

# Optimal Separate Accounting vs. Optimal Formula Apportionment

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**Abstract:** American states and Canadian provinces use formulas to allocate a multi-state company's national profit among the state-level jurisdictions. The formulas typically depend on the company's distribution of revenues and costs. In contrast, most countries allocate a multinational's global profit for the purposes of calculating national tax liabilities by treating each subsidiary as a separate corporation and requiring the multinational to use transfer prices to reflect the value of any trade between its divisions. Motivated in part by recent interest by the EU in replacing the current transfer price system with a formula apportionment system, this paper derives and compares the optimal transfer pricing regulation with the optimal apportionment formula in a model of multinational taxation that allows for both private information and unobservable investments. Conditions are derived under which the optimal transfer price regulation and the optimal apportionment formula are welfare equivalent. In the absence of the equivalence conditions, formula apportionment is shown to create a trade-off between inducing more efficient firm investment and rent extraction. The welfare implications of this trade-off are examined.

**Keywords:** Transfer pricing, formula apportionment, private information, unobservable investment

**JEL Codes:** H21, H73, F23, D82

## 1. Introduction.

With the increase in worldwide economic integration witnessed over the last several decades, national governments have faced pressure to examine the efficacy of their tax systems which may have been designed when international opportunities were less prevalent. The effort to design a tax system that reflects the realities of highly integrated national economies is perhaps best illustrated within the EU which recognizes that "[l]arge EU companies now view the whole EU as their 'home market' and accordingly seek to establish effective pan-European business structures." (EC 2001, p.5) To promote "greater efficiency, effectiveness, simplicity and transparency in company tax systems and remove the hiatuses between national systems which provide fertile ground for avoidance and abuse" (EC 2001, p.10) the European Commission "believes that it is necessary to

- provide companies with a consolidated corporate tax base for their EU-wide activities;
- develop an appropriate apportionment mechanism which can be agreed by all participants;
- and, for Member States, to determine the applicable national corporate tax rates." (EC 2001, p.16)

In 2002, the European Commission (2002) advanced four proposals for an EU-wide policy on the taxation of multinationals.<sup>1</sup> Of these, the first two proposals have received the most serious consideration by the EU. The first proposal, called Home State Taxation, incorporates the last two objectives by allowing an EU company to calculate its EU profit using the tax base definitions of its home country thereby eliminating the need to confront 15 or more different sets of tax rules.<sup>2</sup> The second proposal, called the Common Consolidated Corporate Tax Base (CCCTB), incorporates all three objectives by

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<sup>1</sup>See Mintz and Martens-Weiner (2003), Zodrow (2004), Devereux (2004), Hellerstein and McLure (2004), Mintz (2004) and Sørensen (2004) for descriptions and critiques of these proposals.

<sup>2</sup>In 2005, the EU authorized a pilot program<sup>2</sup> in which small and medium-size EU companies could employ Home State Taxation procedures (EC 2005).

creating a single tax-base definition EU companies may use and using an apportionment formula to calculate the national tax liabilities of EU companies. With formula apportionment, an EU company reports its total EU profit to each country and then a (currently unspecified) formula that could be based upon factors such as revenue shares, wage shares, and/or capital shares will determine the portion of the company's total EU profit that would be taxed in each country.

The idea of using apportionment formulas has been used by some state/provincial governments for multi-state taxation but is a departure from the predominant system used by most countries for multinational taxation, separate accounting. With separate accounting, each subsidiary of a multinational is treated as a separate corporation. To calculate each subsidiary's taxable income, any transactions between subsidiaries of the same multinational are accounted for by a transfer price, a real transfer of funds from the downstream subsidiary to the upstream subsidiary. Differences in national income tax rates create an incentive for multinationals to use their transfer prices to shift profits from high-tax jurisdictions to low-tax jurisdictions, and thus require some form of costly auditing to minimize the amount of profit-shifting. While apportionment formulas can eliminate the need for transfer prices and the related auditing, they too create investment, production, and location distortions (see Gordon and Wilson (1986)). Since both formula apportionment and separate accounting systems can affect a tax system's performance, the purpose of this paper is to compare both systems in a common economic environment with an emphasis on differences in efficiency, tax revenues, and avoidance.

While there exists an extensive literature that studies taxation under separate accounting and a somewhat smaller literature that studies taxation under formula apportionment, few papers have sought to compare the two systems under identical economic assumptions and fewer still have attempted to make comparisons in economic environments that include the fundamental reasons taxing multinationals is more challenging for governments than taxing domestic firms: unobservable cost or revenue complementarities and incomplete information about a multinational's technology and input choices (see

Gresik (2001) for a detailed description of these challenges). For example, Nielsen, Raimondos-Møller, and Schjelderup (2009), Sørensen (2003), and Gérard (2005) compare the economic performance of both systems in a complete information environment in which there are homogeneous firms and all firms face the same reduced form cost to profit-shifting. Kind, Midelfart, and Schjelderup (2005) conduct a similar exercise but include a role for trade costs. None of these studies allow for firm heterogeneity and all are positive analyses that compare the equilibrium outcomes of a specific separate accounting protocol to a specific apportionment formula.

Consistent with the rich literature on optimal income taxation initiated by the seminal work of Mirrlees (1971), Seade (1977), and Stiglitz (1982) and the rich literature on optimal regulation initiated by the seminal work of Baron and Myerson (1982), this paper advances the current literature comparing formula apportionment to separate accounting in two main ways. First, the paper explicitly introduces a model in which a multinational has private information about its costs and can take actions unobservable by governments. The private information explicitly captures the reality of firm heterogeneity which Burbidge, Cuff, and Leach (2006, p.544), in their study of profit tax systems, show can lead to "substantial differences" in economic outcomes relative to complete information analyses while also permitting an analysis of the role of information rent distortions in tax systems based on separate accounting or formula apportionment.

Second, the paper adopts a normative approach whose focus is on the (private-information specific) allocations that can be generated by optimally designed separate accounting rules and by optimally designed apportionment formulas. The focus on allocations consistent with a wide range of possible separate accounting rules and on allocations consistent with a wide range of apportionment formulas has two benefits. By focusing on allocations, one can abstract away from many details of each system that may be useful in implementing a given allocation and it allows one to identify the general economic trade-offs between separate accounting and formula apportionment as opposed to those trade-

offs that may be specific to particular separate accounting rules and apportionment formulas. Thus, the normative analysis of this paper complements the positive (equilibrium) analysis of these two systems.<sup>3</sup>

This optimal taxation exercise begins in section 2 with a description of a model that captures several of the salient characteristics of multinational tax problems: cost and/or revenue complementarities, private (type) information about the multinational's technology, observable and unobservable costs, and unobservable resource decisions. The first two characteristics are necessary for the question of how best to divide up a multinational's profit among the subsidiaries operating in different countries to have a non-trivial answer. If there are no cost or revenue complementarities, the multinational is really just a collection of independent firms, each of which has well-defined taxable income in its country of incorporation. If the tax authorities in each government can perfectly observe all aspects of the multinational's operations and opportunities, first-best tax policy is feasible and can be implemented without using costly procedures whose *raison d'etre* is to gain information about the firm's operations and profitability. The third characteristic reflects the reality of tax policy in using observable cost data for verification reasons. This reality has been emphasized by Hellerstein and McLure (2004) in their evaluation of the EU proposals. In particular, they note that some of the data apportionment formulas use, such as capital valuations, have information problems akin to those associated with transfer prices. Nielsen, Raimondos-Møller, and Schjelderup (2003) show that these information problems can be exacerbated if the multinationals have market power. Finally, if the tax authorities can observe all the actions a multinational makes, it may be able to infer the necessary information it lacks about the multinational. However if, in addition to the multinational having private information about its profits, it

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<sup>3</sup>Gresik (2009) is a companion paper in which both firm heterogeneity and private firm information are introduced into a corporate income tax competition model to compare the positive equilibrium outcomes associated with a specific set of separate accounting rules and a revenue apportionment formula.

also makes decisions that cannot be observed by the tax authorities, then the individual countries will lack sufficient information to distinguish between cost and revenue levels that are due to inefficient technologies versus inefficient resource choices for all but the most trivial cases.

In section 3, I use this model to solve three different optimal tax mechanism problems from the perspective of a tax union that, consistent with the approach being taken by the European Commission outlined above, seeks to agree on a general procedure. The first allows for a completely general non-linear tax function and yields the second-best tax mechanism. The second imposes restrictions on the set of feasible tax functions common to all separate accounting regulations. The third imposes restrictions on the set of feasible tax functions common to all apportionment formulas. The additional restrictions associated with either separate accounting or formula apportionment are imposed because the current debate in the EU is between separate accounting and formula apportionment and has not embraced an examination of more general tax procedures because doing so would require EU countries to give up control over national tax rates. However, by comparing the allocation from the optimal separate accounting mechanism and the allocation from the optimal formula apportionment mechanism to the optimal second-best allocation, we are able to assess the potential welfare losses associated with preserving national tax rate sovereignty.

The taxes a multinational owes in each country are calculated based on each country's corporate tax rate. Since separate accounting and formula apportionment allocate taxable profit differently, the potential for either system to do worse than the optimal second-best mechanism clearly exists. Separate accounting works by dividing up shared costs or revenues and by effectively ignoring country-specific profits. Formula apportionment divides union-wide profits regardless of whether they are country-specific or due to complementarities. The first main result of the paper is however an equivalence result. Theorem 3 provides sufficient conditions under which the optimal second-best mechanism, the optimal separate accounting mechanism, and the optimal formula apportionment mechanism are all welfare

equivalent. This equivalence is referred to as a tax-incentive dichotomy to parallel the terminology used in Laffont and Tirole (1994). The sense in which there is a dichotomy is that the form of the tax procedure is independent of the optimal output and cost distortions. The main economic condition for a tax-incentive dichotomy to exist is that either the multinational's observable costs or its unobservable costs depend on the unobservable resource decisions via a single aggregation function. With what amounts to a one-dimensional resource decision, the ability of the tax union to observe some of the multinational's costs gives it sufficient information to implement the second-best level of taxation.

Section 4 compares the economic differences induced by all three mechanisms in the absence of a tax-incentive dichotomy. Proposition 4 shows that the optimal separate accounting mechanism distorts the multinational's resource decisions in a way that increases observable costs while leaving intact the union's ability to extract rents relative to the second-best direct mechanism. Separate accounting mechanisms are more limited in their ability to influence the multinational's internal resource decisions than the optimal direct mechanism. This limitation encourages the multinational to generate higher observable costs as higher observable costs increase the multinational's capacity for profit-shifting. In the absence of a tax-incentive dichotomy, the optimal separate accounting mechanism generates higher observable costs, lower unobservable costs, and higher total costs than the optimal second-best mechanism.

Formula apportionment mechanisms have the advantage over separate accounting mechanisms of giving the union more influence over the multinational's internal resource decisions and hence over the distribution of observable and unobservable costs. This advantage creates the potential for an apportionment formula to improve firm efficiency consistent with the EC's first objective stated above but comes at the cost of effectiveness through reduced rent extraction. And if the union seeks to use the optimal apportionment formula to enhance effectiveness through better rent extraction than the optimal separate accounting mechanism, it will come at the cost of lower efficiency. Thus, section 4 reveals a

fundamental tension between the stated goals of efficiency and effectiveness associated with a switch from separate accounting to formula apportionment.

This tension between reducing cost distortions and rent extraction is responsible for several welfare differences between the two systems. Proposition 6 shows that it is possible for the worst-type firm to earn strictly positive profit under the optimal formula apportionment mechanism. In contrast, the worst-type firm always earns zero profit under the optimal direct mechanism and the optimal separate accounting mechanism. Proposition 8 reveals how this same tension affects real output and input decisions under both systems.

Perhaps most surprising is that the difference in union welfare can be non-monotonic in the multinational's type parameter. The greatest welfare disadvantage associated with formula apportionment can be associated with both the most efficient firm types (lowest types) and the least efficient firm types (highest types).<sup>4</sup> It is at these two ends of the type distribution that effective tax rates can be the lowest under formula apportionment in order to encourage more efficient resource decisions. It is not surprising that the union might benefit from offering the most efficient firm type a low effective tax rate to encourage more efficient resource allocation. For slightly higher firm types, the union benefits from increasing the effective tax rate to extract more rent. This comes at the cost of greater resource allocation distortions. What is surprising is the fact that the union may also benefit from offering the least efficient firm types a low effective tax rate. When the cost of inefficient resource allocation is sufficiently large, there will exist a firm type at which the marginal union benefits of increased rent extraction via a higher tax rate equals the marginal union costs of less efficient firm decisions. For less efficient firms than this type, union welfare under formula apportionment is maximized by lowering the effective tax rate in order to generate welfare improving resource decisions. Thus, relative to separate accounting, formula apportionment has the potential to generate less union welfare when its rent extraction is the weakest, i.e.

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<sup>4</sup>Similar non-monotonicities arise in the positive equilibrium analysis in Gresik (2009).

with the most efficient and the least efficient firm types.

Finally, the trade-off between cost distortions and rent extraction implies that either mechanism can yield higher expected union welfare. When the statutory tax rate in the country where multinationals locate upstream production is sufficiently low, the optimal separate accounting mechanism induces small cost distortions relative to second-best levels. In this case, any welfare gains from more efficient input decisions is outweighed by increases in the multinational's rents. When the statutory tax rate in the upstream country is large enough, the optimal formula apportionment mechanism can, but need not, yield higher expected welfare than the optimal separate accounting mechanism through the apportionment mechanism's ability to generate significant cost reductions.

I conclude by discussing directions for future research in section 5.

## **2. Model.**

The model I employ is a variation of the regulatory model found in chapter 3 of Laffont and Tirole (1994). There are two countries, denoted by 1 and 2, and one multinational firm. The firm owns and operates a subsidiary in each country.<sup>5</sup> The subsidiary in country 1 earns revenues (net of local selling costs) of  $R_1(q_1)$  by selling  $q_1$  units of a good in country 1 and the subsidiary in country 2 earns revenues (net of local selling costs) of  $R_2(q_2)$  by selling  $q_2$  units of a good in country 2. Production of each final good requires intermediate goods that are produced exclusively by the subsidiary in country 1. One intermediate good is needed for each final good and variety or quality differences between the final goods sold in country 1 and country 2 may necessitate different intermediate goods.

Revenues and sales quantities are observable by both countries but some of the multinational's

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<sup>5</sup>The term "subsidiary" is used to imply that it is incorporated under the laws of its resident country. This distinguishes the organization of the multinational from that of a branch structure as branches operate under different tax rules than subsidiaries. Since this paper abstracts away from repatriation and double taxation issues, it does not matter which unit if either is the parent corporation.

production costs are unobservable for two reasons. First, managers in the firm need to allocate inputs I refer to as effort to the intermediate good production processes. The allocation is unobservable outside the multinational and so too will its opportunity costs.<sup>6</sup> Second, the multinational has private information about its technological capabilities or the opportunity costs of some of its inputs. Neither the inability of the governments to observe the actual input choices nor the multinational's private information precludes the possibility that the countries can observe, or at least verify for tax purposes, the accounting costs associated with the input choices after they have been generated. In fact, knowing these costs may allow the countries to infer some information about the firm's unobservable choices/private information. What is important for the subsequent analysis is that the countries not be able to verify and hence tax the multinational's full economic profit.<sup>7</sup>

Denote the multinational's pre-tax accounting costs associated with intermediate good production by  $\hat{C}(q, e, \theta)$  where  $q = (q_1, q_2)$  is the vector of production/sales quantities,  $e = (e_1, e_2)$  is the non-negative vector of unobservable input choices made within the multinational, and  $\theta$  is the multinational's private information parameter or type. These costs are tax-deductible and hence must be observable by the tax authorities. Denote opportunity costs by  $\hat{K}(q, e, \theta)$ . Typically, at least some of a firm's opportunity costs are not observable to outside parties and as a result are not tax deductible. Without loss of generality, I assume all opportunity costs are unobservable. Distinguishing between tax-deductible and non-tax-deductible costs is a feature common to almost all tax competition analyses. Two output choices is the minimum necessary to capture the phenomena of profit-shifting and production-shifting due to tax incentives while at least two input choices are needed to make the comparison between optimal separate

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<sup>6</sup>Effort in this model could include any productive input whose total employment or distribution across the multinational is not observable by outside parties.

<sup>7</sup>In the absence of non-tax-deductible costs, a profit tax is a non-distortionary pure profit tax whose optimal value is 100%.

accounting mechanisms and optimal formula apportionment mechanisms non-trivial. This last point will be clarified in section 3. Each country treats  $\theta$  as a random variable drawn from  $[\underline{\theta}, \bar{\theta}]$  with distribution function  $F(\theta)$  and continuous, non-zero density,  $f(\theta)$ . The countries hold common beliefs with regard to the firm's type and these beliefs are common knowledge among the countries and the multinational.

Both  $\hat{C}$  and  $\hat{K}$  can be affected by the multinational's output choice as well as its effort choice and type based on the following assumptions. Subscripts denote derivatives.

**Assumption 1.** a.  $\hat{C}$  is non-negative and continuous for all  $q, e$ , and  $\theta$ .

b.  $\hat{C}_{q_i} > 0$ ,  $\hat{C}_{e_i} < 0$ , and  $\hat{C}_{\theta} \geq 0$ .

c.  $\hat{C}$  is weakly convex in  $q$  and weakly convex in  $e$ .  $\hat{C}_{q,\theta} \geq 0$  and  $\hat{C}_{e,\theta} \geq 0$  for  $i=1,2$ .

$\hat{C}_{q,\theta} > 0$  and  $\hat{C}_{e,\theta} > 0$  for some  $i$ .

**Assumption 2.** a.  $\hat{K}$  is non-negative and continuous for all  $q, e$ , and  $\theta$ .

b.  $\hat{K}_{q_i} \geq 0$ ,  $\hat{K}_{e_i} > 0$ , and  $\hat{K}_{\theta} > 0$ .

c.  $\hat{K}$  is strictly convex in  $q$  and weakly convex in  $e$ .  $\hat{K}_{q,\theta} \geq 0$  and  $\hat{K}_{e,\theta} \geq 0$  for  $i = 1,2$ .

$\hat{K}_{q,\theta} > 0$  and  $\hat{K}_{e,\theta} > 0$  for some  $i$ .

d.  $\lim_{e \rightarrow \infty} \hat{K}_{e_i}(q, e, \theta) = \infty$  for all  $\theta$  and for all  $q > 0$ .

Assumption 1b requires that observable costs are decreasing in each type of effort and increasing in the multinational's output and type. Thus, higher type multinationals have higher observable costs.

Assumption 1c requires diminishing returns in effort and increasing marginal costs of production. It also requires higher type firms to exhibit higher marginal costs of production with respect to at least one of the intermediate goods and smaller cost reductions with respect to at least one type of effort. These last two conditions are analogs to the familiar single-crossing property found in most private information models.

The effects of effort and type on unobservable costs are different from the effects on observable costs to emphasize the role of opportunity costs associated with tax distortions. Assumption 2b requires that unobservable costs are increasing in effort, output, and type. A weaker assumption on  $\hat{K}_{e_i}$  would

allow for unobservable costs to first fall with increases in effort and then rise. The stronger assumption of 2b is used because the weaker assumption only adds complexity to the model without altering the results of the analysis. Together Assumptions 1b and 2b imply higher type firms have higher observable costs and higher unobservable costs. Assumption 2c implies that the marginal costs of effort and production are increasing in effort, output, and type, respectively. This assumption also requires that  $\hat{K}$  be strictly convex in  $e$ . Together with Assumption 2d, which is an Inada condition, strict convexity guarantees that  $T\hat{C} + \hat{K}$  is minimized by a unique, finite vector of effort levels for  $0 < T \leq 1$ .

The tax policies of the countries present the multinational with a non-linear tax that can be a function of  $q$  and  $c$ , the level of observable costs. Together these assumptions imply global after-tax profit for the multinational of

$$\pi(q, e, \theta) = R_1(q_1) + R_2(q_2) - \hat{C}(q, e, \theta) - \hat{K}(q, e, \theta) - \hat{\tau}(q, c) \quad (1)$$

where  $c = \hat{C}(q, e, \theta)$  and  $\hat{\tau}(q, c)$  denotes the non-linear tax function used to calculate the multinational's total taxes. Including  $c$  in the tax function implies that some fraction of accounting costs are tax-deductible. Two specific classes of tax functions,  $\hat{\tau}(\cdot, \cdot)$ , are generated by separate accounting and formula apportionments rules. Under separate accounting, the country 1 subsidiary charges the country 2 subsidiary for the intermediate goods it provides. The price charged is called a transfer price. Transfer prices are routinely audited by tax authorities who seek to discourage multinationals from using the transfer prices to shift profits from the high-tax country into the low-tax country. If we let the function  $s(\cdot, \cdot)$  represent the transfer payments the transfer price regulations allow given output,  $q$ , and observable costs,  $c$ , then the multinational's net taxes equal

$$t_1[R_1(q_1) - \hat{C}(q, e, \theta)] + t_2 R_2(q_2) + (t_1 - t_2)s(q, c)$$

where  $t_1$  and  $t_2$  are the statutory profit tax rates for each country.

Under formula apportionment, a formula,  $a(q, c)$ , determines the fraction of profit taxed by country 1 at the rate  $t_1$ . Country 2 will tax the fraction  $1 - a(q, c)$  at the rate  $t_2$  making the multinational's

taxes equal to

$$[\alpha(q,c)t_1 + (1-\alpha(q,c))t_2][R_1(q_1) + R_2(q_2) - \hat{C}(q,e,\theta)].$$

The countries operate as a partial tax union in the sense that they jointly choose a mechanism to maximize their combined welfare

$$W = v(q) + (1+\lambda)\hat{t} + \alpha\pi \quad (2)$$

where  $v(\cdot)$  equals the social benefits of production and is strictly concave in  $q$ ,  $\lambda > 0$  is the social marginal cost of funds to the countries, and  $\alpha \in [0,1]$  is the welfare weight associated with multinational profit. Eq. (2) treats each country's corporate tax rate,  $t_i$ , as exogenous. The focus on union welfare is consistent with the current EC process that seeks to define an EU-wide policy for allocating multinational income among member states while respecting the ability of individual countries to set tax rates. Recall from the introduction that the first two EC proposals do not attempt to control tax rates but do seek to "develop an appropriate allocation mechanism which can be agreed by all parties." (EC 2001, p.16) Ultimately, it will be important to incorporate tax competition incentives in the analysis by endogenizing the tax rates. The results in this paper are a precursor to that type of analysis.

Substituting (1) into (2) implies

$$W = V(q,\lambda) - (1+\lambda)(\hat{C} + \hat{K}) + (\alpha - 1 - \lambda)\pi \quad (3)$$

where  $V(q,\lambda) = v(q) + (1+\lambda)(R_1(q_1) + R_2(q_2))$ .

### 3. A tax-incentive dichotomy.

#### 3.1 Optimal direct taxation.

Designing the optimal direct taxation mechanism requires that we focus on three aspects of multinational behavior: output, observable costs, and taxes paid. Firm heterogeneity will allow for the possibility that different firms produce different quantities, generate different levels of observable costs, and make different tax payments even when all the firms face the same rules and regulations. A direct tax mechanism describes the allocation each type of the multinational and the union will receive when the

firm reacts strategically to the regulations embodied in the mechanism. Thus, denote a direct taxation mechanism by  $\{q(\hat{\theta}), \bar{c}(\hat{\theta}), \tau(\hat{\theta})\}$  where  $q(\cdot)$  is the output vector of the firm,  $\bar{c}(\cdot)$  is the observable cost of the firm,  $\tau(\cdot)$  is the tax paid to the union, and  $\hat{\theta}$  is the firm's type report. By the Revelation Principle we can restrict attention to incentive compatible mechanisms so that the multinational has the incentive to report its type truthfully.

In order to construct the incentive compatibility constraints, it is important to identify the link between the mechanism and the multinational's effort choices. Thus, define

$$\Delta(q, \epsilon, \theta; \phi) = \operatorname{argmin}_e \hat{C}(q, e, \theta) + \hat{K}(q, e, \theta) \text{ subject to } \phi(e) = \epsilon \quad (4)$$

where  $\phi(e)$  is smooth, weakly increasing in  $e$  and strictly increasing in at least one  $e_i$ , and  $\epsilon \geq 0$ . Define  $e^*(q, \epsilon, \theta; \phi) \in \Delta(q, \epsilon, \theta; \phi)$  to be a continuous selection with respect to  $\theta$ . Continuity of  $\hat{C} + \hat{K}$  and Assumptions 2c and 2d imply  $\Delta$  will be non-empty and that a continuous selection exists. Examples of aggregation functions include  $\phi(e) = e_1 + e_2$ , used in Laffont and Tirole (1994), and the functions  $\phi(e) = e_i$  for  $i=1,2$ . The main benefit of beginning with (4) is that it reduces the information content of the multinational's hidden effort choices down to one dimension. As a result, many different aggregation functions can provide the same dimensionality benefits as the linear rule used by Laffont and Tirole.

Solving (4) allows us to define indirect costs  $C(q, \epsilon, \theta) = \hat{C}(q, e^*(q, \epsilon, \theta; \phi), \theta)$  and  $K(q, \epsilon, \theta) = \hat{K}(q, e^*(q, \epsilon, \theta; \phi), \theta)$ . In general,  $C_\epsilon$  or  $K_\epsilon$  can each be positive or negative. If  $e^{**}$  denotes the effort choices that yield the unconstrained minimum of  $\hat{C} + \hat{K}$ , if the Jacobian of  $\phi$  at  $e^{**}$  has full rank, and if  $\epsilon^{**} \equiv \phi(e^{**})$ , then  $C_\epsilon + K_\epsilon$  will be negative for  $\epsilon < \epsilon^{**}$  and positive for  $\epsilon > \epsilon^{**}$ . Comparative statics on (4) also do not allow one to sign  $C_\epsilon$  or  $K_\epsilon$  individually without additional assumptions because changes in  $\epsilon$  can induce an "income" effect as well as a substitution effect. Thus, define  $e^*$  to be  $\phi$ -normal if  $\partial e^* / \partial \epsilon \geq 0$  for all  $q$  and  $\theta$ . If  $e^*$  is  $\phi$ -normal, then  $C_\epsilon$  must be negative and  $K_\epsilon$  must be positive. Define  $\Phi^N(\hat{C}, \hat{K})$  to be the set of all aggregation functions for which there exists  $e^* \in \Delta$  such that  $e^*$  is  $\phi$ -normal.  $\Phi^N(\cdot, \cdot)$  will be non-empty when  $e_1$  and  $e_2$  are weak complements, in the sense that

$\partial^2(\hat{C} + \hat{K})/\partial e_1 \partial e_2$  is non-positive for all  $e$ , as weak complementarity implies  $\phi(e)=e_i$  is  $\phi$ -normal. This discussion prompts the following assumption.

**Assumption 3.**  $\Phi^N(\hat{C}, \hat{K})$  is non-empty.

$\phi$ -normality is a sufficient condition that allows us to define  $E(q, \bar{c}, \theta)$  as the unique value of  $e$  for which  $C(q, E(q, \bar{c}, \theta), \theta) = \bar{c}$  for any feasible value of  $\bar{c}$ .

Given these definitions, multinational profit can be written as

$$\Pi(\hat{\theta}, \theta) \equiv R_1(q_1(\hat{\theta})) + R_2(q_2(\hat{\theta})) - \bar{c}(\hat{\theta}) - K(q(\hat{\theta}), E(q(\hat{\theta}), \bar{c}(\hat{\theta}), \theta), \theta) - \tau(\hat{\theta}). \quad (5)$$

The mechanism,  $\{q(\hat{\theta}), \bar{c}(\hat{\theta}), \tau(\hat{\theta})\}$ , will be incentive compatible and hence induce  $\hat{\theta} = \theta$  only if

$$d\Pi(\theta)/d\theta \equiv d\Pi(\theta, \theta)/d\theta = -K_e E_\theta - K_\theta < 0 \quad (6)$$

and

$$(\partial^2 \Pi(\hat{\theta}, \theta) / \partial \hat{\theta} \partial \theta) |_{\hat{\theta} = \theta} \geq 0. \quad (7)$$

Condition (6), which follows from the Envelope Theorem, indicates that lower types must earn more profit than higher types. Ineq. (7) is the second-order condition for  $\hat{\theta} = \theta$  to maximize (5). An incentive compatible mechanism will be individually rational if, and only if,  $\Pi(\theta) \geq 0$  for all  $\theta$ . Since  $d\Pi(\theta)/d\theta < 0$ , individual rationality will be satisfied as long as  $\Pi(\bar{\theta}) \geq 0$ .

Two final substitutions are used before stating the final version of the union's optimization problem. First, (5) allows one to define the mechanism in terms of  $\Pi(\theta)$  instead of  $\tau(\theta)$  and makes clear that one role of taxes is to determine the post-tax economic rents earned by the multinational. Second, defining  $\bar{\epsilon}(\theta) = E(q(\theta), \bar{c}(\theta), \theta)$  implies  $\bar{c}(\theta) = C(q(\theta), \bar{\epsilon}(\theta), \theta)$  and shows that the ability of the union to infer information about the multinational's unobservable effort choices implies that the ability to observe some costs,  $\bar{c}$ , is equivalent to having the ability to induce specific levels of aggregate effort,  $\bar{\epsilon}$ . Thus, the union will choose the mechanism  $\{q, \bar{\epsilon}, \Pi\}$  to maximize expected union welfare,

$$\mathcal{E}_\theta W = \mathcal{E}_\theta [V(q, \lambda) - (1 + \lambda)(C(q(\theta), \bar{\epsilon}(\theta), \theta) + K(q(\theta), \bar{\epsilon}(\theta), \theta)) + (\alpha - 1 - \lambda)\Pi(\theta)], \quad (8)$$

subject to (6), (7), and  $\Pi(\bar{\theta}) \geq 0$ . As is usual, we will solve this optimization problem without (7). If the

solution to the relaxed problem violates (7), the optimal mechanism may involve pooling among some of the types.

Integrating (6) to recover  $\Pi(\theta)$  then implies that the optimal mechanism can be found by pointwise maximization of

$$\begin{aligned} & V(q, \lambda) - (1 + \lambda)(C(q, \bar{\epsilon}, \theta) + K(q, \bar{\epsilon}, \theta)) \\ & - (\alpha - 1 - \lambda)(F(\theta)/f(\theta))(-K_\epsilon(q, \bar{\epsilon}, \theta)E_\theta(q, C(q, \bar{\epsilon}, \theta), \theta) - K_\theta(q, \bar{\epsilon}, \theta)) \end{aligned} \quad (9)$$

and by setting  $\Pi(\bar{\theta}) = 0$ . Necessary conditions for the optimal output quantities and the aggregate effort associated with an interior solution are

$$V_{q_k}(q, \lambda) - (1 + \lambda)(C_{q_k}(q, \bar{\epsilon}, \theta) + K_{q_k}(q, \bar{\epsilon}, \theta)) - (\alpha - 1 - \lambda)(F(\theta)/f(\theta))(\partial/\partial q_k)(-K_\epsilon E_\theta - K_\theta) = 0 \quad (10)$$

and

$$-(1 + \lambda)(C_\epsilon(q, \bar{\epsilon}, \theta) + K_\epsilon(q, \bar{\epsilon}, \theta)) - (\alpha - 1 - \lambda)(F(\theta)/f(\theta))(\partial/\partial \epsilon)(-K_\epsilon E_\theta - K_\theta) = 0. \quad (11)$$

While the main results of this section do not require that an interior solution exist, they are helpful for conveying the economic intuition of the optimal direct taxation mechanism. First-order conditions (10) and (11) have standard economic interpretations:

$V_{q_k}(q, \lambda) - (1 + \lambda)(C_{q_k}(q, \bar{\epsilon}, \theta) + K_{q_k}(q, \bar{\epsilon}, \theta))$  is the difference between the marginal social benefit of production and the direct marginal social cost of production while  $(\alpha - 1 - \lambda)(F(\theta)/f(\theta))(\partial/\partial q_k)(-K_\epsilon E_\theta - K_\theta)$  equals the marginal information rent the union must pay to induce truthful reporting. Similarly,  $-(1 + \lambda)(C_\epsilon(q, \bar{\epsilon}, \theta) + K_\epsilon(q, \bar{\epsilon}, \theta))$  represents the net welfare gains from inducing higher effort while  $(\alpha - 1 - \lambda)(F(\theta)/f(\theta))(\partial/\partial \epsilon)(-K_\epsilon E_\theta - K_\theta)$  equals the marginal effect on information rents of inducing greater effort.

Let the first-best level of aggregate effort (conditional on  $q$  and  $\theta$ ) be the solution to  $C_\epsilon(q, \bar{\epsilon}, \theta) + K_\epsilon(q, \bar{\epsilon}, \theta) = 0$ . As long as  $-(\partial/\partial \epsilon)(K_\epsilon E_\theta + K_\theta)$  is not zero at this first-best level, the optimal mechanism will distort the firm's effort choices for all but the best-type firm. If in addition to

Assumptions 1, 2, and 3,  $K_{\epsilon\epsilon}$ ,  $K_{\epsilon\theta}$ , and  $C_{\epsilon\epsilon}$  are positive and  $C_{\epsilon\theta}$  is negative, then  $-(\partial/\partial\epsilon)(K_{\epsilon}E_{\theta} + K_{\theta})$  is strictly negative and all types greater than  $\underline{\theta}$  will invest in less than the first-best level of aggregate effort.<sup>8</sup>

Assuming strictly positive output and effort choices, the mechanism defined by (10) and (11) defines a second-best level of welfare for the union under Assumptions 1-3. Denote this mechanism by  $\{q^*(\theta), \bar{\epsilon}^*(\theta), \Pi^*(\theta)\}$  where  $\Pi^*(\theta)$  is defined by (6) and  $\Pi^*(\bar{\theta}) = 0$ . No mechanism can do better. It represents the optimal union tax policy if the member nations were willing to adopt a single harmonized policy that completely replaces existing national tax rules *including tax rates* - what Devereux (2004) refers to as the "single compulsory 'harmonized' tax base" proposal. All accounts of EC deliberations suggest this option is unlikely to be adopted in the foreseeable future. Separate accounting and formula apportionment systems respect national tax rates but as a result impose additional structure on tax rules which may introduce additional welfare distortions.

### 3.2. Optimal separate accounting and formula apportionment mechanisms.

Separate accounting (SA) differs from a general non-linear tax system in two ways: each country applies a profit tax rate to the taxable income of subsidiaries incorporated in that country and transfer prices are used to allocate the multinational's union profit between its subsidiaries. Instead of (1) multinational profit now is

$$\pi(q, e, t_1, \theta) = (1-t_1)[R_1(q_1) - \hat{C}(q, e, \theta) + s(q, c)] + (1-t_2)[R_2(q_2) - s(q, c)] - \hat{K}(q, e, \theta) \quad (12)$$

where  $s(q, c)$  represents the regulatory rule that defines the transfers a multinational can make between subsidiaries via its transfer prices. Any allocation induced by any transfer price rules that generate  $s(\cdot, \cdot)$  can be described by the mechanism  $\{q(\hat{\theta}), \bar{c}(\hat{\theta}), \tau(\hat{\theta})\}$  however now it is important to recognize that

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<sup>8</sup>Even if the mechanism reduces the multinational's rent by distorting effort choices, the optimal mechanism will not extract any additional rent from the multinational via output distortions in market  $k$  if  $(\partial/\partial q_k)(K_{\epsilon}E_{\theta} + K_{\theta}) = 0$ . Laffont and Tirole refer to this situation as an "incentive-pricing dichotomy" as all rent extraction works through effort distortions and not output distortions.

country 1's tax rate will distort the multinational's effort decision.

Under formula apportionment (FA), a formula allocates total union profit between the two countries as a function of each country's profit tax rates and some of the multinational's observable outcomes and choices. Instead of (12), multinational profit now is calculated as

$$\pi(q, e, \theta) = [a(q, c)(1 - t_1) + (1 - a(q, c))(1 - t_2)][R_1(q_1) + R_2(q_2) - \hat{C}(q, e, \theta)] - \hat{K}(q, e, \theta) \quad (13)$$

where  $a(q, c)$  is the formula that defines the proportion of total multinational profit taxed by country 1 at the rate  $t_1$  and  $1 - a(q, c)$  is the proportion of total multinational profit taxed at the rate  $t_2$  and where the multinational's effective tax rate is  $T \equiv at_1 + (1 - a)t_2$ . The three components of any allocation arising from a formula apportionment mechanism are still  $\{q(\hat{\theta}), \bar{c}(\hat{\theta}), \tau(\hat{\theta})\}$  but now the multinational's effort decision is distorted by  $T$ . Given  $q(\cdot)$  and  $\bar{c}(\cdot)$ ,  $\tau(\cdot)$  is now determined by the formula,  $a(q, c)$ , so we can define  $\alpha(\hat{\theta}) = a(q(\hat{\theta}), \bar{c}(\hat{\theta}))$  and we can define  $T(\hat{\theta}) = \alpha(\hat{\theta})t_1 + (1 - \alpha(\hat{\theta}))t_2$ .

The multinational's optimal effort choice under SA is the solution to

$$\min_e (1 - t_1)\hat{C}(q, e, \theta) + \hat{K}(q, e, \theta) \text{ subject to } \phi(e) = \epsilon$$

while its optimal effort choice under FA is the solution to

$$\min_e (1 - T)\hat{C}(q, e, \theta) + \hat{K}(q, e, \theta) \text{ subject to } \phi(e) = \epsilon.$$

Since  $a(\cdot) \equiv 1$  implies  $T(\cdot) \equiv t_1$ , it is possible to study the effort choices induced by both systems together.

For SA and FA, the multinational's optimal effort investment is

$$\Delta^T(q, \epsilon, T, \theta; \phi) = \operatorname{argmin}_e (1 - T)\hat{C}(q, e, \theta) + \hat{K}(q, e, \theta) \text{ subject to } \phi(e) = \epsilon. \quad (14)$$

Denote a continuous selection (in  $\theta$ ) of  $\Delta^T$  by  $e^T(q, \epsilon, T, \theta; \phi)$ .  $e^T$  will be called  $\phi^T$ -normal if  $e^T$  is increasing in  $\epsilon$ . Then define the indirect cost functions  $C^T(q, \epsilon, T, \theta) = \hat{C}(q, e^T(q, \epsilon, T, \theta), \theta)$  and  $K^T(q, \epsilon, T, \theta) = \hat{K}(q, e^T(q, \epsilon, T, \theta), \theta)$ . It is straightforward to show that  $(1 - T)C_T^T + K_T^T = 0$ ,  $C_T^T > 0$ , and  $K_T^T < 0$ . If  $e^T$  is  $\phi^T$ -normal, As in section 3.1, we can define the aggregate effort function  $E^T(q, \bar{c}, T, \theta)$  by  $C^T(q, E^T(q, \bar{c}, T, \theta), T, \theta) = \bar{c}$ .

Proposition 1 describes sufficient conditions for  $e^*$  and  $e^T$  and hence also the indirect cost

functions to be equal.

**Proposition 1.** *If there exists a smooth, weakly increasing aggregation function  $\phi(\cdot)$  such that observable costs can be written as  $\hat{C}(q, \phi(e), \theta)$  or  $T(\theta) < 1$  and unobservable costs can be written as  $\hat{K}(q, \phi(e), \theta)$ , then  $C^T(q, \epsilon, T, \theta) = C(q, \epsilon, \theta)$ ,  $K^T(q, \epsilon, T, \theta) = K(q, \epsilon, \theta)$ , and  $C_\epsilon^T = C_\epsilon < 0$ .*

*Proof.* First, suppose that  $\hat{C}(q, e, \theta) = \hat{C}(q, \phi(e), \theta)$ . Then (4) and (14) are each equivalent to minimizing  $\hat{K}$  subject to  $\phi(e) = \epsilon$ . Since this is true for all  $T$ , the solutions to problems (4) and (14) will be identical.  $C_\epsilon$  must be strictly negative because by assumption  $\hat{C}_{e_i} < 0$  and  $\phi_{e_i} > 0$ .

Second, suppose that  $\hat{K}(q, e, \theta) = \hat{K}(q, \phi(e), \theta)$ . Now (4) and (14) are each equivalent to minimizing  $(1-T)\hat{C}$  subject to  $\phi(e) = \epsilon$ . As long as  $T < 1$ , both  $\Delta$  and  $\Delta^T$  will solve (14) since  $T$  does not distort the effort choices that minimize  $\hat{C}$ . Minimizing  $(1-T)\hat{C}$  subject to  $\phi(e) = \epsilon$  implies the Lagrangian  $\mathcal{L} = (1-T)\hat{C} + \delta(\phi(e) - \epsilon)$  for which  $\delta = -(1-T)\hat{C}_{e_i}/\phi_{e_i} > 0$ . As  $\mathcal{L}_\epsilon = -\delta < 0$ , then  $C_\epsilon^T = C_\epsilon < 0$ . *Q.E.D.*

When either antecedent of Proposition 1 is satisfied, there will be a SA and a FA mechanism that achieve the same output, effort, and tax payments as the optimal direct taxation mechanism as long as  $t_1 \neq t_2$ . A trivial example that satisfies the requirements of Proposition 1 involves a one-dimensional choice. In this case, choosing  $\phi(e) = e$  implies that feasible effort choices in problems (4) and (14) are identical singleton sets,  $e^* = e^s = \epsilon$  for all  $q, \epsilon, t_1$ , and  $\theta$ , and  $e^*$  and  $e^s$  are trivially  $\phi$ -normal and  $\phi^T$ -normal. This is the reason at least two effort choices are needed to make the analysis of optimal SA and FA mechanisms non-trivial. A second example in which the requirements of Proposition 1 are met is when either  $\hat{C}$  or  $\hat{K}$  depend on  $e_1 + e_2$ . This example includes the case in which  $\hat{K}$ , the opportunity cost of effort, depends only on the total amount of effort employed. In addition, Proposition 1 is an interesting result only when  $T > 0$ . For  $T = 0$ , (4) and (14) are identical regardless of the existence of an appropriate aggregation function.

It is again possible to think in terms of the union using choosing an aggregate effort level,

$\bar{e}^T(T, \theta) \equiv E^T(q(\theta), \bar{c}(\theta), T, \theta)$ , instead of observable costs because of the monotonic relationship between the two.

The analog to the indirect profit function, (5), is

$$\Pi^T(\hat{\theta}, \theta) \equiv R_1(q_1(\hat{\theta})) + R_2(q_2(\hat{\theta})) - \bar{c}(\hat{\theta}) - K^T(q(\hat{\theta}), \bar{e}^T(T, \theta), T, \theta) - \tau^T(\hat{\theta}) \quad (15)$$

where  $\tau^T(\hat{\theta}) = t_1[R_1(q_1(\hat{\theta})) - \bar{c}(\hat{\theta})] + t_2 R_2(q_2(\hat{\theta})) + (t_1 - t_2)s(\hat{\theta})$  and  $s(\hat{\theta}) = s(q(\hat{\theta}), \bar{c}(\hat{\theta}))$  under SA while  $\tau^T(\hat{\theta}) = [a(\hat{\theta})t_1 + (1-a(\hat{\theta}))t_2][R_1(q_1(\hat{\theta})) + R_2(q_2(\hat{\theta})) - \bar{c}(\hat{\theta})]$  under FA . Define  $\Pi^s(\theta) \equiv \Pi^T(\theta, \theta)$  to be the multinational's profit when it reports its type truthfully under SA and define  $\Pi^a(\theta) \equiv \Pi^T(\theta, \theta)$  to be the multinational's profit when it reports its type truthfully under FA . We will soon see that even if the effect tax rates are identical under SA and FA, the multinational's rents will in general be different. Similar to (8), expected union welfare can be written as

$$\mathcal{E}_\theta W^j \equiv \mathcal{E}_\theta [\mathcal{V}(q, \lambda) - (1+\lambda)(C^T(q, \bar{e}^T, T, \theta) + K^T(q, \bar{e}^T, T, \theta)) + (\alpha-1-\lambda)\Pi^j(\theta)] \quad (16)$$

and a direct SA or FA mechanism can be defined by  $\{q^j(\theta), \bar{e}^j(\theta), \Pi^j(\theta)\}$  for  $j \in \{s, a\}$  . The specifics of  $\tau^T(\cdot)$  do not appear in (16) as any changes in the total tax paid by the multinational are reflected in the indirect profit function,  $\Pi^T(\hat{\theta}, \theta)$ .

Similar to the derivation of the optimal direct mechanism, the optimal SA or FA mechanism is found by maximizing (16) subject to  $d\Pi(\theta)/d\theta = -K_\epsilon^T E_\theta^T - K_\theta^T$  and  $\Pi^j(\bar{\theta}) \geq 0$  . Denote these optimal mechanisms by  $\{q^{j*}(\theta), \bar{e}^{j*}(T, \theta), \Pi^{j*}(\theta)\}$  for  $j \in \{s, a\}$ .<sup>9</sup>

Two possible differences between  $\{q^*(\theta), \bar{e}^*(\theta), \Pi^*(\theta)\}$  and  $\{q^{j*}(\theta), \bar{e}^{j*}(T, \theta), \Pi^{j*}(\theta)\}$  can arise. The first is due to differences between  $e^*$  as defined by (4) and  $e^T$  as defined by (14). The second is due to limited control over the level of firm profit when  $t_1 = t_2$ . When  $t_1 = t_2$ ,  $\tau^T(\theta)$  depends only on  $q^{j*}(\theta)$  and

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<sup>9</sup>Technically all three components of the optimal SA or FA mechanism will depend on  $t_1$  and  $t_2$  via  $T$ . To minimize notational complexity, only the dependence of  $T$  on  $\bar{e}^{s*}$  is made explicit to remind the reader that  $T$  will affect the effort inferences the union makes upon learning the firm's observable costs.

$\bar{e}^*(T, \theta)$  as transfers between the multinational's subsidiaries have no effect on the multinational's after-tax profit under SA as transfer prices have no effect on union welfare when both countries charge the same tax rate because the multinational faces no profit-shifting incentive and  $T$  is independent of  $a(\cdot)$  under FA.

Proposition 1 implies that, for certain (direct) cost functions, the allocation (not just the effort choice) from a general direct mechanism can also be generated by an SA and an FA mechanism as long as  $t_1 \neq t_2$ . For such cost functions, the optimal SA and FA mechanisms are not only identical but will be second-best if  $t_1 \neq t_2$ . (The optimal SA and FA mechanisms will be identical, but not necessarily second-best, when  $t_1 = t_2$ .) Because the choice between SA and FA has no effect on firm incentives under these conditions, this tax problem exhibits what I call a *tax-incentive dichotomy*. This result is summarized as Theorem 2.

**Theorem 2.** *A tax-incentive dichotomy exists with respect to the choice between a SA and a FA mechanism if  $T(\theta) < 1$  for all  $\theta$  for both the SA and the FA mechanisms and if there exists a smooth, weakly increasing aggregation function  $\phi(\cdot)$  such that the multinational's effort choices affect either observable costs or unobservable costs only via  $\phi(e)$ . In addition, if  $t_1 \neq t_2$ , then the optimal SA and FA mechanisms are equivalent to the optimal direct taxation mechanism.*

The main implication of Theorem 2 is that when there exists a sufficient statistic defining the impact of the multinational's effort choices on either its observable or its unobservable costs, then the only reasons for choosing one system over the other would be compliance costs (Mintz 2004) or tax competition incentives. Note that a tax-incentive dichotomy trivially exists when  $t_1 = t_2 = 0$ .

#### **4. Comparing SA to FA when no tax-incentive dichotomy exists.**

When no tax-incentive dichotomy exists, SA and FA mechanisms introduce additional distortions relative to direct mechanisms. To simplify the exposition, assume  $\phi(e) = e_1 + e_2$  and assume for this

aggregation function that effort choices under direct taxation, SA, and FA are  $\phi$ -normal and  $\phi^T$ -normal.<sup>10</sup>

Throughout this section, I maintain the assumption that  $t_1 \neq t_2$ .

Let us first consider SA distortions. For direct mechanisms,

$$C(q, \epsilon, \theta) + K(q, \epsilon, \theta) = \min_{e_1} \hat{C}(q, e_1, \epsilon - e_1, \theta) + \hat{K}(q, e_1, \epsilon - e_1, \theta) \quad (17)$$

while for SA mechanisms,

$$(1 - t_1)C^T(q, \epsilon, t_1, \theta) + K^T(q, \epsilon, t_1, \theta) = \min_{e_1} (1 - t_1)\hat{C}(q, e_1, \epsilon - e_1, \theta) + \hat{K}(q, e_1, \epsilon - e_1, \theta). \quad (18)$$

By revealed preference, it must be that

$$C(q, \epsilon, \theta) + K(q, \epsilon, \theta) \leq C^T(q, \epsilon, t_1, \theta) + K^T(q, \epsilon, t_1, \theta). \quad (19)$$

Moreover, if we denote the solution to (18) by  $e_1(q, \epsilon, t_1, \theta)$ , then  $e_1(q, \epsilon, 0, \theta)$  solves (17) and

$$\partial e_1 / \partial t_1 = (\hat{C}_{e_1} - \hat{C}_{e_2})^2 / [(1 - t_1)(\hat{C}_{e_1 e_1} - 2\hat{C}_{e_1 e_2} + \hat{C}_{e_2 e_2}) + \hat{K}_{e_1 e_1} - 2\hat{K}_{e_1 e_2} + \hat{K}_{e_2 e_2}] > 0. \quad (20)$$

Thus,  $C^T(q, \epsilon, t_1, \theta) > C(q, \epsilon, \theta)$  and  $K^T(q, \epsilon, t_1, \theta) < K(q, \epsilon, \theta)$ . Because the profit tax rate in the upstream country reduces the marginal benefit of effort investments in observable costs by  $1 - t_1$ , SA mechanisms distort effort choices away from reductions in observable costs and towards reductions in unobservable costs. That is, a positive tax rate effectively subsidizes deductible costs and thus induces the multinational to incur more observable costs on the margin.

**Proposition 3.** *Assume  $t_1 > 0$ . When no tax-incentive dichotomy exists, SA mechanisms create an incentive for the multinational to reallocate effort (relative to the optimal direct mechanism) so that observable costs increase, unobservable costs decrease, and the sum of observable and unobservable costs increases.*

Proposition 3 documents the key source of welfare losses caused by a shift from the optimal direct mechanism to the optimal SA mechanism, higher total production costs due to a reallocation of effort away from observable costs. Related to this change in the cost structure is also a change in the

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<sup>10</sup>Any aggregation function that is strictly increasing in at least one effort component and implies normal effort choices would yield identical results.

information rents the firm must earn. To the extent that  $K_e^T E_\theta^T + K_\theta^T$  and/or its quantity and effort margins are reduced, the welfare losses due to higher total costs can be moderated. (Since SA mechanisms are themselves direct mechanisms, the optimal SA mechanism can never generate higher expected welfare than the optimal direct mechanism.)

Now consider FA distortions. FA mechanisms also create effort distortions but the direction and magnitude may differ relative to SA mechanisms. For  $a(\theta) < 1$ , an apportionment formula will generate smaller effort distortions (relative to a SA mechanism) if  $t_2 < t_1$  and larger distortions if  $t_2 > t_1$ . However, apportionment formulas create an additional source of welfare distortions because the apportionment formula,  $a(\theta)$ , also affects the multinational's rent profile.

This second effect was not present under SA because the profit-shifting function,  $s(\theta)$ , provides the union with an additional degree of freedom to control the multinational's rent in addition to  $q$  and  $\bar{\epsilon}^T$ .

Recall from section 3.2 that the optimal SA mechanism is  $\{q^{s*}(\theta), \bar{\epsilon}^{s*}(t_1, \theta), \Pi^{s*}(\theta)\}$  where

$q^{s*}(\theta) = (q_1^{s*}(\theta), q_2^{s*}(\theta))$ . Given (15),  $s(\theta)$  is set so that

$$\begin{aligned} \Pi^{s*}(\theta) \equiv & (1-t_1)[R_1(q_1^{s*}(\theta)) - C^T(q^{s*}(\theta), \bar{\epsilon}^{s*}(t_1, \theta), t_1, \theta)] \\ & + (1-t_2)R_2(q_2^{s*}(\theta)) - K^T(q^{s*}(\theta), \bar{\epsilon}^{s*}(t_1, \theta), t_1, \theta) + (t_2 - t_1)s(\theta). \end{aligned} \quad (21)$$

The specific choice of  $s(\theta)$  does not affect the multinational's effort decisions as only the tax rate  $t_1$  enters into the multinational's effort decision calculations. With the optimal FA mechanism,

$\{q^{a*}(\theta), \bar{\epsilon}^{a*}(T, \theta), \Pi^{a*}(\theta)\}$ , the formula  $a(\theta)$  controls multinational profit via the equation

$$\Pi^{a*}(\theta) \equiv [1 - T(\theta)][R_1(q_1^{a*}(\theta)) + R_2(q_2^{a*}(\theta)) - C^T(q^{a*}(\theta), \bar{\epsilon}^{a*}(T, \theta), T, \theta)] - K^T(q^{a*}(\theta), \bar{\epsilon}^{a*}(T, \theta), T, \theta) \quad (22)$$

where  $q^{a*}(\theta) = (q_1^{a*}(\theta), q_2^{a*}(\theta))$ . In the absence of a tax-incentive dichotomy, this same formula,  $a(\cdot)$ ,

also determines the cost functions  $C^T$  and  $K^T$  through the effort allocation decision. A simple example

helps illustrate this point. Suppose  $a(\theta) \equiv 1$ . Then indirect cost functions under FA are  $C^T(q, \epsilon, t_1, \theta)$  and

$K^T(q, \epsilon, t_1, \theta)$  which are precisely the indirect cost functions under SA so the FA mechanism with  $a(\cdot) \equiv 1$  creates the same effort allocation incentives as does a SA mechanism. But now  $a(\cdot)$  cannot be used to control the level of firm profit. In equalizing effort incentives, the FA mechanism uses up a degree of freedom that otherwise would have been used to ensure the desired level of firm profit.

This discussion points to a key information-based distinction between SA and FA mechanisms. SA mechanisms introduce an effort distortion through country 1's tax rate while the profit-shifting function controls the overall level of firm profit. FA mechanisms introduce an effort distortion through the apportionment formula,  $a(\cdot)$ , but this formula simultaneously determines the multinational's rent profile. This dual role need not reduce union welfare relative to the optimal SA mechanism. Since the optimal SA mechanism induces third-best levels of effort, the optimal FA mechanism can yield higher expected welfare by inducing effort choices closer to second-best levels.

Deriving these welfare differences requires a more careful examination of the optimal SA and FA mechanisms. For SA, integrating up the incentive compatibility condition,  $d\Pi^s(\theta)/d\theta = -K_\epsilon^T E_\theta^T - K_\theta^T$ , and substituting  $\Pi^s$  out of (16) implies the optimal SA mechanism can be found by setting  $\Pi^s(\bar{\theta}) = 0$  and then choosing  $q$  and  $\epsilon$  pointwise to

$$\max \mathcal{E}_\theta \{v(q) + (1+\lambda)(R_1 + R_2 - C^T - K^T) - (\alpha-1-\lambda)(F(\theta)/f(\theta))\gamma\} \quad (23)$$

subject to a second-order incentive compatibility constraint where  $\gamma(\theta) \equiv -K_\epsilon^T E_\theta^T - K_\theta^T < 0$  equals the multinational's marginal rent and  $\mathcal{E}_\theta$  denotes the expectation with respect to  $\theta$ .

The optimization problem one must solve to find the optimal FA mechanism differs from (23) in several ways. The union's objective can be thought of as choosing  $\{q, \bar{\epsilon}^T, T\}$  to maximize

$$\mathcal{E}_\theta \{v(q) + (1+\lambda)T(R_1 + R_2 - C^T) + \alpha((1-T)(R_1 + R_2 - C^T) - K^T)\} \quad (24)$$

where the choice of  $T(\cdot)$  via the choice of  $a(\cdot)$  affects the firm's indirect costs,  $C^T$  and  $K^T$ , and defines firm rents as

$$(1-T)(R_1 + R_2 - C^T) - K^T = \Pi^s(\bar{\theta}) - \int_{x=\theta}^{\bar{\theta}} \gamma(x) dx. \quad (25)$$

Absent the second-order incentive compatibility constraint, the problem of deriving the optimal FA mechanism is equivalent to maximizing (24) subject to incentive compatibility constraints, (25), and  $\Pi^a(\bar{\theta}) \geq 0$ . To formulate this optimal control problem, define  $\Gamma(\theta) = - \int_{x=\theta}^{\bar{\theta}} \gamma(x) dx$ .  $\Gamma(\theta)$  will be non-negative and  $\Gamma(\bar{\theta}) = 0$ . Then let  $w$  denote union welfare defined by substituting (25) into the second and third terms of (24). Integrating by parts then implies

$$w(q, \epsilon, T, \Pi(\bar{\theta}), \theta) = [v + (1+\lambda)(R_1 + R_2 - C^T - K^T) + (\alpha-1-\lambda)(\Pi(\bar{\theta}) - (F(\theta)/f(\theta))\gamma)]f(\theta) \quad (26)$$

where  $\Gamma(\cdot)$  is the state variable,  $(q, \epsilon, T)$  are the controls,  $\Gamma'(\theta) = \gamma(\theta)$ ,  $\Gamma(\bar{\theta}) = 0$ , and  $\Pi^a(\bar{\theta}) \geq 0$ . Denote the Hamiltonian given this formulation by

$$\mathcal{H} = w + \mu((1-T)(R_1 + R_2 - C^T) - K^T - \Pi^a(\bar{\theta}) - \Gamma) + \eta\gamma \quad (27)$$

where  $\mu(\theta)$  is the multiplier associated with (25) and  $\eta(\theta)$  is the co-state variable.

There are two main differences between (23) and (27). First, remember that the indirect cost functions  $C^T$  and  $K^T$  will be different functions under SA and FA unless FA implies an effective tax rate of  $t_1$ . The effective tax rate is exogenous in (23) and endogenous in (27). Second, since  $\Pi^a(\bar{\theta})$  shows up in constraint (25), we cannot conclude from the monotonic rent structure that minimal firm profit should be zero. Leaving the worst-type firm with positive profit could improve expected union welfare by inducing a higher level of effort among all firm types.

The optimal FA mechanism will be a solution to the following set of Euler and transversality conditions:

$$[v_{q_1} + (1+\lambda)(R_1' - C_{q_1}^T - K_{q_1}^T)]f + \mu[(1-T)(R_1' - C_{q_1}^T) - K_{q_1}^T] = [-\eta + (\alpha-1-\lambda)F]\gamma_{q_1}, \quad (28a)$$

$$[v_{q_2} + (1+\lambda)(R_2' - C_{q_2}^T - K_{q_2}^T)]f + \mu[(1-T)(R_2' - C_{q_2}^T) - K_{q_2}^T] = [-\eta + (\alpha-1-\lambda)F]\gamma_{q_2}, \quad (28b)$$

$$-(1+\lambda)[C_\epsilon^T + K_\epsilon^T]f - \mu[(1-T)C_\epsilon^T + K_\epsilon^T] = [-\eta + (\alpha-1-\lambda)F]\gamma_\epsilon, \quad (28c)$$

$$-(1+\lambda)[C_T^T + K_T^T]f - \mu[R_1 + R_2 - C^T] = [-\eta + (\alpha-1-\lambda)F]\gamma_T, \quad (28d)$$

$$\partial \mathcal{H} / \partial \Gamma = -\mu = -\eta', \quad (28e)$$

$$\Gamma'(\theta) = \gamma(\theta), \quad (28f)$$

$$(25), \eta(\underline{\theta}) = 0, \Pi^a(\bar{\theta}) \geq 0, \text{ and } \Gamma(\bar{\theta}) = 0, \quad (28g)$$

and

$$\alpha - 1 - \lambda - \int_{\theta=\underline{\theta}}^{\bar{\theta}} \mu(\theta) d\theta \leq 0 \quad (28h)$$

where (28h) is the first-order condition with respect to  $\Pi^a(\bar{\theta})$ . Given (28e-f), (28h) can be rewritten as

$$-\eta(\bar{\theta}) \leq 1 + \lambda - \alpha. \quad (28i)$$

The following lemma, proved in the appendix, provides sufficient conditions under which we can bound  $\eta(\cdot)$  and  $\mu(\cdot)$ .

**Lemma 4.** Assume  $t_1 \neq t_2$ . If (a)  $-(R_1 + R_2 - C^T) < \gamma_T \leq 0$  and (b)  $(1+\lambda)C_T^T < R_1 + R_2 - C^T$  for all  $q, \epsilon, T$ , and  $\theta$ , then  $0 > \eta(\theta) \geq -F(\theta)$ . In addition, if  $\alpha \geq \lambda$  or (c)  $\gamma_T > (R_1 + R_2 - C^T - (1+\lambda)TC_T^T)/((\alpha-\lambda)F/f)$ , then  $0 > \mu(\theta) > -f(\theta)$ .

With the common assumption of  $\alpha > 0$  and  $\lambda = 0$ , the shadow price of rent extraction,  $\eta(\cdot)$ , is always decreasing in  $\theta$ .

Conditions (a) and (c) require that an increase in the effective tax rate increase firm rents (by making  $\gamma$  more negative) and that this marginal rent effect is not too large relative to taxable income,  $R_1 + R_2 - C^T$ . Condition (b) requires that taxable income be large enough to cover the change in observable social costs due to a change in  $T$ . The bounds from Lemma 4 on  $\mu$  and  $\eta$ , which are associated with moderate marginal tax rate effects, reveal interesting properties of the optimal FA mechanism. With  $\mu$  negative, (28h) implies a tradeoff between rent extraction and effort investment. Suppose the union were to increase  $\Pi^a(\bar{\theta})$  from 0 to 1. To preserve incentive compatibility, rent for every other type would have to increase by 1. This would decrease expected union welfare by  $1 + \lambda - \alpha$ . At the same time, the only way each multinational type can earn an extra dollar of rent, is for  $(1-T)(R_1 + R_2 - C^T) - K^T$  to increase through more efficient levels of  $q, \epsilon$ , and  $T$ . The shadow price of these efficiency enhancements as captured via (25) is  $-\int \mu(\theta) d\theta$  or  $-\eta(\bar{\theta})$ . If at  $\Pi^a(\bar{\theta}) = 0$ ,  $-\eta(\bar{\theta}) < 1 + \lambda - \alpha$ , then the optimal value of  $\Pi^a(\bar{\theta})$  must be zero. Otherwise, the optimal rent for the worst-type firm implies the marginal welfare

benefit of more efficient production and/or more efficient effort levels will equal the marginal welfare cost of increasing rents for every firm type,  $-\eta(\bar{\theta}) = 1 + \lambda - \alpha$ . In fact, (28i) implies  $\Pi^a(\bar{\theta})$  must be strictly positive when  $\lambda$  is close to zero and  $\alpha$  is close to 1. On the other hand, since Lemma 4 implies  $-\eta < 1$ ,  $\Pi^a(\bar{\theta})$  must always be zero when  $\alpha < \lambda$ . Thus, Lemma 4 implies the following proposition.

**Proposition 5.** *Assume  $t_1 \neq t_2$ . If  $-(R_1 + R_2 - C^T) < \gamma_T \leq 0$  and  $(1 + \lambda)C_T^T < R_1 + R_2 - C^T$ , there exist values of  $\alpha$  and  $\lambda$  that imply the worst-type multinational earns strictly positive rent with the optimal FA mechanism.*

Proposition 5 reveals an important difference between the optimal FA mechanism and the optimal direct and SA mechanisms: endogenizing the effective tax rate can create the incentive for the union to leave all types with positive profit. An immediate implication of Proposition 5 is that as the rent earned by the worst-type multinational increases, the effective tax rate charged this firm must decrease. Yet as long as the optimal FA mechanism does not allow any type firm to earn rent equal to its gross profit,  $R_1 + R_2 - C^T - K^T$ , the optimal value of  $T(\cdot)$  will be strictly bounded away from zero for all  $\theta$ . This would be the case if the union's objective was to maximize expected tax revenues, i.e.  $v(\cdot, \cdot) \equiv 0$  and  $\alpha = 0$ . Moreover, the optimal value of  $T(\cdot)$  is independent of either of the statutory tax rates. Recall from the earlier comparison of the optimal SA mechanism and the optimal direct mechanism that the two are equivalent when  $t_1 = 0$ . Not only does the optimal SA mechanism depend directly on the value of  $t_1$ , as the value of  $t_1$  goes to zero, the optimal SA mechanism converges to the optimal direct mechanism.<sup>11</sup> This means there exist statutory tax rates for which the optimal SA mechanism must be preferred by the union to the optimal FA mechanism.

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<sup>11</sup>Country 2's tax rate plays no role in the welfare effects of either mechanism because it does not distort the firm's effort choices. If some of the observable costs were generated in country 2,  $t_2$  as well as  $t_1$  would have to be 0 in order for the optimal SA mechanism and the optimal direct mechanism to be equivalent.

**Proposition 6.** Assume  $t_1 \neq t_2$ , assume the union's objective is to maximize expected tax revenues, and assume no tax-incentive dichotomy exists. There exists  $t^* > 0$  such that, if  $t_1 < t^*$ , the optimal SA mechanism yields larger expected tax revenues than the optimal FA mechanism.

Later in this section, Example 1 will show that  $t^*$  can be strictly less than one leading to the case in which the optimal FA mechanism is preferred to the optimal SA mechanism.

Lemma 4 also permits a comparison of the output and effort levels under SA and FA. Consider the choice of country 2 output. The Euler condition for  $q_2$ , (28b), can be rewritten as

$$\begin{aligned} \mathfrak{H}_{q_2}(q, \epsilon, T, \theta) &= [v_{q_2} + (1+\lambda)(R_2' - C_{q_2}^T(T) - K_{q_2}^T(T))]f \\ &+ \mu[(1-T)(R_2' - C_{q_2}^T(T)) - K_{q_2}^T(T)] + [\eta - (\alpha-1-\lambda)F]\gamma_{q_2}(T) = 0 \end{aligned} \quad (29)$$

where  $C_{q_2}^T(T)$ ,  $K_{q_2}^T(T)$ , and  $\gamma_{q_2}(T)$  is shorthand notation for  $C_{q_2}^T(q, \epsilon, T, \theta)$ ,  $K_{q_2}^T(q, \epsilon, T, \theta)$ , and  $\gamma_{q_2}(q, \epsilon, \tau, \theta)$  used to emphasize the dependence of costs and marginal rents on the effective tax rate. In order to decompose the effect of  $T$  on rent extraction versus the effect on efficiency, it is possible to rewrite  $\mathfrak{H}_{q_2}$  from (29) as

$$\begin{aligned} \mathfrak{H}_{q_2}(q, \epsilon, T, \theta) &= [v_{q_2} + (1+\lambda)(R_2' - C_{q_2}^T(t_1) - K_{q_2}^T(t_1) + C_{q_2}^T(t_1) - C_{q_2}^T(T) + K_{q_2}^T(t_1) - K_{q_2}^T(T))]f \\ &+ \mu[(1-T)(R_2' - C_{q_2}^T(t_1)) - K_{q_2}^T(t_1) + (1-T)(C_{q_2}^T(t_1) - C_{q_2}^T(T)) + K_{q_2}^T(t_1) - K_{q_2}^T(T)] \\ &+ \eta\gamma_{q_2}(T) - (\alpha-1-\lambda)F[\gamma_{q_2}(t_1) + \gamma_{q_2}(T) - \gamma_{q_2}(t_1)] \end{aligned} \quad (30)$$

where terms such as  $C_{q_2}^T(t_1)$  mean  $C_{q_2}^T(q, \epsilon, t_1, \theta)$ . For the optimal SA mechanism,

$$[v_{q_2} + (1+\lambda)(R_2' - C_{q_2}^T(t_1) - K_{q_2}^T(t_1))]f - (\alpha-1-\lambda)F\gamma_{q_2}(t_1) = 0 \quad (31)$$

evaluated at  $(q^{s*}, \bar{\epsilon}^{s*}, t_1, \theta)$ . Combining (30) and (31) implies that

$$\begin{aligned}
\mathcal{H}_{q_2}(q^{s*}, \bar{\epsilon}^{s*}, T, \theta) &= (1+\lambda)[C_{q_2}^T(t_1) - C_{q_2}^T(T) + K_{q_2}^T(t_1) - K_{q_2}^T(T)]f \\
&+ \mu[-TK_{q_2}^T(t_1) + (1-T)(-v_2 + (\alpha-1-\lambda)F\gamma_{q_2}(t_1)/f)/(1+\lambda) \\
&+ (1-T)(C_{q_2}^T(t_1) - C_{q_2}^T(T)) + K_{q_2}^T(t_1) - K_{q_2}^T(T)] \\
&+ \eta\gamma_{q_2}(T) - (\alpha-1-\lambda)F[\gamma_{q_2}(T) - \gamma_{q_2}(t_1)].
\end{aligned} \tag{32}$$

Let  $T^{a*}(\theta)$  denote the optimal effective tax rate under formula apportionment and suppose  $T^{a*}(\theta) = t_1$ . At  $\theta = \underline{\theta}$ , (32) simplifies down to  $\mathcal{H}_{q_2}(q^{s*}, \bar{\epsilon}^{s*}, t_1, \underline{\theta}) = -\mu[t_1 K_{q_2}^T(t_1) + (1-t_1)v_2/(1+\lambda)] > 0$  as  $\eta(\underline{\theta})=0$ . The term in brackets is equal to  $-[(1-t_1)(R_2' - C_{q_2}^T(t_1)) - K_{q_2}^T(t_1)]$ , the negative of marginal after-tax profit at  $t_1$ , and is positive. With marginal after-tax profits less than zero at SA values, FA gives the union the incentive to reduce the rent earned by the lowest-type multinational by increasing country 2 output beyond the optimal SA level when  $t_1$  is equal to the optimal effective tax rate levied on the lowest-type multinational.

Note also that

$$\partial \mathcal{H}_{q_2}(q^{s*}, \bar{\epsilon}^{s*}, T, \theta) / \partial t_1 = (1+\lambda)[C_{q_2}^T(t_1) + K_{q_2}^T(t_1)]f - \mu TK_{q_2}^T(t_1) + \mu[(1-T)C_{q_2}^T(t_1) + K_{q_2}^T(t_1)] \tag{33}$$

Since  $(1-T)C_{q_2}^T(q, \epsilon, T, \theta) + K_{q_2}^T(q, \epsilon, T, \theta) = 0$  for all  $q, \epsilon, T$ , and  $\theta$ , it will also be true that

$$(1-T)C_{q_2}^T(q, \epsilon, T, \theta) + K_{q_2}^T(q, \epsilon, T, \theta) = 0 \text{ for all } q, \epsilon, T, \text{ and } \theta \text{ and (33) is equivalent to}$$

$$\partial \mathcal{H}_{q_2}(q^{s*}, \bar{\epsilon}^{s*}, T, \theta) / \partial t_1 = C_{q_2}^T(t_1)[(1+\lambda)t_1 f + \mu T(1-t_1) + \mu(t_1 - T)]. \tag{34}$$

Lemma 4 implies the bracketed term in (34) is positive for all  $t_1 \geq T$  (because  $(1+\lambda)f + \mu > 0$ ) which in turn implies that  $\partial \mathcal{H}_{q_2}(q^{s*}, \bar{\epsilon}^{s*}, T^{a*}, \theta) / \partial t_1$  is positive for all  $t_1 \geq T^{a*}(\theta)$  if  $C_{q_2}^T \geq 0$ . While the above analysis focused on country 2 production for the sake of concreteness, the same analysis can be done with respect to  $q_1$  and  $\epsilon$ . Doing so establishes the following result.

**Proposition 7.** *Assume  $t_1 \neq t_2$  and assume for all  $q, \epsilon, T$ , and  $\theta$  that conditions (a)-(c) from Lemma 5 are satisfied. For each  $i \in \{q_1, q_2, \epsilon\}$ , if for all  $q, \epsilon$ , and  $T$ ,  $K_i^T > 0$ ,  $C_{iT}^T \geq 0$ , and  $\gamma_i \leq 0$  at  $\underline{\theta}$ , then*

$$\mathcal{H}_i(q^{s*}, \bar{\epsilon}^{s*}, T^{a*}, \underline{\theta}) > 0 \text{ if } t_1 \geq T^{a*}(\underline{\theta}).$$

Under the conditions of Proposition 7, if the optimal FA mechanism presents the best-type multinational with an effective tax rate no larger than  $t_1$ , the union will face an initial incentive to increase production in each country and to increase aggregate effort for the  $\underline{\theta}$  firm as long as three conditions are met. First, positive marginal unobservable costs,  $K_i^T$ , (with respect to  $q_1$ ,  $q_2$ , or  $\epsilon$  depending on the variable under discussion) allows us to sign marginal after-tax profits and determine the direction in which firm rents will fall. Second, a lower effective tax implies lower marginal observable costs. That is, as  $T^{a*}(\underline{\theta})$  drops below  $t_1$ , the efficiency effect present at  $t_1 = T^{a*}(\underline{\theta})$  is strengthened. And third, larger production or effort values increase the magnitude of the information rents earned by this firm (recall that  $\gamma$  is negative). Since Lemma 4 implies the shadow price of increased rents,  $\mu$ , is smaller in magnitude than the weight put on lower costs,  $f$ , this third effect moderates the cost efficiency effect but does not reverse it.

In extending this analysis to make comparisons for  $\theta > \underline{\theta}$ , three caveats must be mentioned. First, for  $t_1$  enough smaller than  $T^{a*}(\underline{\theta})$ , the third (information rent) effect can be the dominant effect. Second, the terms involving  $\gamma_{q_2}$  (or  $\gamma_j$  more generally) in (32) are no longer zero. While the  $\gamma_{q_2}(t_1)$  terms have positive coefficients by Lemma 4, the  $\gamma_{q_2}(T)$  terms have negative coefficients and the net effect can be negative. Finally, Proposition 7 only evaluates the derivatives of the Hamiltonian near  $(q^{s*}, \epsilon^{s*})$ . Even if  $\mathcal{H}_i$  is positive at  $(q^{s*}, \epsilon^{s*})$  for all  $i \in \{q_1, q_2, \epsilon\}$ , it can still be possible for the optimal FA mechanism to result in lower production or effort than SA for some (but not all) of these variables. Example 1 which follows provides evidence that the tax rate and marginal rent terms can be large enough to reverse the predictions of Proposition 7 when  $t_1 < T^{a*}(\underline{\theta})$  and  $\theta > \underline{\theta}$ . In so doing, Example 1 illustrates the competing effects of efficient effort investment and rent extraction. The example also provides evidence that the difference in production and effort levels between the optimal SA and the optimal FA mechanisms can be consistent with the local effects identified in Proposition 7.

*Example 1.* Let  $R_i(q_i) = (10 - q_i)q_i$  and let  $\hat{C} = (2 - e_1)q_1 + (\theta - e_2)q_2$  and let  $\hat{K} = e_1^2 q_1 / 2 + e_2^2 q_2 / 2 + 16$ . The

fixed cost term in  $\hat{K}$  is included for calibration reasons discussed below. These cost functions imply that

$$e_2^T(T, \theta) = 1 - T, \quad C^T(q, \epsilon, T, \theta) = (2 - \epsilon)q_1 + (\theta + T - 1)q_2, \quad E^T(q, \bar{c}, \theta) = 2 - \bar{c}/q_1 + (\theta + T - 1)(q_2/q_1),$$

$$K^T(q, \epsilon, T, \theta) = \epsilon^2 q_1 / 2 + (1 - T)^2 q_2 / 2 + 16, \quad \text{and} \quad \gamma(\theta) = -\epsilon(\theta)q_2(\theta). \quad \text{Information rents then equal}$$

$$\int_{w=\theta}^{\bar{\theta}} \epsilon(w)q_2(w)dw.$$

$\theta$  is uniformly distributed on  $[3/2, 5/2]$  so  $f(\theta) = 1$  and  $F(\theta) = \theta - 3/2$ . This example considers the case of tax revenue maximization which implies  $\alpha = 0$  and  $v(q) = 0$ .  $\lambda$  is set to zero. It turns out that  $w + \eta\gamma$  (the Hamiltonian without constraint (25)) is not globally concave in  $(q, \epsilon, T, \Gamma)$ . For  $q_1$  small or  $q_2$  small, the Hessian conditions associated with the second-order and higher principal minors will not be satisfied. Fortunately, Arrow's sufficiency condition (see Theorem 2.5 in Seierstad and Sydsæter (1987)) can be verified. This is done in the appendix. As a result, there will be a unique solution to (28a-h) and this solution describes the optimal FA mechanism.

For this example, (28d) implies  $\mu = -(C_T^T + K_T^T)/(R_1 + R_2 - C^T)$ . In accord with Lemma 4,  $\mu$  is greater than -1 and so (28h) implies  $\Pi^a(\bar{\theta}) = 0$ .

The graphs in Figure 1 describe the optimal FA mechanism. Production levels, country 1 effort, multinational rent, and union welfare are all decreasing in type. Country 2 effort and effective tax rates are not monotonic in this example although they will be strictly decreasing if country revenues are sufficiently large. The monotonic structure is most important for  $q_2$  and  $\epsilon$  (which is equal to  $e_1$ ) as it means the second-order incentive compatibility constraint is satisfied. The optimal SA mechanism is qualitatively very similar.

Two features of the optimal apportionment formula worth noting are related to the range of effective tax rates,  $T(\cdot)$ , in Figure 1d. First, at  $\theta = \bar{\theta}$ , the after-tax return is defined by the zero profit condition. Hence,  $1 - T = K^T/(R_1 + R_2 - C^T)$ . Without a fixed cost term in  $\hat{K}$ ,  $T(\bar{\theta})$  would be very close to one simply because unobservable indirect costs are small relative to observable operating profit. By including a fixed cost term in unobservable costs, a more realistic range for  $T(\cdot)$  arises without increasing

the analytic complexity of the example. In this example, the range of effective tax rates is .48 to .51. The hump-shaped curve reflects the tradeoff between rent extraction and effort incentives. For low types, the dominant effect is the rent extraction effect. As firm type increases above  $\underline{\theta}$ , the union can extract more rent from the multinational by imposing a higher effective tax rate while distorting country 1 effort down. The lower investment in country 1 effort in turn reduces the marginal information rent the multinational earns. For high types, the dominant effect is the effort incentive effect. At some point, the union will offer higher types a lower tax rate in order to encourage more investment in country 1 effort at the expense of larger rents. While offering higher types a lower tax rate increases the marginal rent earned by these types, it does so off of a relatively lower rent level and at the same time encourages those types to invest in more effort than they would at a higher tax rate.<sup>12</sup> Second, the value of  $T(\cdot)$  is also independent of the statutory tax rates,  $t_1$  and  $t_2$ , because the formulation of the optimal FA mechanism does not require *a priori* that  $T$  fall between  $t_1$  and  $t_2$ . Imposing this restriction would add a bias against FA mechanisms. Thus, the optimal control formulation used to solve for the best FA mechanism gives FA the best chance of dominating SA. What will depend on both  $t_1$  and  $t_2$  is the apportionment formula,

$$a(\theta) = (T(\theta) - t_2)/(t_2 - t_1).$$

As noted before, the optimal SA mechanism depends on  $t_1$  via its effect on the indirect cost functions. To compare the optimal FA mechanism with the optimal SA mechanism in a way that reflects this sensitivity, the graphs in Figure 2 compare the optimal FA mechanism to each of three optimal SA mechanisms corresponding to three different values of  $t_1$ : a lower rate than that implied by the optimal FA mechanism,  $t_1 = .3$  (red lines), the minimum rate from the optimal FA mechanism,  $t_1 = .48$  (green lines), and the maximum rate from the optimal FA mechanism,  $t_1 = .51$  (blue lines). The relationship between the optimal FA mechanism and the optimal direct mechanism will be qualitatively similar to the

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<sup>12</sup>For other examples, this trade-off can manifest itself in the form of non-monotonic production quantities or effort levels while still satisfying the second-order incentive constraint.

relationship between the optimal FA mechanism and the optimal SA mechanism when  $t_1=.3$ . For each component, the lines describe the difference between the FA mechanism value minus the SA mechanism value. These graphs reveal that for this example the optimal FA mechanism induces more production in country 1, more cost-reducing effort in country 1, and larger rents than the optimal SA mechanism. The blue and green lines in Figures 2a-c confirm the predictions of Proposition 7 for  $q$  and  $\epsilon$ .

The differences in country 2 production, cost-reducing effort in country 2, and union welfare depend on country 1's tax rate,  $t_1$ . The differences in country 2 production provide evidence of the reversal possibilities indicated by the discussion following Proposition 7. For  $t_1$  close to  $T^{a*}(\underline{\theta})$ , FA results in more country 2 production at  $\underline{\theta}$ . But for larger values of  $\theta$ , country 2 production will be smaller under FA. Country 2 effort must also be larger under FA for some types and smaller for others as long as  $t_1$  falls within the range of effective tax rates defined by  $T^{a*}(\cdot)$  since  $e_2^T(q, \epsilon, T, \theta) = T$ . In this example, country 2 effort will be lower under FA for extreme types, because those are the types that face the lowest effective tax rates on country 1 effort and it will be higher for intermediate types.

Union welfare follows a similar pattern to country 2 effort as seen in Figure 2e. In this example, union welfare equals union tax revenues. For intermediate types that invest in more cost-reducing effort in both countries and who face a higher effective tax rate under FA, tax revenues will be higher under FA because these types have a larger tax base and a higher tax rate. Extreme type firms face two types of trade-offs. One is the usual tax rate/tax base trade-off. The other involves substitution between cost-reducing effort investments in country 1 and in country 2. The example shows that the net effect is lower tax revenues from these extreme types under FA. Figure 2f shows the net effect on the union's expected welfare is consistent with Proposition 6. For  $t_1$  above approximately .5, the optimal FA mechanism generates a higher level of expected welfare whereas for  $t_1$  below approximately .5, the optimal SA mechanism is better. This comparison confirms the intuition expressed above that FA can lead to an increase in expected union welfare over the optimal SA mechanism by inducing more efficient effort

investments and more efficient production in country 2. However, this efficiency enhancing effect only exists when the optimal FA mechanism generates an effective tax rate that is lower than the country 1 rate for enough intermediate firm types.

Finally, suppose we modify the union's objective by putting positive weight on firm profit. Figure 3 plots the profit paid to a multinational of type  $\bar{\theta}$  under the optimal FA mechanism as a function of  $\alpha$ . Consistent with Proposition 5, this profit will be strictly positive for  $\alpha$  greater than approximately .85 and it increases with  $\alpha$ . For instance, when  $\alpha = .9$ , the worst-type firm earns profit of 14.40 and the best-type firm earns a profit of 21. In contrast, profits under the optimal SA mechanism range from 0 to 4. Thus, all types earn significantly higher profits under the optimal FA mechanism and expected union welfare is lower.

## 5. Concluding remarks.

Separate accounting and formula apportionment are the two main approaches governments use to tax multijurisdictional firms. The former has been used almost universally by national governments to tax multinational firms while the latter has been used by states or provinces to tax multi-state/multi-province firms. Recent interest by the EU in advancing formula apportionment to tax companies active in several EU countries has prompted comparative economic analyses of various apportionment formulas to separate accounting rules. The goal of this paper has been to first derive the optimal separate accounting and apportionment formula mechanisms from the perspective of a tax union in a framework that explicitly models the informational disadvantages that prompt national and state governments to employ these approaches and second to compare the economic differences between these optimal mechanisms. The analysis identifies conditions under which a tax-incentive dichotomy exists that implies the choice of a tax system is independent of the specific distortions and level of union welfare. In the absence of a tax-incentive dichotomy, a shift from separate accounting to formula apportionment is shown to create a tension between the use of the apportionment formula to encourage efficient investment within the firm

and the use of separate accounting to extract rents. Depending on how these competing effects balance out, either system can dominate the other in terms of expected union welfare and differences in union welfare by type can be non-monotonic. One key indication of the limited rent extraction capability of apportionment formulas is the result that the union may have to leave even the worst-type firm with positive rent.

The current model can be extended to incorporate several other aspects of multinational taxation. Depending on how a multinational's home country handles repatriated earnings, double taxation policies will influence the extent to which apportionment formulas and separate accounting regulations distort unobservable effort decisions. The current model assumes no repatriation motive for the multinational and that each government exempts foreign-source profits from taxation (consistent with the double taxation policies of a number of EU countries). If a multinational's home country used a credit rule for double taxation and the multinational was in an excess credit position, income repatriation issues could confound effort incentives under both formula apportionment and separate accounting.

At several points in the paper, it was noted that country 2's statutory tax rate played no role in comparing the optimal separate accounting mechanism to the optimal direct mechanism. This is due to the fact that all observable costs affected by the multinational's effort choices are realized in country 1. If these costs were distributed between both countries, optimal effort choices under separate accounting would depend on both tax rates. The decision to locate all observable costs in country 1 was made to simplify the presentation of the tax-incentive dichotomy arguments. Distributing observable costs across both countries would not alter the conditions under which a tax-incentive dichotomy exists nor would it reverse the qualitative properties described in Propositions 5 and 6.

The model also assumes that the union welfare function weights tax revenues collected in each country equally while distributional concerns about the location of production and revenues can be reflected in the  $V(q, \lambda)$  as long as these concerns imply  $V$  is concave. Introducing welfare weights to

reflect distributional preferences for tax revenues as well as a more explicit modelling of revenue and production distribution preferences would not change the basic form of the analysis nor the qualitative trade-offs between rent extraction and efficiency identified in this paper. However, their introduction could identify new trade-offs not captured in this model as well as add substantially to the paper's length. As a result, I leave the study of these distributional questions to future research.

There are two issues studied in the international tax literature which this paper does not address. The first is the effect of tax competition. In order to focus on the informational differences between separate accounting and apportionment formulas, it was assumed that the countries' statutory tax rates were exogenous. The results of this paper can be viewed as a first step in understanding the broader tax competition implications of these two types of tax systems. The second issue concerns the possible adoption of a hybrid system in which a union allows multinationals to choose between separate accounting and formula apportionment as is currently done in Canada. (See Mintz and Smart (2004)). The non-monotonic pattern of effective tax rates evidenced in Example 1 suggest that such a hybrid system could create interesting, and as of yet, unanticipated self-selection patterns. Both issues are important topics for future research.

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## Appendix

*Proof of Lemma 4.*

Rewrite (28d) as  $(R_1 + R_2 - C^T)\eta' - \gamma_T \eta = \beta$  where  $\beta = -(1 + \lambda)(C_T^T + K_T^T)f - (\alpha - 1 - \lambda)F\gamma_T$ . Since  $(1 - T)C_T^T + K_T^T = 0$ ,  $C_T^T + K_T^T = TC_T^T \geq 0$ . Thus,  $\gamma_T \leq 0$  implies  $\beta \leq 0$ . Given  $\eta(\underline{\theta}) = 0$ , the solution to (28d) is

$$\eta(\theta) = e^{\int_{w=\underline{\theta}}^{\theta} \gamma_T / (R_1 + R_2 - C^T) dw} \cdot \int_{z=\underline{\theta}}^{\theta} e^{-\int_{w=\underline{\theta}}^z \gamma_T / (R_1 + R_2 - C^T) dw} \cdot \beta(z) / (R_1 + R_2 - C^T) dz. \quad (\text{A.1})$$

Since  $\beta(\theta) \leq 0$  for all  $\theta$ , (A.1) implies  $\eta(\theta) \leq 0$  for all  $\theta$ . Moreover,  $\gamma_T \leq 0$  implies

$$\eta(\theta) \geq e^{\int_{w=\underline{\theta}}^{\theta} \gamma_T / (R_1 + R_2 - C^T) dw} \cdot \int_{z=\underline{\theta}}^{\theta} e^{-\int_{w=\underline{\theta}}^z \gamma_T / (R_1 + R_2 - C^T) dw} \cdot \beta(z) / (R_1 + R_2 - C^T) dz \geq \int_{z=\underline{\theta}}^{\theta} \beta(z) / (R_1 + R_2 - C^T) dz. \quad (\text{A.2})$$

The assumptions,  $|\gamma_T| < R_1 + R_2 - C^T$  and  $(1 + \lambda)C_T^T < R_1 + R_2 - C^T$ , mean  $\beta / (R_1 + R_2 - C^T) > -1$  and  $\eta(\theta) \geq -F(\theta)$ .

Differentiating  $\eta(\theta)$  from (A.1) implies

$$\mu(\theta) = \gamma_T \eta / (R_1 + R_2 - C^T) + \beta(\theta) / (R_1 + R_2 - C^T) \geq -(1 + \lambda)TC_T^T f / (R_1 + R_2 - C^T) - (\alpha - \lambda)F\gamma_T / (R_1 + R_2 - C^T). \quad (\text{A.3})$$

If  $\alpha \geq \lambda$ ,  $(1 + \lambda)C_T^T < R_1 + R_2 - C^T$  and  $\gamma_T \leq 0$  implies  $\mu(\theta) > -f(\theta)$ . If  $\alpha < \lambda$  and

$$\gamma_T > (R_1 + R_2 - C^T - (1 + \lambda)TC_T^T) / ((\alpha - \lambda)F/f), \text{ then again (A.3) implies } \mu(\theta) > -f(\theta). \quad Q.E.D.$$

*Arrow sufficiency condition.*

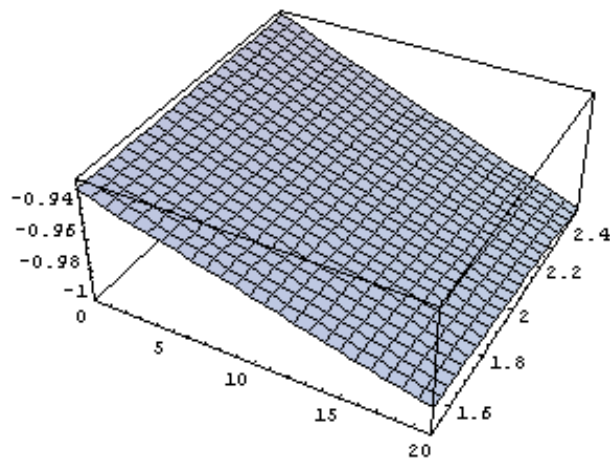
Define  $\tilde{\mathcal{H}}(q, \epsilon, T, \Gamma, \theta, \eta) = w + \eta\gamma$  (which by construction is independent of  $\Gamma$ ) and define  $T^*(q, \epsilon, \Gamma, \theta)$  to be the solution to (25). Then define  $\hat{\mathcal{H}}(\Gamma, \theta, \eta) = \max_{q, \epsilon} \tilde{\mathcal{H}}(q, \epsilon, T^*, \Gamma, \theta, \eta)$ . Arrow's sufficiency condition

requires that, for some  $\eta$ ,  $\hat{\mathcal{H}}$  be concave in  $\Gamma$  for all  $\theta$ . By the envelope theorem,

$$\hat{\mathcal{H}}_{\Gamma}(\Gamma, \theta, \eta) = \tilde{\mathcal{H}}_{\Gamma}(q, \epsilon, T^*, \Gamma, \theta, \eta) = \tilde{\mathcal{H}}_{\Gamma}(q, \epsilon, T, \Gamma, \theta, \eta) T_{\Gamma}^*(q, \epsilon, \Gamma, \theta)$$

where the last equality arises because  $\tilde{\mathcal{H}}$  is independent of  $\Gamma$ . For Example 1,  $\hat{\mathcal{H}}_{\Gamma} = T^* C_T^T f / (R_1 + R_2 - C^T)$  evaluated at  $T^*$  and at the values of  $q$  and  $\epsilon$

that maximize  $\tilde{\mathcal{H}}$  given the solution to  $\eta(\theta)$  from (28a-h). We need to show that  $\hat{\mathcal{H}}_\Gamma$  is decreasing in  $\Gamma$ . This was done in *Mathematica* by numerically maximizing  $\tilde{\mathcal{H}}$  over  $q_i \in [0,10]$  and  $\epsilon \in [0,2]$ . Outside these bounds  $\tilde{\mathcal{H}}$  is clearly strictly concave. Then  $\hat{\mathcal{H}}_\Gamma$  was evaluated at these optimal values. The results are presented in Figure A1 in which  $\Gamma$  ranges from 0 to 20. The graph shows  $\hat{\mathcal{H}}$  is clearly concave in  $\Gamma$ .



**Figure A1:  $\hat{\mathcal{H}}_\Gamma$**

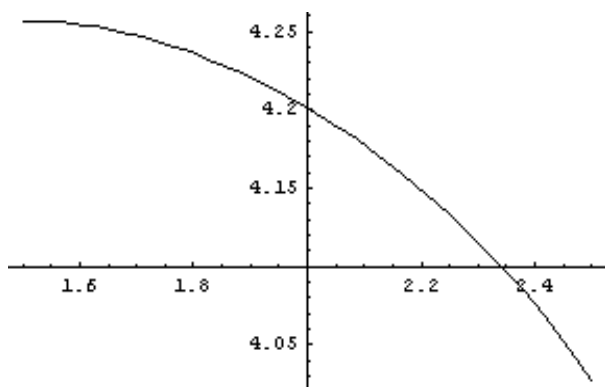


Figure 1a: Country 1 production

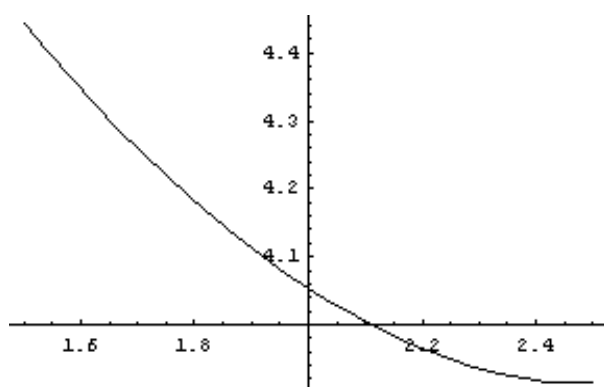


Figure 1b: Country 2 production,  $q_2(\theta)$

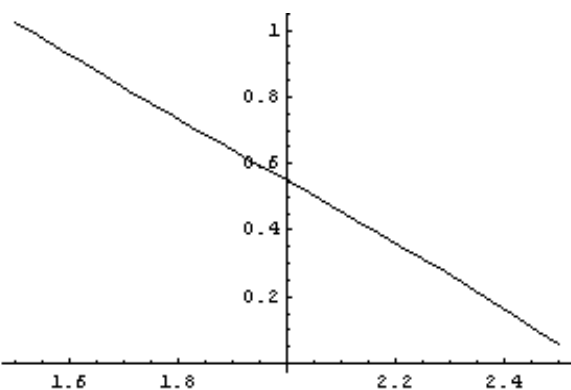


Figure 1c: Cost-reducing effort in country 1,  $e_1(\theta)$

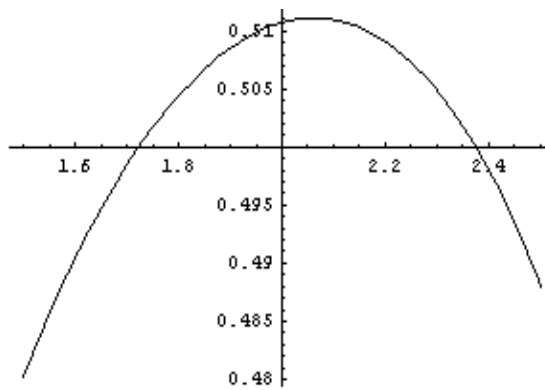


Figure 1d: Effective tax rate,  $T(\theta)$  and  $1-e_2(\theta)$

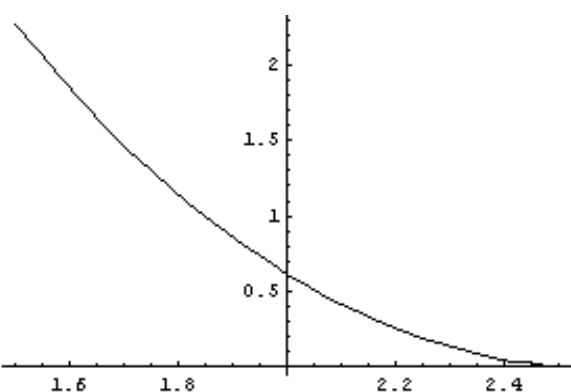


Figure 1e: Multinational rent,  $\Gamma(\theta)$

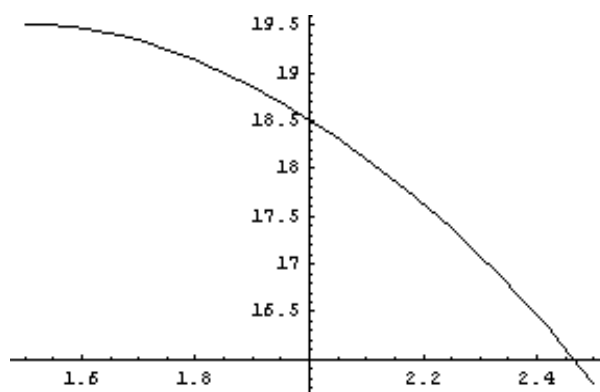
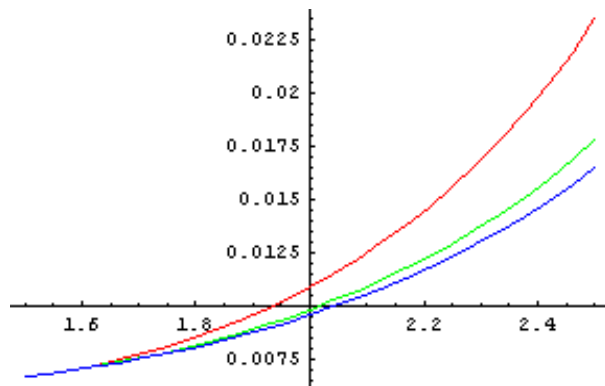
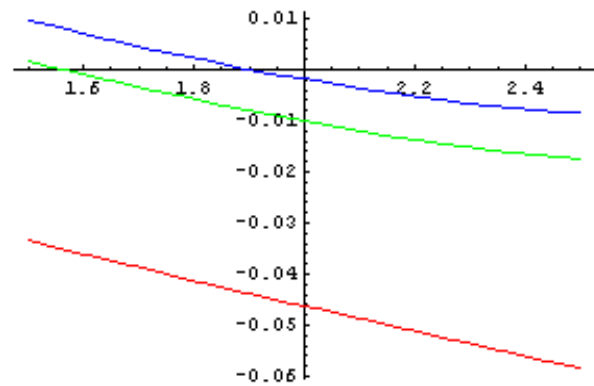


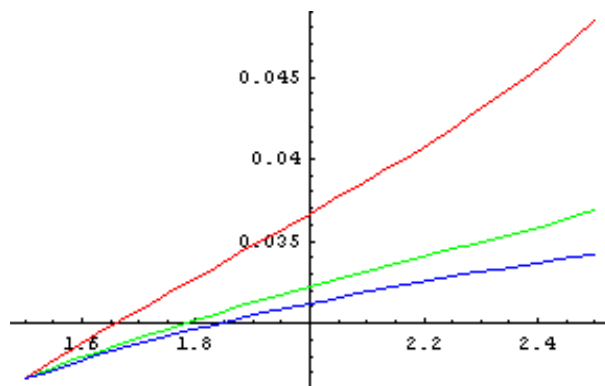
Figure 1f: Union welfare,  $w(\theta)$



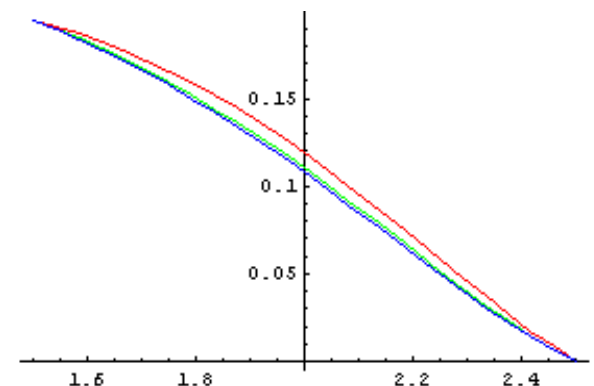
**Figure 2a: Differences in country 1 production (FA-SA)**



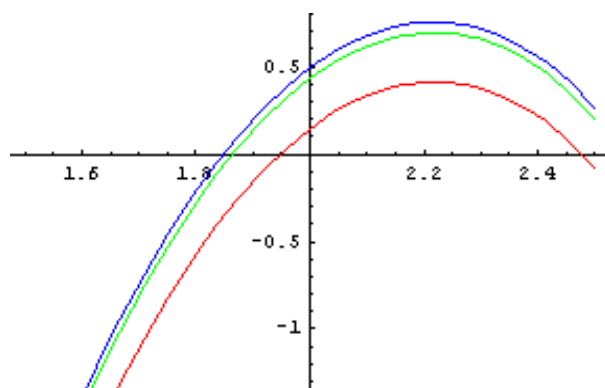
**Figure 2b: Differences in country 2 production (FA-SA)**



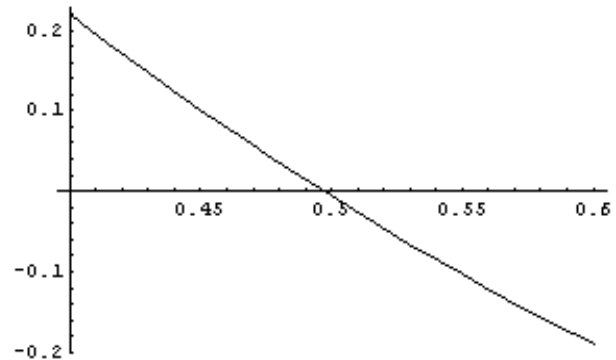
**Figure 2c: Differences in country 1 effort investment (FA-SA)**



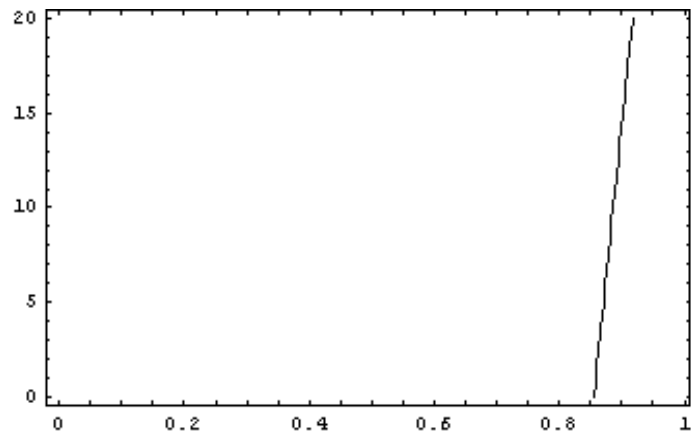
**Figure 2d: Differences in multinational rent (FA-SA)**



**Figure 2e: Differences in union welfare (FA-SA)**



**Figure 2f: Differences in expected union welfare as a function of  $t_1$  (FA-SA)**



**Figure 3: The rent earned by the worst-type firm in the optimal formula apportionment mechanism as a function of  $\alpha$ .**