*) - C(q^*, q^*, \theta_j)) = -(\Pi_j(t, \theta_j) + K(q^*, q^*)) / (1-t) < 0 \quad (3.9)$$

and

$$\partial^2 \Pi_j / \partial t \partial \theta_j = 2q^*g - 2(1-t)g \partial q^* / \partial t > 0. \quad (3.10)$$

Eq. (3.9) shows that equilibrium firm profit is decreasing in the common tax rate and (3.10) shows that this tax effect is weaker on high cost firms than on low cost firms.

For identical tax rates, define $TR^i(t, \theta_i) = 2TR_i^i(t, \theta_i)$ where $TR_i^i(t, \theta_i)$ can be written as

$$TR^i(t, \theta_i) = (t(1-t))[\Pi_i(t, \theta_i) + K(q^*(t, \theta_i), q^*(t, \theta_i))]. \quad (3.11)$$

Lemma 1 records several key properties of $TR^i(t, \theta_i)$ that will be used to compare FA and SA equilibria.

Lemma 1.a. $TR^i(t, \theta_i)$ is decreasing in θ_i .

b. $\partial TR^i / \partial t$ is decreasing in θ_i and $TR^i(t, \theta_i)$ is strictly concave in t .

c. Suppose $|\partial^2 q^*(t, \theta_i) / \partial \theta_i^2|$ is close to zero. If $g'(\theta_i)$ is negative, then $TR^i(t, \theta_i)$ will be locally convex in θ_i . If $g'(\theta_i)$ is sufficiently positive, then $TR^i(t, \theta_i)$ will be locally concave in θ_i .

Lemma 1 is important for two reasons. First, coupled with (3.11), it documents the ways marginal information rents (captured by $\partial \Pi_i / \partial \theta_i$) influence tax revenues. Lemmas 1a and 1c show that the tax revenue profile inherits its shape primarily from each firm's rent profile, $\Pi_i(t, \theta_i)$. Second, part (b) implies that an increase in the common tax rate reduces tax revenues collected from high-cost firms before it reduces tax revenues collected from low-cost firms. As a result, one can define two cutoff tax rates, \underline{T} and \bar{T} , which are defined so that $\partial TR^i(\underline{T}, +\infty) / \partial t = 0$ and $\partial TR^i(\bar{T}, -\infty) / \partial t = 0$. \underline{T} maximizes tax revenues collected from the highest cost firm and \bar{T} maximizes revenues from the lowest cost firm.

Lemma 1b implies $0 < \underline{T} < \bar{T} < 1$ and leads to Proposition 2.

Proposition 2. Let t^f denote a symmetric equilibrium tax rate of the FA game. Then,

a. $t^f < \underline{T}$ and $\partial TR^i / \partial t > 0$ for all θ_i , or

b. $\underline{T} < t^f < \bar{T}$ and there exists $\tilde{\theta}$, so that for all $\theta_i > \tilde{\theta}$, $\partial TR^i / \partial t < 0$ and for all $\theta_i < \tilde{\theta}$, $\partial TR^i / \partial t > 0$.

To see how Proposition 2 will be used in comparing SA and FA equilibria in section 5, suppose the switch from SA to FA causes the symmetric equilibrium tax rate to increase. Proposition 2 implies that $TR^i(t, \theta_i)$ is increasing in t for all θ_i (case a) or it will rotate in a clockwise direction so that $TR^i(t, \theta_i)$ is

increasing in t for low values of θ_i and decreasing in t for high values of θ_i (case b). Case (b) generates the non-monotonic tax revenue differences mentioned in the introduction.

4. The Separate Accounting Game.

4.1 Game Definition.

The SA game is a 3-stage game. In stage 1, the two countries simultaneously and independently choose their tax rates. In stage 2, each multinational j chooses its home and foreign production and its transfer price, ρ_j , which is the unit price the foreign subsidiary pays the home subsidiary for the intermediate good. In stage 3, the countries jointly apply a noisy auditing technology to decide if each multinational set an acceptable transfer price. If not, each country adjusts the firm's tax base and penalties are levied for underpayment. Define $P_H(\rho_j, \theta_j)$ to be the expected per unit tax base adjustment for firm j if its transfer price is deemed to be too high and define $P_L(\rho_j, \theta_j)$ to be the expected per unit tax base adjustment for firm j if its transfer price is deemed to be too low. $P_H(\rho_j, \theta_j)$ is non-negative and strictly increasing in ρ_j . $P_L(\rho_j, \theta_j)$ is non-negative and strictly decreasing in ρ_j . These functions represent the outcome of stage 3 auditing and result in expected global after-tax profit for firm j of

$$\begin{aligned} \pi_j^s(q_j^j, q_j^i, \rho_j, \theta_j) = & (1-t^j)(R(q_j^j) - C(q_j^j, q_j^i, \theta_j) + \rho_j q_j^i) + (1-t^i)(R(q_j^i) - \rho_j q_j^i) - K(q_j^j, q_j^i) \\ & - q_j^i((1+\eta)t^i - t^j)P_H(\rho_j, \theta_j) - q_j^i((1+\eta)t^j - t^i)P_L(\rho_j, \theta_j). \end{aligned} \quad (4.1)$$

π_j^s is strictly concave in ρ_j , q_j^j and q_j^i . The exact form of these tax base adjustments will be derived in the next subsection.

The first line of (4.1) shows how a firm's transfer price can be used to shift taxable income between countries. The second line of (4.1) captures the compliance costs from the transfer price auditing. When an audit reveals income shifting, a new tax liability is calculated and a penalty is imposed for underpayment. The parameter η denotes this penalty. If ρ_j is deemed to be too high, any recalculation will, in expectation, reduce the firm's home taxable income and increase its foreign taxable income by $q_j^i P_H(\rho_j, \theta_j)$. This increases the firm's foreign tax liability by $(1+\eta)t^i q_j^i P_H(\rho_j, \theta_j)$ and it decreases the firm's home tax liability by $t^j q_j^i P_H(\rho_j, \theta_j)$ which incurs no penalty. The net effect of these two adjustments is reflected in the first expression in line 2 of (4.1). Similarly, if firm j 's transfer price is deemed to be too low, its home tax liability will increase and its foreign tax liability will decrease. Now the home country imposes a penalty and the expected net effect is represented by the last term of (4.1).

Since each country receives revenues from two sources, tax receipts and penalty payments, define the pre-audit taxes i collects from its domestic firm by

$$TR_i^{si}(t^i, t^j, \theta_i) = t^i [R(q_i^i(t^i, t^j, \theta_i)) - C(q_i^i(t^i, t^j, \theta_i), q_i^j(t^i, t^j, \theta_i), G(\theta_i)) + \rho_i(t^i, t^j, \theta_i) q_i^j(t^i, t^j, \theta_i)], \quad (4.2)$$

define the post-audit penalty payments i receives from its domestic firm by

$$\begin{aligned} Z_i^i(t^i, t^j, \theta_i) &= t^i q_i^j(t^i, t^j, \theta_i) [(1+\eta) P_L(q_i^i(t^i, t^j, \theta_i), q_i^j(t^i, t^j, \theta_i), \rho_i(t^i, t^j, \theta_i), \theta_i) \\ &\quad - P_H(q_i^i(t^i, t^j, \theta_i), q_i^j(t^i, t^j, \theta_i), \rho_i(t^i, t^j, \theta_i), \theta_i)], \end{aligned} \quad (4.3)$$

define the pre-audit revenues i collects from the foreign firm by

$$TR_j^{si}(t^i, t^j, \theta_j) = t^i [R(q_j^i(t^j, t^i, \theta_j)) - \rho_j(t^j, t^i, \theta_j) q_j^i(t^j, t^i, \theta_j)], \quad (4.4)$$

and define the post-audit penalty payments i receives from the foreign firm by

$$\begin{aligned} Z_j^i(t^i, t^j, \theta_j) &= t^i q_j^i(t^j, t^i, \theta_j) [(1+\eta) P_H(q_j^j(t^j, t^i, \theta_j), q_j^i(t^j, t^i, \theta_j), \rho_j(t^j, t^i, \theta_j), \theta_j) \\ &\quad - P_L(q_j^j(t^j, t^i, \theta_j), q_j^i(t^j, t^i, \theta_j), \rho_j(t^j, t^i, \theta_j), \theta_j)]. \end{aligned} \quad (4.5)$$

Then, total revenue for country i equals

$$TR^{si}(t^i, t^j, \theta_i, \theta_j) = TR_i^{si}(t^i, t^j, \theta_i) + Z_i^i(t^i, t^j, \theta_i) + TR_j^{si}(t^i, t^j, \theta_j) + Z_j^i(t^i, t^j, \theta_j).$$

Under SA, country i chooses t^i to maximize $\mathcal{E}_{\theta_i, \theta_j} TR^{si}(t^i, t^j, \theta_i, \theta_j)$ given t^j .

4.2. A Stochastic Auditing Technology.

In practice, tax authorities evaluate a company's transfer price by comparing it to an arm's-length standard based on transaction prices for similar items traded by independent companies. Because it is difficult to adjust for all economic differences between the audited and comparison firms, standard regulations use the comparison data to define a range of arm's-length prices. If an audited firm's transfer price falls outside this range, its transfer price is considered to be either too high or too low. In such instances, the firm's transfer price is set equal to the mean in the comparison data. In this subsection, I derive the P_H and P_L functions consistent with these common attributes of transfer price regulation.

The auditing technology each government uses consists of three components: a signal, a transfer price standard, and a compliance probability. One can think of the adjustments a tax authority makes to comparable data to account for unobservable differences among firms in the comparison cohort as producing a noisy signal but unbiased signal μ_j of the audited firm's true type, θ_j . Denote the signal distribution by $F(\mu_j|\theta_j)$ and denote its density by $f(\mu_j|\theta_j)$. Since neither government knows θ_j , each will make an inference about the firm's type from the signal μ_j which is described by the conditional distribution of θ_j given μ_j , $H(\theta_j|\mu_j)$, and the conditional density, $h(\theta_j|\mu_j)$. I assume the auditing technology is automatically used jointly by A and B rather than assuming that only the country from which profits are

being shifted conducts the audit. This assumption not only reflects the high degree of information sharing among tax authorities of different countries (European Commission (2005a)) and the common policy of “competent authority” which essentially requires two countries to agree on what constitutes an appropriate transfer price, it also is motivated by the fact that any transfer price adjustments made by one country automatically trigger adjustments in a multinational's tax returns in the other country. Thus, any information learned by one country is available to the other.

To derive specific properties of the expected penalty functions, assume θ_j is normally distributed with mean 0 and standard deviation 1 and that conditional on θ_j , μ_j is normally distributed with mean θ_j and standard deviation σ . Thus, the auditing signal will have precision $1/\sigma$. Conditional on μ_j , θ_j is normally distributed with mean, $\Sigma(\mu_j) = \mu_j/(\sigma^2 + 1)$, and standard deviation, $\sigma/\sqrt{\sigma^2 + 1}$. Note that for all θ_j , $H(\theta_j|\mu_j) = H(\theta_j - \Sigma(\mu_j)|0)$ and $\Sigma'(\cdot) > 0$.

Given linear intermediate good costs, the long-run equilibrium competitive price for the intermediate good is $G(\theta_j)$. $G(\theta_j)$ is also equal to the Shapley-Shubik cost-sharing value. Since the rationale for calculating an arm's-length price is to approximate a competitive market price, $G(\theta_j)$ is the standard against which a firm's transfer price will be judged. If one instead were to adopt a short-run equilibrium perspective, the standard would equal $G(\theta_j) + \lambda(R(q_i^j)/q_i^j - k)$ for $\lambda(\cdot)$ between 0 and 1 to reflect profit-sharing instead of cost-sharing. Adopting this different standard does not change the qualitative results of this paper.

To define the range of compliant transfer prices, let $\beta < 1/2$ denote the probability a firm's type falls into a tail of the distribution $H(\theta_j|\mu_j)$. For a given probability, β , define the type cutoffs $\theta^+(\beta)$ and $\theta^-(\beta)$ such that $H(\theta^+(\beta)|0) = 1 - \beta$ and $H(\theta^-(\beta)|0) = \beta$. Normality implies that $\theta^+(\beta) = -\theta^-(\beta)$. Then for all μ_j , the compliance region will be defined by

$$\theta^+(\mu_j, \beta) = \theta^+(\beta) + \Sigma(\mu_j) \quad (4.6)$$

and

$$\theta^-(\mu_j, \beta) = \theta^-(\beta) + \Sigma(\mu_j). \quad (4.7)$$

Since $\Sigma(\cdot)$ is strictly monotonic, $\theta^+(\mu_j; \beta) = x$ implies $\mu_j = \Sigma^{-1}(x - \theta^+(\beta))$. Similarly, if $\theta^-(\mu_j; \beta) = x$, then $\mu_j = \Sigma^{-1}(x - \theta^-(\beta))$. Note that $\beta \geq 1/2$ implies that a truthtelling firm with $\theta_j = 0$ would always be non-compliant.

It is now possible to see the significance of defining the firm's type over $(-\infty, \infty)$. Suppose instead that θ_j was distributed on a compact interval and suppose the firm's type was close to the lower endpoint of the support. If the conditional signal distribution has the same variance in the tails of the unconditional support as it does near the unconditional median, it would either have positive mass at the lower endpoint

or it would have to be asymmetric about the firm's type. For a fixed β , compliance regions for firms with extreme types would be very asymmetric. For example, all firm types within the top β tail of the unconditional distribution would be penalized with high probability even if the auditing signal is accurate, $\mu_j = \theta_j$, and ρ_j was very close to $G(\theta_j)$. The current model is designed to avoid such outcomes. For any θ_j , there exists an open set of signals, μ_j , centered about θ_j such that the firm will not be penalized if its transfer price is close to its true marginal cost. In effect, this design captures the idea that the tax authority can learn more from an audit of an extreme firm than from a “middle of the pack” firm for the same amount of resources. Thus, for a given shift in unit profit of $\rho_j - G(\theta_j)$, an extreme type firm will be more likely to be penalized than will a median type firm.

Since $G(\cdot)$ is strictly increasing in firm j 's type, firm j 's transfer price is considered in compliance (with the arm's-length standard) if

$$G(\theta^-(\mu_j, \beta)) \leq \rho_j \leq G(\theta^+(\mu_j, \beta)). \quad (4.8)$$

If firm j is audited and found to be non-compliant, the firm's tax liability is restated based upon the auditing signal μ_j and a penalty is imposed. Firm j 's transfer price is too high if $\rho_j > G(\theta^+(\mu_j, \beta))$ or if $\theta^+(\mu_j, \beta) < G^{-1}(\rho_j)$ or if the auditing signal is too low, i.e.,

$$\mu_j < \mu^0(\rho_j) \equiv \Sigma^{-1}(G^{-1}(\rho_j) - \theta^+(\beta)). \quad (4.9)$$

In this situation, firm j 's taxable income in country j will be restated downward by $(\rho_j - G(\mathcal{E}(\theta_j | \mu_j)))q_j^i$ while firm j 's taxable income reported in country i needs to be restated upward by the same amount.¹¹

Since the signal distribution is conditioned by firm j 's actual type, (4.9) implies that

$$P_H(\rho_j, \theta_j) = \int_{\mu_j = -\infty}^{\mu^0(\rho_j)} [\rho_j - G(\mathcal{E}(\theta_j | \mu_j))] f(\mu_j | \theta_j) d\mu_j. \quad (4.10)$$

Firm j 's transfer price is too low if $\rho_j < G(\theta^-(\mu_j, \beta))$ or if

$$\mu_j > \mu^1(\rho_j) \equiv \Sigma^{-1}(G^{-1}(\rho_j) - \theta^-(\beta)) = \Sigma^{-1}(G^{-1}(\rho_j) + \theta^+(\beta)). \quad (4.11)$$

Now firm j 's home taxable income needs to be restated upward by $(G(\mathcal{E}(\theta_j | \mu_j)) - \rho_j)q_j^i$ while its host taxable income needs to be restated downward by the same amount. Therefore,

¹¹The standard procedure in OECD countries is to restate a non-compliant firm's transfer price equal to the transfer price charged by the median comparable firm. The conditional mean is used as a proxy for this value since we have a single observation. This procedure differs from one in which the non-compliant firm's transfer price is set equal to the median (which would be proxied by $\mathcal{E}(G(\theta_j) | \mu_j)$).

$$P_L(\rho_j, \theta_j) = \int_{\mu_j = \mu^1(\rho_j)}^{\infty} [G(\mathcal{E}(\theta_j | \mu_j)) - \rho_j] f(\mu_j | \theta_j) d\mu_j. \quad (4.12)$$

Eqs. (4.10) and (4.12) imply that P_H is non-negative and strictly increasing in ρ_j while P_L is non-negative and strictly decreasing in ρ_j . Note also that, consistent with Baron and Besanko (1984), the ability of the countries to detect income shifting and impose penalties is type-dependent because auditing gives each government information that allows it to update its beliefs about a firm's type and each firm's expected probability of being penalized will fall between 0 and 1. Thus, auditing limits but does not eliminate the multinational's ability to earn information rents.

4.3 SA Equilibrium Properties.

For the second stage of the SA game, denote firm j 's optimal quantities and transfer price by $Q_j^j(t^j, t^i, \theta_j)$, $Q_j^i(t^j, t^i, \theta_j)$, and $\rho_j^*(t^j, t^i, \theta_j)$. With equal tax rates, home production is $Q^h(t, \theta_j) = Q_j^j(t, t, \theta_j)$, foreign production is $Q^f(t, \theta_j) = Q_j^i(t, t, \theta_j)$, and each firm sets the transfer price $\rho^*(t, \theta_j) = \rho_j^*(t^j, t^i, \theta_j)$. Let $\Pi_j^s(t^j, t^i, \theta_j)$ denote the indirect profit of firm j given its optimal quantity and transfer price choices and let $\Pi_j^s(t, \theta_j)$ denote firm j 's indirect profit when both countries choose a common tax rate, t . Therefore, we can write

$$\begin{aligned} \Pi_j^s(t, \theta_j) = & (1-t)(R(Q^h(t, \theta_j)) + R(Q^f(t, \theta_j)) - C(Q^h(t, \theta_j), Q^f(t, \theta_j), G(\theta_j))) - K(Q^h(t, \theta_j), Q^f(t, \theta_j)) \\ & - \eta t Q^f(t, \theta_j) (P_H(\rho^*(t, \theta_j), \theta_j) + P_L(\rho^*(t, \theta_j), \theta_j)). \end{aligned} \quad (4.13)$$

By the Envelope Theorem,

$$\begin{aligned} d\Pi_j^s(t, \theta_j)/d\theta_j = & -(1-t)(Q^h(t, \theta_j) + Q^f(t, \theta_j))g(\theta_j) \\ & - \eta t Q^f(t, \theta_j) [\partial P_H(\rho^*(t, \theta_j), \theta_j)/\partial \theta_j + \partial P_L(\rho^*(t, \theta_j), \theta_j)/\partial \theta_j]. \end{aligned} \quad (4.14)$$

Eq. (4.14) is the analog to (3.8) and describes the marginal information rents of a multinational under SA and equal tax rates. The differences between these two equations will be discussed in the next section.

Q^h , Q^f , and ρ^* must satisfy the first-order conditions

$$(1-t)(R'(Q^h) - G(\theta_j)) = k, \quad (4.15)$$

$$(1-t)(R'(Q^f) - G(\theta_j)) - \eta t (P_H(\rho^*, \theta_j) + P_L(\rho^*, \theta_j)) = k, \quad (4.16)$$

and

$$\partial P_H(\rho^*, \theta_j)/\partial \rho_j + \partial P_L(\rho^*, \theta_j)/\partial \rho_j = 0. \quad (4.17)$$

Comparing (3.7) with (4.15) and (4.16) reveals that $Q^h(t, \theta_j) = q^*(t, \theta_j)$ while $Q^f(t, \theta_j) \leq q^*(t, \theta_j)$ for all

$t > 0$. In equilibrium auditing distorts foreign production down because it increases the cost of profit-shifting which is proportional to the amount of foreign production. Eq. (4.17) implies that $\rho^*(t, \theta_j)$ is independent of t so that we can henceforth write $\rho^*(\theta_j)$.

Now consider the first stage tax competition. A symmetric SA equilibrium tax rate, t^s , will satisfy $\partial \mathcal{E}_{\theta, \theta_j} TR^{si}(t^i, t^s, \theta, \theta_j) / \partial t^i_{i=t^s} = 0$ given Q^h, Q^f , and ρ^* as defined by (4.15)-(4.17). Given (4.10) and (4.12), a symmetric equilibrium will exist with linear demand. More general existence conditions are not known.

When $t^i \neq t^j$, the optimal transfer price balances the marginal gains from profit-shifting against the marginal expected penalties which depend on differences in the tax rates and the precision of the auditing signal. Yet when $t^i = t^j$, there are no marginal gains or marginal penalties due to tax rate differences but the marginal expected penalties linked to the precision of the auditing signal remain and are equal to the left-hand side of (4.17). Since in the absence of a noisy auditing technology, the multinationals have no strict incentive to use their transfer prices for profit-shifting, should the countries elect not to audit at all and avoid the output distortions from noisy auditing? Consider two cases. With identical tax rates and no auditing, there exist a continuum of best responses for each multinational because any transfer price strategy is optimal. With different tax rates and no auditing, the optimal response of the multinationals is to engage in maximal profit-shifting, even if the difference in tax rates is infinitesimal. This latter response would then encourage auditing in any subgame with $t^i \neq t^j$ and the optimal transfer prices would be defined by

$$t^i - t^j - ((1+\eta)t^i - t^j) \partial P_H(\rho^*, \theta_j) / \partial \rho_j - ((1+\eta)t^j - t^i) \partial P_L(\rho^*, \theta_j) / \partial \rho_j = 0. \quad (4.18)$$

In the limit as $t^i - t^j$ goes to zero, (4.18) converges to (4.17). As a result, (4.17) most accurately captures the marginal effects of tax competition with noisy auditing. It also reflects very different equilibrium output distortions relative to complete information models of tax competition which most often assume a reduced form penalty function that imposes zero (expected) penalties on firms that report transfer prices equal to their true marginal cost. With multiple firm types, such a penalty function is consistent only with a perfect auditing technology. Differences between $G(\theta_j)$ and $H(\theta_j | \mu_j)$, which will exist for all $\mu_j \neq 0$, must imply positive expected penalties even for a truth-telling firm. Thus, the standard reduced form models do not accurately capture the distortions created by less than perfect auditing.

Since a noisy auditing technology has not been used before in tax competition models, it is useful to report some of its equilibrium properties.

Proposition 3. *a. $\rho^*(\theta_j) = G(\theta_j)$ if $\theta_j = 0$. $\rho^*(\theta_j) > G(\theta_j)$ if $\theta_j < 0$ and $\rho^*(\theta_j) < G(\theta_j)$ if $\theta_j > 0$.*

b. $\rho^(\theta_j)$ is increasing in θ_j .*

Proposition 3 establishes that in any symmetric equilibrium, the distortions due to noisy auditing induce a firm with higher than average costs to understate its true cost information, i.e. $G(\theta_j)$, and they induce a firm with lower than average costs to overstate its true cost information. Only a firm with average costs will set its transfer price equal to its marginal cost. In addition, Proposition 3b reveals that higher type firms will report higher transfer prices.

Proposition 4. *The expected adjustment to a firm's taxable profit in a symmetric equilibrium,*

$P_H(\rho^*(\theta_j), \theta_j) + P_L(\rho^*(\theta_j), \theta_j)$, *is increasing in θ_j for $\theta_j < 0$ and decreasing in θ_j for $\theta_j > 0$.*

Proposition 4 reveals that the types that incur the highest expected adjustments are not the least common types but rather the most common types (θ_j near 0). With normally distributed types, it is easier for types close to 0 to distort their transfer prices without the prices seeming unusual because the tax authority will most likely receive a signal that does not allow for much updating. To offset this opportunity, the equilibrium expected adjustments need to be higher for average type firms. This means the largest output distortions will be associated with intermediate firm types. In fact, Proposition 5 reports that in the limit the most extreme types pay no penalties.

Proposition 5. $\lim_{\theta_j \rightarrow \pm\infty} P_H(\rho^*(\theta_j), \theta_j) + P_L(\rho^*(\theta_j), \theta_j) = 0$

Proposition 5 implies that, in a symmetric equilibrium, for all $t > 0$, $\lim_{\theta_j \rightarrow \pm\infty} Q^j(t, \theta_j) = q^*(t, \theta_j)$.

5. Comparing Equilibrium Tax Revenues and Firm Profit Under SA and FA

5.1 Equilibrium tax rates and expected tax revenues.

As a benchmark for comparing t^s to t^f , consider the optimal tax rate when the two countries form a single tax area and hence no longer need to allocate pre-tax income. The optimal tax rate will maximize $\mathcal{E}_{\theta_j} TR^i(t, \theta_j)$. Denote this optimal tax rate by t^{opt} and denote the quantity produced by multinational i in each country by $q^*(t^{opt}, \theta_j)$.

With FA, the fact that $\partial TR^i / \partial t^j > 0$ means that when one country raises its tax rate it imposes a positive strategic externality on the other country. Thus, in equilibrium, we must have $t^f < t^{opt}$.

With SA, the sign of $\partial TR^{st} / \partial t^j$ can be either positive or negative as in Sørensen (2003, 2004) and auditing, which is not needed in a single tax area, also creates a tax competition distortion. However, in the limit as the auditing technology perfectly reveals firm type, that is as σ goes to 0, t^s converges to t^{opt} . This means that for σ sufficiently small, equilibrium tax rates and expected tax revenues will be greater under SA. In the limit as σ converges to ∞ , the signal μ_j provides no information to update beliefs. In this case, the penalty range is independent of μ_j so each type of firm will choose the highest or lowest possible transfer price that is not subject to a penalty (either $G(\theta^+(\beta))$ or $G(\theta^-(\beta))$) no matter how small $|t^s - t^f|$ is. Thus, t^s will be less than t^f and expected tax revenues will be lower under SA. Given the continuous

structure of the SA game, there must then exist a value for σ such that the equilibrium tax rates under SA and FA are equal. Denote this value by σ^* . This result is analogous to results in Nielsen et al. (2009), Sørensen (2003,2004), and Kind et al (2005). What one can assess in this model, and not in the others, is how equilibrium firm profit and equilibrium tax revenues differ by type.

To illustrate the potential differences in equilibrium tax revenues for the two regimes, consider the following example in which $R(q) = (3-q)q$, $k=.2$, $\beta=.1$, and $\eta=.3$. For this example, $t^f = .763$ and $\sigma^* \approx .85$. For $\sigma = .5$, $t^s = .77$ and SA yields higher expected tax revenues. For $\sigma = 1$, $t^s = .756$ and expected tax revenues are higher under FA. As σ increases further, the equilibrium tax rate and the equilibrium tax revenues under SA continue to decrease.

5.2 Comparisons when SA and FA induce identical equilibrium tax rates.

With $t^s=t^f$, any differences in firm profit between SA and FA equilibria will be due to differences in information rents. The shift from SA to FA has two effects on the firms' information rents holding the common tax rate fixed. First, the marginal information rent due to production incentives under SA (line 1 of (4.14)) is smaller than under FA (from (3.8)), i.e.,

$$Q^h(t, \theta_j) + Q^f(t, \theta_j) = q^*(t, \theta_j) + Q^f(t, \theta_j) \leq 2q^*(t, \theta_j). \quad (5.1)$$

Second, the marginal information rent effect due to the auditing technology (line 2 of (4.14)), can be positive or negative. If $\partial P_H/\partial \theta_j + \partial P_L/\partial \theta_j$ is positive, then for all $t > 0$,

$$d\Pi_j^s(t, \theta_j)/d\theta_j < -(1-t)(Q^h(t, \theta_j) + Q^f(t, \theta_j))g(\theta_j) \quad (5.2)$$

so the production and the auditing effects work in opposite directions. If $\partial P_H/\partial \theta_j + \partial P_L/\partial \theta_j$ is negative, then for all $t > 0$,

$$d\Pi_j^s(t, \theta_j)/d\theta_j > -(1-t)(Q^h(t, \theta_j) + Q^f(t, \theta_j))g(\theta_j) \quad (5.3)$$

and the two effects work in the same direction so that

$$d\Pi_j^s(t, \theta_j)/d\theta_j > d\Pi_j(t, \theta_j)/d\theta_j. \quad (5.4)$$

By Proposition 4, $\partial P_H/\partial \theta_j + \partial P_L/\partial \theta_j$ is negative when $\theta_j > 0$ and it is positive when $\theta_j < 0$. By Proposition 5, $\Pi_i^s(t, \pm\infty) = \Pi_i(t, \pm\infty)$. As θ_j falls from $+\infty$, $\Pi_i(t, \theta_j) - \Pi_i^s(t, \theta_j)$ will be positive and by (5.4) is decreasing in θ_j . This pattern will persist as θ_j drops below zero. For θ_j sufficiently small, the auditing effect (5.2) will dominate the quantity effect (5.1) and $\Pi_i(t, \theta_j) - \Pi_i^s(t, \theta_j)$ will decrease to zero in the limit at $-\infty$. So although the sign of $\partial P_H/\partial \theta_j + \partial P_L/\partial \theta_j$ depends only on properties of the auditing technology, Proposition 6, which relies only on the concavity of the revenue function, shows that it is possible to uniformly rank firm profit under SA and FA.

Proposition 6. $\Pi_i^s(t, \theta_i) \leq \Pi_i(t, \theta_i)$ for all $t > 0$ and for all θ_i .

Proof. This proof relies on differences in q^* and Q^f and hence exploits the relationship between the auditing distortions and firm production. Substituting (4.16) into (4.13) implies

$$\begin{aligned} \Pi_i^s(t, \theta_i) &= (1-t)(R(q^*) + R(Q^f) - C(q^*, Q^f, G(\theta_i))) - K(q^*, Q^f) \\ &\quad - Q^f[(1-t)(R'(Q^f) - C_2(q^*, Q^f, G(\theta_i))) - k] \end{aligned} \quad (5.5)$$

while

$$\Pi_i(t, \theta_i) = (1-t)(R(q^*) + R(q^*) - C(q^*, q^*, G(\theta_i))) - K(q^*, q^*). \quad (5.6)$$

Subtracting (5.6) from (5.5) and using (4.15) then implies

$$\Pi_i^s(t, \theta_i) - \Pi_i(t, \theta_i) = (1-t)[R(Q^f) - Q^f R'(Q^f) - (R(q^*) - R'(q^*)q^*)]. \quad (5.7)$$

Since $R(q) - R'(q)q$ is increasing in q and $Q^f \leq q^*$, the right-hand side of (5.7) must be non-positive.

Q.E.D.

Proposition 6 shows that for the same tax rate, all firms prefer to operate under FA because the indirect way in which FA uses a firm's private information results in higher rents. This ranking will also be preserved if $t^s > t^f$.

Tax revenue differences can be calculated using (4.2)-(4.5) by defining

$$DTR^{si}(t, \theta_i) \equiv TR_i^{si}(t, \theta_i) + Z_i^i(t, \theta_i) = t(R(q^*) - G(\theta_i)(q^* + Q^f) + \rho^* Q^f + Q^f((1+\eta)P_L - P_H)) \quad (5.8)$$

and

$$FTR^{si}(t, \theta_i) \equiv TR_j^{si}(t, \theta_i) + Z_j^i(t, \theta_i) = t(R(Q^f) - \rho^* Q^f + Q^f((1+\eta)P_H - P_L)) \quad (5.9)$$

so that

$$\begin{aligned} TR^{si}(t, \theta_i) &\equiv DTR^{si}(t, \theta_i) + FTR^{si}(t, \theta_i) \\ &= t[R(q^*) + R(Q^f) - G(\theta_i)(q^* + Q^f)] + t\eta Q^f(P_H + P_L) \end{aligned} \quad (5.10)$$

where $Z_i^i(t, \theta_i) = Z_i^i(t, t, \theta_i)$ and $Z_j^i(t, \theta_i) = Z_j^i(t, t, \theta_i)$. Eq. (5.8) calculates the tax revenue country i collects from its home multinational of type θ_i . Eq. (5.9) calculates the tax revenue country i collects from multinational j 's foreign subsidiary doing business in country i that has type θ_i . Eq. (5.10) adds together the domestic-source and foreign-source tax revenues from a domestic subsidiary and a foreign subsidiary of the same type even though they are different multinationals. For expected revenue calculations, there is no loss of generality in using (5.10), otherwise one has to use (5.8) and (5.9) separately.

Eq. (5.10) captures the two ways tax revenues from SA and FA differ holding firm type fixed:

the difference between Q^f and q^* and the impact of the penalty payments. The first-order condition for Q^f , (4.16), links these two effects together. If $P_H(\rho^*(\theta_i), \theta_i) + P_L(\rho^*(\theta_i), \theta_i) = 0$, then $TR^{si}(t, \theta_i) = TR^i(t, \theta_i)$. If $P_H + P_L > 0$, the quantity effect implies $TR^{si}(t, \theta_i) < TR^i(t, \theta_i)$ while the penalty effect implies $TR^{si}(t, \theta_i) > TR^i(t, \theta_i)$. To get a sense for the net effect, note that substituting (4.16) into (5.10) allows one to write TR^{si} as a function of t , q^* , Q^f , and θ_i instead of just t and θ_i and yields

$$TR^{si}(t, q^*, Q^f, \theta_i) \equiv t(R(q^*) + R(Q^f) - C(q^*, Q^f, G(\theta_i))) + (1-t)(R'(Q^f) - G(\theta_i)) - k. \quad (5.11)$$

Evaluating the partial derivative of $TR^{si}(t, q^*, Q^f, \theta_i)$ with respect to Q^f at q^* then implies

$$\partial TR^{si} / \partial Q^f |_{Q^f=q^*} = kt / (1-t) + (1-t)R''(q^*)q^* \quad (5.12)$$

which can be positive or negative. Proposition 7 describes whether the positive or negative effects will dominate and hence the conditions under which each system can generate higher expected tax revenues.

Proposition 7. There exist tax rates t^- and t^+ with $t^- < t^+$ such that

- (a) for all $t < t^-$ and for all θ_i , $TR^{si}(t, \theta_i) > TR^i(t, \theta_i)$,
- (b) for all $t > t^+$ and for all θ_i , $TR^{si}(t, \theta_i) < TR^i(t, \theta_i)$, and
- (c) for $t^- < t < t^+$, there exists $\theta'(t)$ such that for all $\theta_i < \theta'(t)$, $TR^{si}(t, \theta_i) > TR^i(t, \theta_i)$ and for all $\theta_i > \theta'(t)$, $TR^{si}(t, \theta_i) < TR^i(t, \theta_i)$.

Proof. Concave marginal revenue ensures that $TR^{si}(t, q^*, Q^f, \theta_i)$ is concave in Q^f . For t small, (5.12) will be negative. Since $Q^f < q^*$, higher tax revenues will be collected under SA. For t large, (5.12) will be positive and higher tax revenues will be collected under FA. For intermediate values of t , the concavity of R' and the fact that q^* is decreasing in θ_i means SA will yield higher tax revenues for small values of θ_i and FA will yield higher tax revenues for large values of θ_i . **Q.E.D.**

Figure 1 plots $TR^{si}(t, \theta_i) - TR^i(t, \theta_i)$ for θ_i between -6 and 6 assuming the intermediate case, 7 (c). In the example illustrated in Figure 1, expected tax revenues will be higher under SA. By using a sufficiently larger common tax rate, the opposite would be true.

What happens when we consider the separate effects on domestic and foreign tax revenues at a common tax rate? The change in domestic tax revenues using FA as the baseline is the difference between (5.8) and (3.4),

$$\Delta DTR(t, \theta_i) \equiv DTR^{si}(t, \theta_i) - TR_i^i(t, \theta_i) = tQ^f[\rho^* - G(\theta_i) + (1+\eta)P_L - P_H], \quad (5.13)$$

and the change in foreign tax revenues is the difference between (5.9) and (3.4),

$$\begin{aligned} \Delta FTR(t, \theta_i) &\equiv FTR^{si}(t, \theta_i) - TR_i^i(t, \theta_i) \\ &= t[G(\theta_i) - \rho^* + (1+\eta)P_H - P_L] + t[R(Q^f) - R(q^*) + G(\theta_i)(q^* - Q^f)]. \end{aligned} \quad (5.14)$$

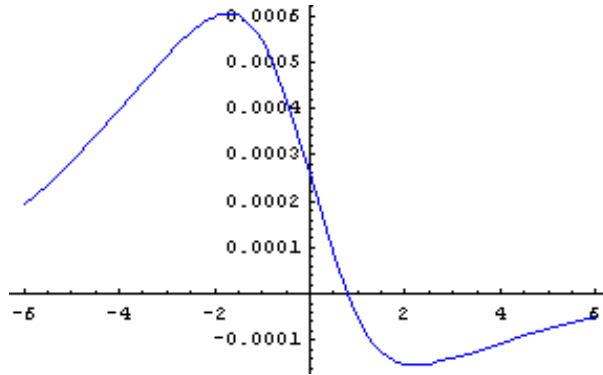


Figure 1: $TR^{si}(t, \theta_i) - TR^i(t, \theta_i, 0)$ when $t^- < t < t^+$

According to (5.13), the change in domestic tax revenues is solely a function of the auditing technology. According to (5.14), the change in foreign tax revenues is a function of an analogous auditing term and a quantity distortion term (the expression in the second set of brackets). No quantity term appears in (5.13) because domestic production is unchanged by a shift between SA and FA for the same tax rate. Given (4.15) and (4.16), the quantity term in (5.14) is negative when $q^* \neq Q^f$.

Propositions 3 and 4 imply in general that ΔDTR will be positive for low-type firms and negative for high-type firms. For instance if $\theta_i = 0$, $\rho^* = G(0)$ but $(1+\eta)P_L$ will be larger than P_H . Figure 2a demonstrates the overall shape of ΔDTR implied by Propositions 3 and 4. If it were not for the extra quantity term, ΔFTR would be the mirror image of ΔDTR reflected on the horizontal axis. The negative quantity term shifts the mirror image of ΔDTR to the right. Figure 2b illustrates the shape of ΔFTR implied by Propositions 3 and 4. As long as the shift from SA to FA does not change the equilibrium tax rate, each country will see reduced tax revenues from low cost domestic subsidiaries and high cost foreign subsidiaries and increased tax revenues from the other subsidiaries.

5.3 Comparisons when SA and FA induce different equilibrium tax rates.

Recall from Proposition 6 that all firm types prefer FA to SA at the same tax rate and that this preference becomes stronger when $t^s > t^f$ as in the example when $\sigma = .5$. Recall also from Proposition 5 that $\Pi_i^s(t, \pm\infty) = \Pi_i(t, \pm\infty)$. If $t^s < t^f$, the lower tax rate with SA will now imply $\Pi_i^s(t^s, \pm\infty) > \Pi_i(t^f, \pm\infty)$ or that extreme types will now earn larger profit with SA while intermediate types may still earn higher profit with FA. This observation establishes Proposition 8.

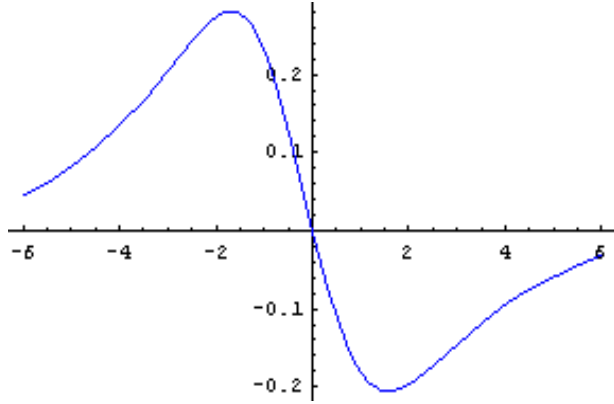


Figure 2a: $\Delta DTR(t, \theta_i)$

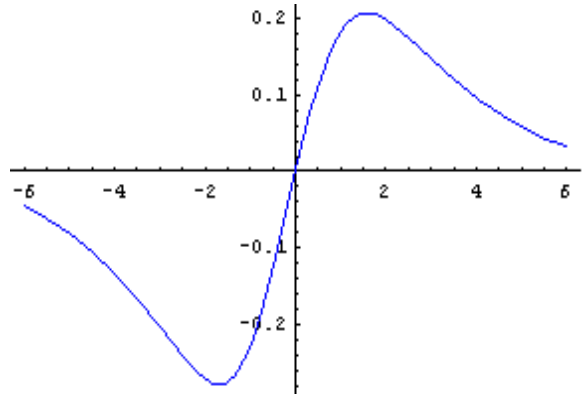


Figure 2b: $\Delta FTR(t, \theta_i)$

Proposition 8. Consider a small increase in σ above σ^* so that $t^s < t^f$. There will exist types $\mu^1 < 0 < \mu^2$ such that expected after-tax firm profit will be smaller under FA if, and only if, $\theta_i < \mu^1$ or $\theta_i > \mu^2$. For σ sufficiently large, expected after-tax firm profit will be smaller under FA for all firm types.

Figure 3 illustrates the effect on firm profit described in Proposition 8 using the example from section 5.1 with $\sigma = .86 > \sigma^* \approx .85$ which implies $t^s = .7626$ and $t^f = .763$. With $\sigma = 1$, the difference in tax rates is actually sufficient to imply that all firm types earn lower after-tax profit under FA.

When the two systems generate different equilibrium tax rates, differences in domestic and foreign tax revenues are now defined by

$$\Delta DTR(t^s, t^f, \theta_i) \equiv DTR^{si}(t^s, \theta_i) - TR_i^i(t^f, \theta_i) = \Delta DTR(t^s, \theta_i) + TR_i^i(t^s, \theta_i) - TR_i^i(t^f, \theta_i) \quad (5.15)$$

and

$$\Delta FTR(t^s, t^f, \theta_i) \equiv FTR^{si}(t^s, \theta_i) - TR_i^i(t^f, \theta_i) = \Delta FTR(t^s, \theta_i) + TR_i^i(t^s, \theta_i) - TR_i^i(t^f, \theta_i). \quad (5.16)$$

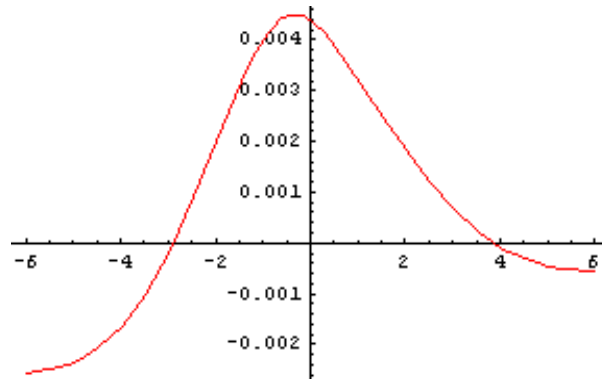


Figure 3: $\Pi_j(t^f, \theta_j) - \Pi_j^s(t^s, \theta_j)$ for $\sigma = .86$

Eqs. (5.15) and (5.16) differ from (5.13) and (5.14) by a common term that measures the difference in FA tax revenues at t^s and at t^f . As long as t^s is less than t^f or t^s is not too much larger than t^f , Proposition 2 tells us that this difference must either be strictly decreasing in t^f for all θ_i , or decreasing in t^f for low values of θ_i and increasing in t^f for high values. For the example reported above in which $\sigma = 1$, the latter case arises and (5.15) is shown in Figure 4. Notice that an increase in the equilibrium tax rate when moving to FA shifts the curve in Figure 2a down and may cause it to rotate counterclockwise. This implies that each country will earn higher tax revenues under FA from its lowest cost domestic firms. These lowest cost firms would pay equal domestic taxes under SA and FA when $t^s = t^f$ but when a shift to FA raises the equilibrium rate, these firms will by Proposition 2 pay greater taxes. This will be true regardless of whether case (a) or case (b) from Proposition 2 arises. If case (b) arises, then each country may also collect fewer tax revenues under FA from its highest cost domestic firms. For a small decrease in the equilibrium tax rate when moving to FA, the negative region in Figure 2a will arise for fewer types. The type representing the lower bound for this region will increase and the upper bound ($+\infty$ in Figure 2a) may decrease. Regardless of the Proposition 2 case that arises, a shift to FA that does not lower the equilibrium tax rate too much will generate a non-monotonic relationship between firm type and the change in domestic tax revenues that is due to the fact that an increase in the tax rate reduces profit for high cost firms first. For a large decrease in the equilibrium tax rate, FA will yield uniformly lower domestic tax revenues.

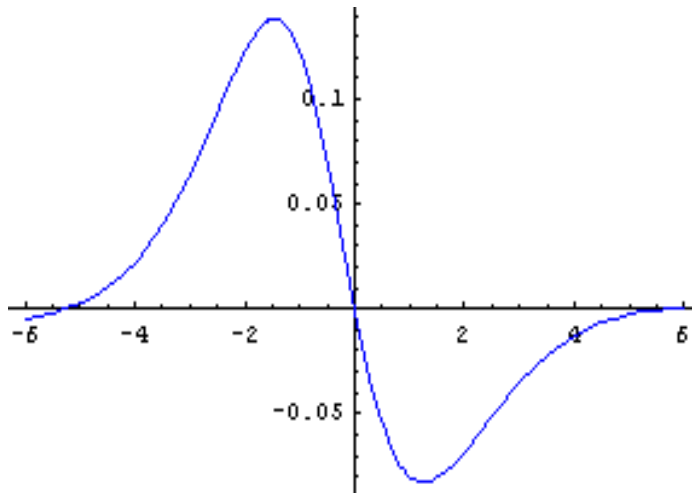


Figure 4: $\Delta DTR(t^s=.756, t^f=.763, \theta_i)$

Proposition 9. Domestic Tax Revenues.

- a) If $t^s < t^f$, then there exist types $\delta^1 < \delta^2 < \delta^3 \leq \infty$ such that each country will earn higher tax revenues under FA from its domestic firms if, and only if, $\theta_i < \delta^1$ or $\delta^2 < \theta_i < \delta^3$.
- b) If t^s is not too much larger than t^f , then there exist types $\delta^4 < \delta^5 < \infty$ such that each country will

earn higher tax revenues under FA from its domestic firms if, and only if, $\delta^4 < \theta_i < \delta^5$.

Figure 4 illustrates Proposition 9a where $\delta_1 \approx -4.6$, $\delta_2 \approx 0$, and $\delta_3 \approx 5.5$.

To assess tax revenue changes from foreign-owned subsidiaries first suppose $t^s < t^f$. If Proposition 2a applies, the total change in foreign tax revenues is described by shifting ΔFTR in Figure 2b down for all θ_i . This must increase the set of foreign subsidiary types that yield more tax revenue under FA and will now include the highest-cost foreign subsidiaries. If Proposition 2b applies, the left side of ΔFTR will shift down and the right side will shift up. Each country will still generate higher tax revenues from some low-cost foreign subsidiaries but not from as many of the highest-cost ones. Second, suppose t^s is a little larger than t^f . If Proposition 2a applies, the total change in foreign tax revenues corresponds to an upward shift in ΔFTR for all θ_i . This shift decreases the set of foreign types that pay more taxes under FA. If Proposition 2b applies, the left side of ΔFTR still shifts up but the right side will shift down. Each country may now collect higher taxes from the highest cost foreign subsidiaries and will collect lower taxes from the lowest cost foreign subsidiaries. Proposition 10 summarizes this analysis.

Proposition 10. Foreign Tax Revenues.

- a) If $t^s < t^f$, then there exist types $0 < \phi^1 < \phi^2 \leq \infty$ such that each country will earn higher tax revenues under FA from its foreign firms if, and only if, $\theta_i < \phi^1$ or $\theta_i > \phi^2$.
- b) If t^s is not too much larger than t^f , then there exist types $\phi^3 < \phi^4 < \phi^5 \leq \infty$ such that each country will earn higher tax revenues under FA from its foreign firms if, and only if, $\phi^3 < \theta_i < \phi^4$ or $\theta_i > \phi^5$.

Propositions 8 - 10 indicate that the welfare implications of a shift from SA to FA will vary based on firm costs. As long as $t^s < t^f$ or t^s is not too much larger than t^f , the only types that will earn higher profit, and from which the countries will see an increase in both domestic and foreign tax revenues, fall in an intermediate range of low to average costs (δ^2 to ϕ^1 or δ^4 to ϕ^4). In general, a shift to FA creates a trade-off between domestic and foreign sources of tax revenues. Expected tax revenues will be larger under FA unless t^s is sufficiently smaller than t^f , something that requires very precise auditing.

6. Conclusion

The problem of how best to apportion multinational profits between countries is inherently a private information problem. Nonetheless, most of the literature comparing SA and FA either assumes complete information or assumes private information only in the SA analysis. To the best of my knowledge, this is the first paper to formally incorporate private information in an equilibrium comparison of both tax systems. In addition, I have introduced actual compliance activity in the SA model in the form of noisy auditing. (While government compliance costs have not been modeled, the adjustment functions do reflect some of the compliance costs firms face. Any model in which overall

compliance costs are proportional to $g(\cdot)$ will generate similar results.) By focusing on the private information effects for **both** SA and FA, I can identify how a change from SA to FA will affect firm profit and tax revenues collected from domestic and foreign firms as a function of firm costs. Multinational profits and country tax revenues not only exhibit type-specific differences, the tax revenue differences depend on the location of the each firm's parent company. Domestic tax revenues and foreign tax revenues are shown to respond to a change from SA to FA in different ways. Moreover, the tax revenue changes do not predict changes in firm profit.

The model has three limitations that will be important to address in future research: symmetric markets and countries, the lack of general equilibrium effects via input markets, and the restriction to two firms. Because of the model's symmetries, no profit-shifting or production-shifting incentives persist in equilibrium. Small differences in either market revenues or country preferences would not alter the marginal tax competition incentives significantly but they may introduce level effects that could bias performance towards either system. While the current paper does not incorporate market or country asymmetries, it does provide a framework for investigating such effects. For instance, Mintz and Smart (2004) study the behavior of multiprovince firms in Canada which allows firms to choose between SA and FA. A model in which differential equilibrium tax rates arise would be needed to understand whether allowing firms to self-select the tax system under which they will operate generates adverse or positive screening effects for the countries. The framework of the model can also allow one to incorporate general equilibrium effects via input markets. Input markets were not formally modeled in this paper to reduce the complexity of the analysis. With regard to the number of firms, the model does permit a straightforward extension by allowing for a continuum of multinationals, each operating in distinct product markets. All of the paper's results generalize to this case simply by reinterpreting each country's expected tax revenues as now equaling total tax revenues. Introducing competition among the multinationals adds a dimension to the analysis that was beyond the scope of this paper.

Finally, one advantage of using the revenue formula in this paper is that it allowed for an explicit derivation of tax competition effects. The fact that many of the same non-monotonicities appear in both this paper and in Gresik (2008), which studies optimal SA and FA rules, suggests that the observed patterns will be robust to the choice of a different formula and to the introduction of a more complex information structure. Studying the tax competition effects of different formulas and introducing multidimensional incomplete information into a tax competition analysis represents yet another important direction for future research.

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Appendix

Proof of Lemma 1.

a. Eq. (3.8) implies that Π_i is clearly decreasing in θ_i and (3.7) implies that $\partial q^*/\partial \theta_i = g/R'' < 0$. Given (3.11), $TR^i(t, \theta_i)$ must be decreasing in θ_i .

b. Differentiating (3.11) implies

$$\partial TR^i(t, \theta_i)/\partial t = (1/(1-t)^2)[\Pi_i(t, \theta_i) + 2kq^*(t, \theta_i)] + (t/(1-t))[\partial \Pi_i(t, \theta_i)/\partial t + 2k\partial q^*(t, \theta_i)/\partial t].$$

Thus,

$$\frac{\partial^2 TR^i(t, \theta_i)}{\partial t \partial \theta_i} = \frac{1}{(1-t)^2} \left[\frac{\partial \Pi_i}{\partial \theta_i} + 2k \frac{\partial q^*}{\partial \theta_i} \right] + \frac{t}{1-t} \left[\frac{\partial^2 \Pi_i}{\partial t \partial \theta_i} + 2k \frac{\partial^2 q^*}{\partial t \partial \theta_i} \right]. \quad (\text{A.1})$$

Using (3.8), (A.1) simplifies to

$$\frac{\partial^2 TR^i}{\partial t \partial \theta_i} = -2q^*g + \frac{2k}{(1-t)^2} \frac{\partial q^*}{\partial \theta_i} - 2tg \frac{\partial q^*}{\partial t} + \frac{2tk}{1-t} \frac{\partial^2 q^*}{\partial t \partial \theta_i}. \quad (\text{A.2})$$

Since $\partial q^*/\partial t = k/(1-t)^2 R'' < 0$, the difference of the middle two terms in (A.2) is negative. Comparative statics also implies $\partial^2 q^*/\partial t \partial \theta_i = -kgR'''/((1-t)^2(R'')^3)$ which will be negative as long as R''' is negative (which is implied by the concavity of R'). Therefore, (A.2) is negative.

It is also the case that

$$\partial^2 TR^i/\partial t^2 = 2(\Pi_i + K)/(1-t)^3 + 2[\partial \Pi_i/\partial t + 2k\partial q^*/\partial t]/(1-t)^2 + (t/(1-t))[2k\partial^2 q^*/\partial t^2 + \partial^2 \Pi_i/\partial t^2]. \quad (\text{A.3})$$

Since, $\partial \Pi_i/\partial t = -(\Pi_i + K)/(1-t)$, (A.3) simplifies down to

$$\partial^2 TR^i/\partial t^2 = [(2-t)/(1-t)^2] \cdot [2k\partial q^*/\partial t] + (t/(1-t))2k\partial^2 q^*/\partial t^2 \quad (\text{A.4})$$

where

$$\partial^2 q^*/\partial t^2 = -[k/(1-t)^2(R'')^3] \cdot [-(R'')^2 + kR''']$$

is negative when R' is concave. Hence, (A.4) is negative which means $TR^i(t, \theta_i)$ is strictly concave in t .

c. With $TR^i(t, \theta_i) = (t/(1-t))[\Pi_i(t, \theta_i) + 2kq^*(t, \theta_i)]$, differentiating twice with respect to θ_i yields

$$\frac{\partial^2 TR_i^i(t, \theta_i)}{\partial \theta_i^2} = \frac{t}{2(1-t)} \left[\frac{\partial^2 \Pi_i}{\partial \theta_i^2} + 2k \frac{\partial^2 q^*}{\partial \theta_i^2} \right]. \quad (\text{A.5})$$

Eq. (A.5) suggests that if $q^*(t, \theta_i)$ is approximately linear in θ_i , then the curvature of $TR^i(t, \theta_i)$ will be inherited primarily from $\Pi_i(t, \theta_i)$. Differentiating (3.9) twice with respect to θ_i implies that, when $g'(\cdot)$ is negative, equilibrium firm profit and equilibrium tax revenues must be locally convex. When $g'(\cdot)$ is sufficiently positive, equilibrium firm profit and equilibrium tax revenue will be locally concave. **Q.E.D.**

Proof of Proposition 2. The proof involves showing that for any $t^i = t^j = t$, $\partial TR^i(t^i, t^j, \theta_i)/\partial t^j > 0$ for all

θ_i . Recall that

$$TR^i(t^i, t^j, \theta_i) = 2TR_i^i(t^i, t^j, \theta_i) = \frac{2t^i R(q_i^i)}{R(q_i^i) + R(q_i^j)} [R(q_i^i) + R(q_i^j) - C(q_i^i, q_i^j, G(\theta_i))] \quad (\text{A.6})$$

where it is understood that $q_i^i = q_i^i(t^i, t^j, \theta_i)$ and $q_i^j = q_i^j(t^i, t^j, \theta_i)$. Differentiating the right-hand side of (A.6) with respect to q_i^i yields

$$2t^i [R'(q_i^i)R(q_i^j)(R(q_i^i) + R(q_i^j) - C)/(R(q_i^i) + R(q_i^j))^2 + R(q_i^i)(R'(q_i^i) - C_1)/(R(q_i^i) + R(q_i^j))] \quad (\text{A.7})$$

which is positive and differentiating the right-hand side of (A.6) with respect to q_i^j yields

$$2t^i [-R(q_i^i)R'(q_i^j)(R(q_i^i) + R(q_i^j) - C)/(R(q_i^i) + R(q_i^j))^2 + R(q_i^i)(R'(q_i^j) - C_2)/(R(q_i^i) + R(q_i^j))]. \quad (\text{A.8})$$

The bracketed term in (A.8) equals

$$\frac{R(q_i^i)G(\theta_i)}{(R(q_i^i) + R(q_i^j))^2} [R'(q_i^j)(q_i^i + q_i^j) - R(q_i^i) - R(q_i^j)] \quad (\text{A.9})$$

which must be negative given the concavity of the revenue function at $q_i^i = q_i^j = q^*$.

With $C_{12} = 0$, comparative statics reveal that $\partial q_i^i / \partial t^j > 0$ and $\partial q_i^j / \partial t^j < 0$. Combining these signs with the signs of (A.7) and (A.8) implies that $\partial TR^i(t^i, t^j, \theta_i) / \partial t^j$ is strictly positive at $t^i = t^j = t$.

Recall from Lemma 1b, that TR^i is concave in t and $\partial TR^i / \partial t$ is decreasing in θ_i . Moreover, TR^i is increasing in t for all θ_i at $t = 0$ and decreasing in t for all θ_i for t sufficiently close to 1. Because

$$\partial TR^i(t, \theta_i) / \partial t = 2(\partial TR_i^i(t^i, t, \theta_i) / \partial t^i)|_{t^i=t} + 2(\partial TR_i^i(t, t^j, \theta_i) / \partial t^j)|_{t^j=t}, \quad (\text{A.10})$$

part (a) follows from the fact that both derivatives in (A.10) are positive for all θ_i if $t < \underline{T}$. To establish

part (b), if $t > \bar{T}$, the left-hand side of (A.10) must be negative for all θ_i while the second term on the right-hand side of (A.10) must be positive for all θ_i . Therefore the first term on the right-hand side of (A.10) must be negative for all θ_i . In order for t to be a symmetric equilibrium tax rate, (3.5) requires the expected value of $\partial TR_i^i(t^i, t, \theta_i) / \partial t^i$ to be zero at $t^i = t$. Thus, by contradiction, t must be no greater than \bar{T} . **Q.E.D.**

Proof of Proposition 3.

a. At $\rho_j = G(\theta_j)$, (4.10) and (4.12) implies

$$\partial P_H / \partial \rho_j = F[\Sigma^{-1}(\theta_j - \theta^+(\beta))|\theta_j] + (G(\theta_j) - G[\theta_j - \theta^+(\beta)])F[\Sigma^{-1}(\theta_j - \theta^+(\beta))|\theta_j](\Sigma^{-1})'(G^{-1})'(G(\theta_j)) \quad (\text{A.11})$$

and

$$\begin{aligned} \partial P_L / \partial \rho_j &= -(1 - F[\Sigma^{-1}(\theta_j + \theta^+(\beta))|\theta_j]) \\ &\quad - (G[\theta_j + \theta^+(\beta)] - G(\theta_j))F[\Sigma^{-1}(\theta_j + \theta^+(\beta))|\theta_j](\Sigma^{-1})'(G^{-1})'(G(\theta_j)). \end{aligned} \quad (\text{A.12})$$

Due to the symmetry of the normal distribution, (A.11) and (A.12) sum to zero when $\theta_j = 0$. For $\theta_j > 0$, each term in (A.11) is greater in magnitude than its counterpart in (A.12). Hence, $\partial P_H / \partial \rho_j + \partial P_L / \partial \rho_j > 0$. Since profit-maximization implies that $P_H + P_L$ is minimized, $\rho^*(\theta_j) < G(\theta_j)$. A similar argument applied to the case when $\theta_j < 0$ implies $\rho^*(\theta_j) > G(\theta_j)$.

b. Let $\theta'_j > \theta_j > 0$. Evaluating $\partial P_H / \partial \rho_j + \partial P_L / \partial \rho_j$ for θ'_j at $\rho_j = \rho^*(\theta_j)$ results in

$$\begin{aligned} \frac{\partial P_H(\rho^*(\theta_j), \theta'_j)}{\partial \rho_j} + \frac{\partial P_L(\rho^*(\theta_j), \theta'_j)}{\partial \rho_j} &= F(\Sigma^{-1}(G^{-1}(\rho^*(\theta_j)) - \theta^+) | \theta'_j) - 1 + F(\Sigma^{-1}(G^{-1}(\rho^*(\theta_j)) + \theta^+) | \theta'_j) \\ &+ [\rho^*(\theta_j) - G(G^{-1}(\rho^*(\theta_j)) - \theta^+)] f(\Sigma^{-1}(G^{-1}(\rho^*(\theta_j)) - \theta^+) | \theta'_j) (\Sigma^{-1})'(G^{-1})'(\rho^*(\theta_j)) \\ &- [G(G^{-1}(\rho^*(\theta_j)) + \theta^+) - \rho^*(\theta_j)] f(\Sigma^{-1}(G^{-1}(\rho^*(\theta_j)) + \theta^+) | \theta'_j) (\Sigma^{-1})'(G^{-1})'(\rho^*(\theta_j)). \end{aligned} \quad (\text{A.13})$$

By definition, the right-hand side of (A.13) equals zero when $\theta'_j = \theta_j$.

First, note that $F(x | \theta_j) - F(x | \theta'_j) = \Phi((x - \theta_j) / \sigma) - \Phi((x - \theta'_j) / \sigma)$ where $\Phi(\cdot)$ is the standardized normal distribution. For $\theta'_j > \theta_j$, this difference is positive and the first line of (A.13) is decreasing in θ'_j .

Second, $\rho^*(\theta_j) - G(G^{-1}(\rho^*(\theta_j)) - \theta^+)$ and $G(G^{-1}(\rho^*(\theta_j)) + \theta^+) - \rho^*(\theta_j)$ are both positive. Since Σ^{-1} is linear and increasing, the second and third lines of (A.13) represent the difference between two strictly positive terms at any θ'_j .

Suppose this difference is negative at $\theta'_j = \theta_j$ and consider the marginal effect of increasing θ'_j at $\theta'_j = \theta_j$. The first line of (A.13) must be positive under this assumption. The only way this can be true is if $F(\Sigma^{-1}(G^{-1}(\rho^*(\theta_j)) + \theta^+) | \theta_j) > 1/2$ or $\Sigma^{-1}(G^{-1}(\rho^*(\theta_j)) + \theta^+) > \theta_j$. But this means $f(\Sigma^{-1}(G^{-1}(\rho^*(\theta_j)) + \theta^+) | \theta'_j)$ must be increasing in θ'_j at $\theta'_j = \theta_j$. If $\Sigma^{-1}(G^{-1}(\rho^*(\theta_j)) - \theta^+)$ is also greater than θ_j , then $f(\Sigma^{-1}(G^{-1}(\rho^*(\theta_j)) - \theta^+) | \theta'_j)$ will also be increasing in θ'_j at $\theta'_j = \theta_j$ but at a slower rate than $f(\Sigma^{-1}(G^{-1}(\rho^*(\theta_j)) + \theta^+) | \theta'_j)$. In this case, the last two lines in (A.13) together must be decreasing in θ'_j at $\theta'_j = \theta_j$. Alternatively, if $\Sigma^{-1}(G^{-1}(\rho^*(\theta_j)) - \theta^+)$ is less than θ_j , then $f(\Sigma^{-1}(G^{-1}(\rho^*(\theta_j)) - \theta^+) | \theta'_j)$ is decreasing in θ'_j at $\theta'_j = \theta_j$. Now each line of (A.13) is decreasing in θ'_j at $\theta'_j = \theta_j$. Therefore, when the last two lines of (A.13) sum to a negative value, the right-hand side of (A.13) is strictly decreasing in θ'_j at $\theta'_j = \theta_j$. Thus, $\rho^*(\theta'_j)$ must be greater than $\rho^*(\theta_j)$ for θ'_j near θ_j . Since θ_j was chosen arbitrarily, $\rho^{*\prime}(\theta_j)$ must be positive.

If instead this difference is positive, the first line of (A.13) must be negative. Analogous arguments again imply that $\rho^{*\prime}(\theta_j)$ must be positive. Similar arguments hold as well when $\theta'_j < \theta_j < 0$.

Q.E.D.

Proof of Proposition 4.

To simplify the forthcoming expressions, let $P(\rho, \theta_j) = P_H(\rho, \theta_j) + P_L(\rho, \theta_j)$. By the Envelope Theorem, $dP(\rho^*(\theta_j), \theta_j)/d\theta_j = P_2(\rho^*(\theta_j), \theta_j)$. Since $\rho^*(\cdot)$ is strictly increasing, $P_{12}(\rho, \theta_j)$ is strictly negative at $\rho = \rho^*(\theta_j)$. For $\theta_j > 0$, this negative cross-partial implies that $P_2(\rho^*(\theta_j), \theta_j) < P_2(1/2, \theta_j)$. For $\theta_j < 0$, this negative cross-partial implies that $P_2(\rho^*(\theta_j), \theta_j) > P_2(1/2, \theta_j)$. To prove the desired result, we will show that $\theta_j \cdot P_2(1/2, \theta_j) < 0$. Note that

$$P_2(1/2, \theta_j) = \int_{\mu=-\infty}^{-\Sigma^{-1}(\theta^*)} [1/2 - G(\Sigma(\mu))] f_2(\mu|\theta_j) d\mu + \int_{\mu=\Sigma^{-1}(\theta^*)}^{\infty} [G(\Sigma(\mu)) - 1/2] f_2(\mu|\theta_j) d\mu.$$

Changing the variable of integration in the second integral to $-\mu$ implies that

$$P_2(1/2, \theta_j) = \int_{\mu=-\infty}^{-\Sigma^{-1}(\theta^*)} [1/2 - G(\Sigma(\mu))] (f_2(\mu|\theta_j) + f_2(-\mu|\theta_j)) d\mu. \quad (\text{A.14})$$

Since $f_2(\mu|\theta_j) = f_2(-\mu + 2\theta_j|\theta_j)$ and since $\mu < 0$ in (A.14), $f_2(\mu|\theta_j) < f_2(-\mu|\theta_j)$ when $\theta_j > 0$ and $f_2(\mu|\theta_j) > f_2(-\mu|\theta_j)$ when $\theta_j < 0$. Consequently, $\theta_j \cdot P_2(1/2, \theta_j) < 0$. **Q.E.D.**

Proof of Proposition 5.

Consider the limit as θ_j tends to $+\infty$. Clearly, P_L goes to zero. Thus, we need to show that P_H converges to zero as well. From (4.10), note that for θ_j sufficiently large,

$$P_H(\rho^*(\theta_j), \theta_j) \leq \Gamma(\theta_j) \equiv \int_{\mu=-\infty}^{\Sigma^{-1}(\theta_j)} (G(\theta_j) - G(\Sigma(\mu))) f(\mu|\theta_j) d\mu. \quad (\text{A.15})$$

Alternatively,

$$\Gamma(\theta_j) = G(\theta_j) F(\Sigma^{-1}(\theta_j)|\theta_j) - \int_{\mu=-\infty}^{\Sigma^{-1}(\theta_j)} \int_{t=-\infty}^{\Sigma(\mu)} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} \cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(\mu-\theta_j)^2/2\sigma} dt d\mu. \quad (\text{A.16})$$

If one changes the first variable of integration in (A.16) to $w = (\mu - \theta_j)/\sigma$, then (A.16) can be written as

$$G(\theta_j) F(\Sigma^{-1}(\theta_j)|\theta_j) - \int_{w=-\infty}^{\sigma\theta_j} \int_{t=-\infty}^{\Sigma(\sigma w + \theta_j)} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} \cdot \frac{1}{\sqrt{2\pi}} e^{-w^2/2} dt d\mu. \quad (\text{A.17})$$

As θ_j goes to ∞ , $\sigma\theta_j$ and $\Sigma(\sigma w + \theta_j)$ both converge to ∞ . The integral in (A.17) thus converges to 1.

Since the first term in (A.17) can be no greater than 1, (A.17) converges to 0. Thus,

$\lim_{\theta_j \rightarrow \infty} P_H(\rho^*(\theta_j), \theta_j) \leq 0$. But since P_H must be non-negative, the limiting value must be zero. A similar argument applies to the limit as θ_j tends to $-\infty$. **Q.E.D.**