

Optimal Profit Allocation Mechanisms for Economic Unions

Under Adverse Selection and Moral Hazard

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Abstract: Economic unions, such as American states, Canadian provinces, or EU countries, face the problem of how to allocate union-wide corporate profits among its individual members when each member retains some taxing authority. This paper studies the normative properties of two common types of allocation mechanisms, separate accounting and formula apportionment, when firms have private cost information and make investment decisions union members cannot observe. The optimal separate accounting and formula apportionment mechanisms are derived. Their differences reflect a trade-off between firms earning fewer rents versus productive efficiency. The welfare implications of this trade-off are examined. The optimal separate accounting mechanism generates higher expected union welfare when upstream tax rates are low.

Keywords: Mechanism design, private information, unobservable investment, separate accounting, transfer pricing, formula apportionment

JEL Codes: D82, H21, H73, F23

1. Introduction.

The increase in worldwide economic integration over the last several decades has prompted national governments to examine the efficacy of their tax systems which were designed when international opportunities were less prevalent. The effort to design a tax system reflecting the realities of highly integrated economies is perhaps best illustrated by the EU which recognizes that "Large EU companies now view the whole EU as their 'home market' and accordingly seek to establish effective pan-European business structures." (EC 2001, p.5) To promote "greater efficiency, effectiveness, simplicity and transparency in company tax systems and remove the hiatuses between national systems which provide fertile ground for avoidance and abuse" (EC 2001, p.10) the European Commission "believes that it is necessary to provide companies with a consolidated corporate tax base for their EU-wide activities; develop an appropriate apportionment mechanism which can be agreed by all participants; and, for Member States, to determine the applicable national corporate tax rates." (EC 2001, p.16)

Recently the European Commission (2011) forwarded a proposal to the Council of the European Union to adopt a formula apportionment plan referred to as the Common Consolidated Corporate Tax Base or CCCTB (EC 2011).¹ The CCCTB defines a single tax-base definition, calculates a company's national tax liabilities with an apportionment formula, and preserves each member country's right to set its corporate tax rate. For any company doing business within the EU that elects to be taxed under the CCCTB, the fraction of its total EU profit allocated to each member country will be based on a weighted average of its share of EU sales (1/3), compensation (1/6), number of employees (1/6), and assets (1/3).

What is notable about this EC recommendation is that while formula apportionment has long been used by some state/provincial governments for multi-state taxation, the predominant system for

¹See Mintz and Martens-Weiner (2003), Zodrow (2004), Devereux (2004), Hellerstein and McLure (2004), Mintz (2004) and Sørensen (2004) for descriptions and critiques of the CCCTB and alternative proposals that were considered.

multinational firms is separate accounting which relies on transfer prices set by the firm to allocate its total income to each tax jurisdiction. Differences in national income tax rates create an incentive for multinationals to use their transfer prices to shift profits from high-tax jurisdictions to low-tax jurisdictions, and thus require some form of costly auditing based on the arm's-length pricing principle to limit the amount of profit-shifting. In contrast to the European Commission, OECD (2010,1.21) states, "OECD member countries ... do not consider global formulary apportionment a realistic alternative to the arm's-length principle."² Among the objections to formula apportionment particularly relevant to EC (2011) are that "predetermined formulae are arbitrary and disregard market conditions (1.25), "[c]ontrary to the assertions of its advocates, global formulary apportionment may in fact present intolerable compliance costs" (1.27), and "[d]ifficulties would also arise in determining the sales of each member and in the valuation of assets" (1.28). The first critique recognizes that while apportionment formulas can eliminate the need for transfer prices and the related auditing, they too create investment, production, and location distortions (see Gordon and Wilson (1986)). The second critique denies the common claim, as in Mintz (2004), that formula apportionment generates lower compliance costs. The third critique echoes the concern raised by Hellerstein and McLure (2004) that some formula factors such as assets can be subject to the same valuation manipulation/validation issues as are transfer prices.

These opposing positions suggest the need to evaluate both systems in a common economic environment with an emphasis on differences in efficiency, the distribution of factors, and tax revenues. While there exists an extensive literature on separate accounting and a somewhat smaller literature on formula apportionment, few papers have sought to compare the two systems under identical economic

²OECD (2010) is categorized as a **Recommendation** of the Council. The Council consists of one ambassador from each member country and the EC. Recommendations are not legally binding but the expectation is that member countries who do not abstain are expected to "do their utmost to fully implement a Recommendation." (*OECD Legal Instruments - "The Acts"* at http://www.oecd.org/document/46/0,3746,en_21571361_38481278_40899182_1_1_1_1,00.html).

assumptions and fewer still have attempted to make comparisons in economic environments that include the fundamental reasons taxing multinationals is more challenging for governments than taxing domestic firms: unobservable cost or revenue complementarities and incomplete information about a multinational's technology and input choices (see Gresik (2001) for a detailed description of these challenges). For example, Nielsen, Raimondos-Møller, and Schjelderup (2009), Sørensen (2003), Gérard (2005), and Eichner and Runkel (2011) compare the economic performance of both systems in a complete information environment in which there are homogeneous firms (or a representative firm) and all firms face the same reduced form cost to profit-shifting. Kind, Midelfart, and Schjelderup (2005) conduct a similar exercise but include a role for trade costs. None of these studies allow for firm heterogeneity and all are positive analyses that compare the equilibrium outcomes of a specific separate accounting protocol to a specific apportionment formula.³

Consistent with optimal income taxation literature initiated by Mirrlees (1971), Seade (1977), and Stiglitz (1982) and the optimal regulation literature initiated by Baron and Myerson (1982), this paper advances existing analyses of formula apportionment and separate accounting in two main ways. First, the paper explicitly introduces a model in which a multinational has private information about its productivity and can take actions unobservable by governments. The private information explicitly captures the reality of firm heterogeneity which Burbidge, Cuff, and Leach (2006, p.544), in their study of profit tax systems, show can lead to "substantial differences" in economic outcomes relative to complete information analyses, directly addresses the Hellerstein and McClure (2004) critique, and permits an analysis of the role of information rent distortions with separate accounting and formula apportionment.

³Eichner and Runkel (2011) consider a parameterized family of formulas that depend on capital shares, wage shares, and revenue shares but do not consider the optimal or equilibrium choice of the parameters.

Second, the paper adopts a normative approach whose focus is on the (private-information specific) allocations that can be generated by optimally designed separate accounting rules and by optimally designed apportionment formulas. The normative focus on optimal allocations allows one to abstract away from many details of each system that may be useful in implementing a given allocation but do not play a role in identifying the key welfare trade-offs embodied in each approach. It also complements the above positive (equilibrium) studies that rely on specific separate accounting rules and apportionment formulas by deriving upper bounds on the expected welfare of each class of rules.⁴ In so doing, the normative analysis avoids the possibility of analyzing either the equilibrium of an apportionment formula or of a separate accounting system with suboptimal welfare properties.

Section 2 describes a model that captures several of the salient characteristics of multinational tax problems: cost complementarities, private (type) information about the multinational's technology, observable and unobservable costs, and unobservable resource decisions. The first two characteristics are necessary for the question of how best to divide up a multinational's profit among the subsidiaries operating in different countries to have a non-trivial answer. If there are no cost (or revenue) complementarities, the multinational is really just a collection of independent firms, each of which has well-defined taxable income in its country of incorporation. If the tax authorities in each government can perfectly observe all aspects of the multinational's operations and opportunities, first-best tax policy is feasible and can be implemented without using costly procedures whose *raison d'etre* is to gain information about the firm's operations and profitability. The third characteristic reflects the reality of tax policy in using observable cost data for verification reasons since some apportionment formulas rely on factor valuations that are subject to manipulation. Nielsen, Raimondos-Møller, and Schjelderup (2003)

⁴Gresik (2010) compares the equilibrium outcomes associated with a specific set of separate accounting rules and a revenue apportionment formula with heterogeneous firms and private firm information.

show that these information problems can be exacerbated if the multinationals have market power.

Finally, if the tax authorities can observe all the actions a multinational makes, it may be able to infer the necessary information it lacks about the multinational. However if, in addition to the multinational having private information about its profits, it also makes decisions that cannot be observed by the tax authorities, then the individual countries will lack sufficient information to distinguish between cost and revenue levels that are due to inefficient technologies versus tax-induced effort or production shifting choices for all but the most trivial cases. For example, decisions regarding time and effort allocations across firm projects are generally not observable outside the firm.

In section 3, I use this model to solve three different optimal tax problems from the perspective of a tax union that, consistent with the approach being taken by the European Commission outlined above, seeks to agree on a general procedure taking each country's tax rates as given. The first allows for a completely general non-linear tax function and yields the second-best tax allocation. The second imposes restrictions on the set of feasible tax functions common to all separate accounting regulations. The third imposes restrictions on the set of feasible tax functions common to all apportionment formulas. The additional restrictions associated with either separate accounting or formula apportionment are imposed because the current debate among entities such as the EU and the OECD is between separate accounting and formula apportionment and has not embraced an examination of more general tax procedures because doing so would effectively require EU countries to give up control over national tax rates. However, by comparing the allocation from the optimal separate accounting regulations and the allocation from the optimal apportionment formula to the second-best allocation, we are also able to assess the potential welfare losses associated with preserving national tax rate sovereignty.

Both formula apportionment and separate accounting ignore relevant economic information (a point the OECD criticisms of formula apportionment de-emphasize). Separate accounting regulations focus more on dividing shared costs or revenues and tend to ignore country-specific profits.

Apportionment formulas seek to divide union-wide profits without regard for whether they are country-specific or due to complementarities. Since both systems rely on each country's corporate tax rate and both create profit-shifting, production, and resource distortions, the potential for either system to do worse than the second-best mechanism clearly exists. However, it turns out that when either the firm's observable or unobservable costs constitute a sufficient statistic for the firm's unobservable resource decisions, the tax union can use either system to implement the second-best allocation. I refer to this result as a tax-incentive dichotomy to parallel the terminology in Laffont and Tirole (1994) since the general form of the union's tax procedure is independent of the optimal output and cost distortions.

Section 4 compares the economic differences between the second-best, optimal separate accounting, and optimal formula apportionment allocations in the absence of a tax-incentive dichotomy. Proposition 3 shows that, relative to the second-best mechanism, the optimal separate accounting regulations distort the multinational's resource decisions in a way that decreases marginal after-tax observable costs while leaving intact the union's ability to extract rents. By re-allocating resources towards observable costs, the multinational's capacity for profit-shifting increases but this is partially offset by the ability of the union to extract some of these additional profit-shifting gains via its separate accounting procedures. In order to generate higher observable costs, the firm ends up with lower unobservable costs and higher total costs relative to the second-best.

Apportionment formulas have an advantage over separate accounting regulations as they can change the multinational's internal resource margins and hence change its observable and unobservable costs. This advantage creates the potential for an apportionment formula to improve a firm's productive efficiency consistent with the EC's consolidated tax-base objective although at the cost of less effective rent extraction. Thus, section 4 reveals a fundamental tension between the stated goals of productive efficiency and rent extraction associated with a switch from separate accounting to formula apportionment.

This trade-off between reducing cost distortions and rent extraction is responsible for several welfare differences between the two systems. Proposition 5 shows that it is possible for the lowest productivity firm to earn strictly positive profit under the optimal apportionment formula. In contrast, this type firm always earns zero profit from the second-best and optimal separate accounting allocations. Proposition 6 describes how this trade-off affects real output and input decisions. Surprisingly, the optimal apportionment formula generates the greatest increase (or the smallest decrease) in union welfare relative to the optimal separate accounting regulations from the least productive firm types (highest types). This occurs because the effective tax rate under the optimal apportionment formula exhibits an inverted U-shape pattern: the most productive firm types and the least productive firm types face lower effective tax rates while intermediate productivity types face higher effective tax rates. As a result, it is the lowest and the highest productivity types that generate the fewest cost distortions. At the same time, with the smallest marginal rent effects coming from the least productive types, any welfare advantage one might find from using an apportionment formula will come from the more favorable incentives it provides the least productive firms!

Across all types, the trade-off between productive efficiency and rent extraction implies that either separate accounting or formula apportionment can yield higher expected union welfare. When the statutory tax rate in the countries where multinationals locate upstream production is sufficiently low, the optimal separate accounting regulations induce small cost distortions relative to second-best levels. In this case, the optimal apportionment formula results in less downstream production, and hence less union welfare than the optimal separate accounting regulations. When the statutory tax rate in the upstream countries is large enough, the optimal apportionment formula induces fewer cost distortions for enough of the lowest productivity types to generate larger expected union welfare.

I conclude by discussing directions for future research in section 5.

2. Model.

The model I employ is a variation of the regulatory model found in chapter 3 of Laffont and Tirole (1994). There are two countries, denoted by 1 and 2, and one multinational firm. The firm owns and operates a subsidiary in each country.⁵ The subsidiary in country 1 earns revenues (net of local selling costs) of $R_1(q_1)$ by selling q_1 units of a good in country 1 and the subsidiary in country 2 earns revenues (net of local selling costs) of $R_2(q_2)$ by selling q_2 units of a good in country 2. Production of each final good requires intermediate goods that are produced exclusively by the subsidiary in country 1. One intermediate good is needed for each final good and variety or quality differences between the final goods sold in country 1 and country 2 may necessitate differentiated intermediate goods.

Revenues and sales quantities are observable by both countries but some of the multinational's production costs are unobservable for two reasons. First, managers in the firm need to allocate inputs I refer to as effort to the intermediate good production processes. The allocation, and its associated opportunity costs, are unobservable outside the multinational.⁶ Second, the multinational has private information about its technological capabilities or the opportunity costs of some of its inputs. Neither the inability of the governments to observe the actual input choices nor the multinational's private information precludes the possibility that the countries can observe, or at least verify for tax purposes, the accounting costs associated with the input choices after they have been generated. In fact, knowing these costs may allow the countries to infer some information about the firm's unobservable choices/private information. What is important for the subsequent analysis is that the countries not be able to verify and

⁵The term "subsidiary" is used to imply that it is incorporated under the laws of its resident country. This distinguishes the organization of the multinational from that of a branch structure as branches operate under different tax rules than subsidiaries. Since this paper abstracts away from repatriation and double taxation issues, it does not matter which unit if either is the parent corporation.

⁶Effort in this model could include any productive input whose total employment or distribution across the multinational is not observable by outside parties.

hence tax the multinational's full economic profit.⁷

Denote the multinational's pre-tax accounting costs associated with intermediate good production by $\hat{C}(q,e,\theta)$ where $q = (q_1, q_2)$ is the vector of production/sales quantities, $e = (e_1, e_2)$ is the non-negative vector of unobservable input choices made within the multinational, and θ is the multinational's private information parameter or type. These costs are tax-deductible and hence their realized values are observable by the tax authorities. Denote the multinational's non-deductible costs by $\hat{K}(q,e,\theta)$. Typically, at least some of a firm's non-deductible costs are not observable. As Hellerstein and McClure (2004) point this may be due to the inability of parties outside the multinational to determine capital costs. These costs may also include opportunity costs or they may include costs incurred by the multinational outside the union such as in a tax haven. Without loss of generality, I assume that all non-deductible costs are unobservable. Two output choices is the minimum necessary to capture the phenomena of profit-shifting and production-shifting due to tax incentives while at least two input choices are needed to make the comparison between optimal separate accounting and optimal formula apportionment non-trivial. This last point will be clarified in section 3. Each country treats θ as a random variable drawn from $[\underline{\theta}, \bar{\theta}]$ with distribution function $F(\theta)$ and continuous, non-zero density, $f(\theta)$. The countries hold common beliefs with regard to the firm's type and these beliefs are common knowledge among the countries and the multinational.

Both \hat{C} and \hat{K} are affected by the multinational's output choices, its effort choices, and its type based on the following assumptions. Subscripts denote derivatives.

Assumption 1. a. \hat{C} is non-negative and continuous for all q , e , and θ .

⁷In the absence of non-deductible costs, a profit tax is a non-distortionary pure profit tax whose optimal value is determined solely by distributional welfare concerns. Non-deductible costs are a common feature in most international tax papers precisely because it reflects the inability of governments to levy pure profit taxes.

- b. $\hat{C}_{q_i} > 0$, $\hat{C}_{e_i} < 0$, and $\hat{C}_\theta \geq 0$.
- c. \hat{C} is weakly convex in q and weakly convex in e . $\hat{C}_{q,\theta} \geq 0$ and $\hat{C}_{e,\theta} \geq 0$ for $i=1,2$ while $\hat{C}_{q,\theta} > 0$ and $\hat{C}_{e,\theta} > 0$ for some i .
- d. $\hat{C}_{e_1, e_2} \leq 0$.

Assumption 2. a. \hat{K} is non-negative and continuous for all q, e , and θ .

- b. $\hat{K}_{q_i} \geq 0$, $\hat{K}_{e_i} > 0$, and $\hat{K}_\theta > 0$.
- c. \hat{K} is strictly convex in q and weakly convex in e . $\hat{K}_{q,\theta} \geq 0$ and $\hat{K}_{e,\theta} \geq 0$ for $i = 1, 2$ while $\hat{K}_{q,\theta} > 0$ and $\hat{K}_{e,\theta} > 0$ for some i .
- d. $\hat{K}_{e_1, e_2} \leq 0$.
- e. $\lim_{e \rightarrow \infty} \hat{K}_{e_i}(q, e, \theta) = \infty$ for all θ and for all $q > 0$.

Assumption 1b implies that observable costs are decreasing in each type of effort and increasing in the multinational's output and type. Thus, effort can be thought of as tax planning investments by the firm that can have real effects. Assumption 1b also implies that higher type multinationals have higher observable costs. Assumption 1c implies diminishing returns in effort and increasing marginal costs of production. It also implies that higher type firms exhibit higher marginal costs of production with respect to at least one of the intermediate goods and smaller cost reductions with respect to at least one type of effort. These last two conditions are analogs to the familiar single-crossing property found in most private information models. Assumption 1d implies that the effort choices are weak complements with respect to the benefits of cost reduction. As long as the effort choices are not overly strong substitutes all of the analysis goes through.

The effects of effort and type on unobservable costs are different from the effects on observable costs to emphasize the role of opportunity costs and tax planning activities associated with a firm's cost distortions. Assumption 2b requires that unobservable costs are increasing in effort, output, and type. A weaker assumption on \hat{K}_{e_i} would allow for unobservable costs to first fall with increases in effort and then

rise. The stronger assumption of 2b is used because the weaker assumption only adds complexity to the model without altering the results of the analysis. Together Assumptions 1b and 2b imply that tax planning efforts come at an economic resource cost to the firm and that higher type firms are inherently less productive as they have higher observable costs and higher unobservable costs. Assumption 2c implies that the marginal costs of effort and production are increasing in effort, output, and type, respectively. This assumption also requires that \hat{K} be strictly convex in e . Together with Assumption 2e, which is an Inada condition, strict convexity guarantees that $T\hat{C} + \hat{K}$ is minimized by a unique, finite vector of effort levels for $0 < T \leq 1$. Assumption 2d implies that the effort choices are also weak complements with respect to the costs of cost reduction. As with assumption 1d, all of the results of this paper go through as long as the effort choices are not overly strong substitutes.

At the most general level, if the two countries were to act as a union, their fully coordinated tax policy would correspond to a non-linear function of q and c , the level of observable costs. Denoting such a policy by the tax function $\hat{\tau}(q, c)$ implies global after-tax profit for the multinational of

$$\pi(q, e, \theta) = R_1(q_1) + R_2(q_2) - c - \hat{K}(q, e, \theta) - \hat{\tau}(q, c) \quad (1)$$

where $c = \hat{C}(q, e, \theta)$. Including c in the tax function implies that some fraction of observable costs are tax-deductible. With full coordination, the countries' choice of $\hat{\tau}(\cdot, \cdot)$ could, but need not, be defined in terms of either country's tax rate

Separate accounting (SA) and formula apportionment (FA) are two classes of tax policies, each of which generates a set of tax functions, $\hat{\tau}(\cdot, \cdot)$, whose definitions directly depend on each country's tax rate. These two classes of policies allow the countries to operate as a union with respect to its rules for allocating union profits between the two countries while retaining sovereignty over other aspects such as tax rate determination. They are not the only two classes of tax policies with these features but they are the two widely used or considered in practice.

Under SA, the country 1 subsidiary charges the country 2 subsidiary for the intermediate goods it

provides. The price charged is called a transfer price. Transfer prices are routinely audited by tax authorities who use observable information about a multinational to determine allowable transfer prices in order to prevent the multinational from shifting excessive profits from the high-tax country into the low-tax country. While all SA tax functions rely on transfer prices to allocate union profit among member countries, what distinguishes one SA function from another are the specific regulations used to determine allowable transfer prices. If we let the function $s(\cdot, \cdot)$ represent the transfer payments the transfer price regulations allow given output, q , and observable costs, c , then this most basic feature common to all transfer price regulation implies that the multinational's union taxes are defined by the tax function

$$\hat{\tau}^s(q, c) = t_1[R_1(q_1) - c] + t_2R_2(q_2) + (t_1 - t_2)s(q, c)$$

where t_1 and t_2 are the statutory profit tax rates for each country.⁸

Under FA, a formula $a(q, c)$ determines the fraction of profit taxed by country 1 at the rate t_1 while country 2 will tax the fraction $1-a(q, c)$ at the rate t_2 .⁹ Given this basic feature of all FA policies, the multinational's union taxes are defined by the tax function

$$\hat{\tau}^a(q, c) = [a(q, c)t_1 + (1-a(q, c))t_2][R_1(q_1) + R_2(q_2) - c].$$

What distinguishes one FA tax function from another is the specific formula that is used.

The countries are assumed to operate as a partial tax union in the sense that they jointly choose a tax function $\hat{\tau}(\cdot, \cdot)$ (either from among all feasible functions, or among all SA functions, or among all FA

⁸The specific structure of $s(q, c)$ is intentionally general. Commonly used features of separate accounting regulations such as commensurate-with-income standards or competent authority provisions are important for implementing a particular allocation but not for the normative analysis in this paper which seeks to derive an optimal allocation.

⁹In a more general model, the union would be able to observe different categories of costs such as capital or wage costs. In this case, the apportionment formula $a(\cdot, \cdot)$ (as well as the transfer pricing function $s(\cdot, \cdot)$ and the general non-linear tax function $\hat{\tau}(\cdot, \cdot)$) could be defined in terms of each type of cost. For the sake of exposition, I focus on only one category of costs.

functions) to maximize their combined welfare

$$W = v(q) + (1+\lambda)\hat{t} + \alpha\pi \quad (2)$$

where $v(\cdot)$ equals the social benefits of production and is strictly concave in q , $\lambda > 0$ is the social marginal cost of funds to the countries, and $\alpha \in [0,1]$ is the welfare weight associated with multinational profit. Eq. (2) treats each country's corporate tax rate, t_i , as exogenous. The focus on union welfare is consistent with the current EC process that seeks to define an EU-wide policy for allocating multinational income among member states while respecting the ability of individual countries to set tax rates. Recall from the introduction that the first two EC proposals do not attempt to control tax rates but do seek to "develop an appropriate allocation mechanism which can be agreed by all parties." (EC 2001, p.16) Ultimately, it will be important to incorporate tax competition incentives in the analysis by endogenizing the tax rates. The results in this paper are a precursor to that type of analysis.

Substituting (1) into (2) implies

$$W = v(q) + (1+\lambda)(R_1 + R_2 - \hat{C} - \hat{K}) + (\alpha - 1 - \lambda)\pi. \quad (3)$$

3. Three optimal tax mechanisms.

Given a specific tax function, a multinational will choose the production and effort levels that maximize its profit. These optimal choices will depend on the firm's private information. Thus, a single tax function applied to a heterogeneous set of firms (or types) can yield production levels, effort levels, and observable and unobservable costs that can also vary with the firm's type as will firm profit and taxes paid. A direct tax mechanism records the type-specific allocation of output, effort, costs, profit, and taxes generated by a single tax function. To derive a welfare-optimal tax function, it is easier to optimize with respect to tax mechanisms instead of tax functions. Thus, this section derives three optimal tax mechanisms: the second-best mechanism, the optimal SA mechanism, and the optimal FA mechanism.

Because a firm's type is private information, a direct tax mechanism must elicit type information from the multinational. Let $\{q(\hat{\theta}), \bar{c}(\hat{\theta}), \tau(\hat{\theta})\}$ denote a direct tax mechanism where $q(\cdot)$ is the output

vector of the firm, $\bar{c}(\cdot)$ is the observable cost of the firm, $\tau(\cdot)$ is the tax paid to the union, and $\hat{\theta}$ is the firm's type report. Denote the tax function, $\tau(\hat{\theta})$, for a general mechanism by $\hat{\tau}(q(\hat{\theta}), \bar{c}(\hat{\theta}))$; for an SA mechanism by $\hat{\tau}^s(q(\hat{\theta}), \bar{c}(\hat{\theta}))$; and for an FA mechanism by $\hat{\tau}^a(q(\hat{\theta}), \bar{c}(\hat{\theta}))$. Given the tax functions, $\hat{\tau}^s$ and $\hat{\tau}^a$, one can back out $s(\hat{\theta}) = s(q(\hat{\theta}), \bar{c}(\hat{\theta}))$, which is the amount of profit-shifting a multinational with reported type $\hat{\theta}$ is allowed, and $\alpha(\hat{\theta}) = \alpha(q(\hat{\theta}), \bar{c}(\hat{\theta}))$, which is the fraction of union profits that will be taxed at the rate t_1 for a multinational with reported type $\hat{\theta}$.

A direct mechanism is incentive compatible if the multinational optimal type report is truthful, that is $\hat{\theta} = \theta$. By the Revelation Principle there is no loss of generality in restricting attention to incentive compatible direct tax mechanisms. In addition, we will require the optimal incentive compatible direct tax mechanism to be individually rational so that the multinational's profit given its type is non-negative for all types.

Each of the optimal direct mechanisms can be derived in two steps. In step 1, indirect observable and unobservable cost functions are derived because they summarize the economically relevant information about the firm's unobservable effort choices. These indirect cost functions will allow us to see how each tax mechanism distorts the multinational's internal effort decisions. In step 2, the indirect cost functions are used to derive the information rents the multinational must earn under each type of tax mechanism which then allows one to derive the optimal mechanisms. For each optimal tax mechanism, this step will identify the trade-off between the union's ability to limit cost inefficiencies through the firm's internal distortions and the firm's ability to earn information rents.

3.1 Indirect observable and unobservable costs.

Following Laffont and Tirole (1994), let $\phi(e) \equiv e_1 + e_2$ denote the firm's aggregate effort investment. Then define the firm's indirect after-tax costs by

$$\Delta(q, \epsilon, T, \theta) = \underset{e}{\operatorname{argmin}} (1-T)\hat{C}(q, e, \theta) + \hat{K}(q, e, \theta) \text{ subject to } \phi(e) = \epsilon \quad (4)$$

where $\epsilon \geq 0$ and T denotes the effective tax rate. With the optimal second-best tax mechanism, the tax bill

can be set independently of t_1 and t_2 . Thus, $T = 0$. Under SA, observable costs are tax-deductible only in country 1 so $T = t_1$. Under FA, observable costs end up being apportioned so that $T = at_1 + (1-a)t_2$. The aggregate effort constraint function allows one to summarize the firm's effort choices in terms of a single parameter, ϵ . Continuity of $\hat{C} + \hat{K}$ and Assumptions 2c and 2d imply Δ will be non-empty and that a continuous selection exists. So define an optimal effort choice by the multinational under each tax mechanism that is continuous in θ by $e^*(q, \epsilon, T, \theta) \in \Delta(q, \epsilon, T, \theta)$.¹⁰

From the solution to (4), define indirect costs $C(q, \epsilon, T, \theta) = \hat{C}(q, e^*(q, \epsilon, T, \theta), \theta)$ and $K(q, \epsilon, T, \theta) = \hat{K}(q, e^*(q, \epsilon, T, \theta), \theta)$. By varying T based on whether one is considering a general tax mechanism, an SA mechanism, or an FA mechanism, these indirect cost functions will reflect the economic link between the type of tax mechanism adopted by the union and any internal cost distortions it generates. Proposition 1 describes two cases in which the decision to restrict attention to either an SA or an FA mechanism introduces no additional tax distortions relative to the second best. The proof is presented in Appendix A.

Proposition 1. *If the firm's observable costs depend only on aggregate effort, $\phi(e) = e_1 + e_2$, or if $T < 1$ and unobservable costs depend only on aggregate effort, $\phi(e) = e_1 + e_2$, then the firm's indirect observable costs and its indirect unobservable costs do not depend on the effective tax rate, and as long as a finite solution to (4) exists, indirect observable costs are strictly decreasing in ϵ for all $T < 1$.*

If observable costs depend only on aggregate effort, then the firm can minimize its (constrained) after-tax costs only by minimizing its unobservable costs, \hat{K} , which are independent of T . If

¹⁰The specific aggregate effort function, $\phi(e) \equiv e_1 + e_2$, is used to simplify the presentation of the results in this section. More generally, one can use any function $\phi(e)$ that is smooth and strictly increasing in e . I will indicate where allowing for a broader set of aggregation functions is needed. The function $\phi(e) = e_1$ is the easiest function to use when $e = (e_1, e_2)$ and is used in the example in section 4. Its use does not extend to problems in which effort has more than 2 components so I use $\phi(e) = e_1 + e_2$ in this part of the paper to illustrate how the techniques presented can generalize.

unobservable costs depend only on aggregate effort, then the firm can minimize its (constrained) after-tax costs only by minimizing \hat{C} . However, in this case T is a pure tax that cannot distort the firm's effort choices. When the conditions of Proposition 1 fail to exist, restricting attention to SA and FA mechanisms can introduce additional cost distortions. A trivial example that satisfies the conditions of Proposition 1 arises when there is only one effort variable. With $\phi(e_1) = e_1$, the set of feasible effort choices in (4) is a singleton and the solution to (4) must be independent of T . This is the reason at least two effort choices are needed to make the analysis of optimal SA and FA mechanisms non-trivial.

Assumptions 1d and 2d imply that $0 \leq \partial e_i^* / \partial \epsilon \leq 1$ for $i = 1, 2$ which in turn implies that C_ϵ is negative and K_ϵ is positive. As a result, we can define $E(q, \bar{c}, T, \theta)$ to be the unique value of ϵ for which $C(q, E(q, \bar{c}, T, \theta), T, \theta) = \bar{c}$ for any feasible value of \bar{c} . $E(\cdot, \cdot, \cdot, \cdot)$ represents the information about the aggregate effort choices the union can infer a firm of type θ would make given q , \bar{c} , and T .

3.2 Optimal tax mechanisms.

The direct tax mechanism $\{q(\hat{\theta}), \bar{c}(\hat{\theta}), \tau(\hat{\theta})\}$ implies that multinational profit can be written as

$$\Pi(\hat{\theta}, \theta) \equiv R_1(q_1(\hat{\theta})) + R_2(q_2(\hat{\theta})) - \bar{c}(\hat{\theta}) - K(q(\hat{\theta}), E(q(\hat{\theta}), \bar{c}(\hat{\theta}), T(\hat{\theta}), \theta), T(\hat{\theta}), \theta) - \tau(\hat{\theta}) \quad (5)$$

and the multinational's indirect profit from truthful reporting is $\Pi(\theta) \equiv \Pi(\theta, \theta)$. The effective tax rate $T(\cdot)$ is endogenous only under FA. In this case, given t_1 and t_2 the choice of $\tau(\cdot)$ corresponds to the choice of an apportionment formula $a(q(\cdot), \bar{c}(\cdot))$ which in turn defines $T(\cdot)$. Eq. (5) implies that the mechanism, $\{q(\hat{\theta}), \bar{c}(\hat{\theta}), \tau(\hat{\theta})\}$, will be incentive compatible, if and only if,

$$d\Pi(\theta)/d\theta \equiv d\Pi(\theta, \theta)/d\theta = -K_\epsilon E_\theta - K_\theta < 0 \quad (6)$$

and

$$(\partial^2 \Pi(\hat{\theta}, \theta) / \partial \hat{\theta} \partial \theta) |_{\hat{\theta} = \theta} \geq 0. \quad (7)$$

Condition (6), which follows from the Envelope Theorem, indicates that lower types must earn more profit than higher types and, because the union can regulate observable costs, the only source of rents for the multinational comes through its unobservable costs. Ineq. (7) is the second-order condition for $\hat{\theta} = \theta$

to maximize (5). In addition, an incentive compatible mechanism will be individually rational if, and only if, $\Pi(\theta) \geq 0$ for all θ . Individual rationality ensures that a multinational of type θ can choose to exit the union if its indirect profit is negative. Since $d\Pi(\theta)/d\theta < 0$, individual rationality will be satisfied as long as $\Pi(\bar{\theta}) \geq 0$.

Two final substitutions are used before stating the final version of the union's optimization problem. First, (5) allows one to define the mechanism in terms of $\Pi(\theta)$ instead of $\tau(\theta)$ and makes clear that one role of taxes is to determine the post-tax economic rents earned by the multinational. Second, defining $\bar{\epsilon}(\theta) = E(q(\theta), \bar{c}(\theta), T(\theta), \theta)$ implies $\bar{c}(\theta) = C(q(\theta), \bar{\epsilon}(\theta), T(\theta), \theta)$ and shows that the ability of the union observe some costs, \bar{c} , is equivalent to having the ability to induce specific levels of aggregate effort, $\bar{\epsilon}$. The ability to make these substitutions means one can describe the union's tax mechanism in terms of either $\{q, \bar{c}, \tau\}$ or in terms of $\{q, \bar{\epsilon}, \Pi\}$. Thus, the union's problem will be to choose $\{q, \bar{\epsilon}, \Pi\}$ to maximize expected union welfare,

$$\begin{aligned} \mathcal{E}_\theta W = \mathcal{E}_\theta [& v(q(\theta)) + (1+\lambda)(R_1(q_1(\theta)) + R_2(q_2(\theta)) - C(q(\theta), \bar{\epsilon}(\theta), T(\theta), \theta) - K(q(\theta), \bar{\epsilon}(\theta), T(\theta), \theta)) \\ & + (\alpha-1-\lambda)\Pi(\theta)], \end{aligned} \quad (8)$$

subject to (6), (7), and $\Pi(\bar{\theta}) \geq 0$. As is usual, we will solve this optimization problem without (7). If the solution to the relaxed problem violates (7), the optimal mechanism may involve pooling among some of the types.

Integrating (6) to recover $\Pi(\theta)$ then implies that the optimal mechanism can be found by pointwise maximization of

$$\begin{aligned} & v(q) + (1+\lambda)(R_1(q_1) + R_2(q_2) - C(q, \bar{\epsilon}, T, \theta) - K(q, \bar{\epsilon}, T, \theta)) \\ & - (\alpha-1-\lambda)(F(\theta)/f(\theta))(-K_\epsilon(q, \bar{\epsilon}, T, \theta)E_\theta(q, C(q, \bar{\epsilon}, T, \theta), T, \theta) - K_\theta(q, \bar{\epsilon}, T, \theta)) \end{aligned} \quad (9)$$

and by setting $\Pi(\bar{\theta}) = 0$. Necessary conditions for the optimal output quantities and the aggregate effort

associated with an interior solution are

$$v_{q_k}(q) + (1+\lambda)(R'_k(q_k) - C_{q_k}(q, \bar{\epsilon}, T, \theta) - K_{q_k}(q, \bar{\epsilon}, T, \theta)) - (\alpha-1-\lambda)(F(\theta)/f(\theta))(\partial/\partial q_k)(-K_\epsilon E_\theta - K_\theta) = 0 \quad (10)$$

and

$$-(1+\lambda)(C_\epsilon(q, \bar{\epsilon}, T, \theta) + K_\epsilon(q, \bar{\epsilon}, T, \theta)) - (\alpha-1-\lambda)(F(\theta)/f(\theta))(\partial/\partial \epsilon)(-K_\epsilon E_\theta - K_\theta) = 0. \quad (11)$$

While the main results of this section do not require that an interior solution exist, they are helpful for conveying the economic intuition associated with any of the three optimal tax mechanisms.

First-order conditions (10) and (11) have standard economic interpretations:

$v_{q_k}(q) + (1+\lambda)(R'_k(q_k) - C_{q_k}(q, \bar{\epsilon}, T, \theta) - K_{q_k}(q, \bar{\epsilon}, T, \theta))$ is the difference between the marginal social benefit of production and the direct marginal social cost of production while

$(\alpha-1-\lambda)(F(\theta)/f(\theta))(\partial/\partial q_k)(-K_\epsilon E_\theta - K_\theta)$ equals the marginal information rent the union must pay to induce truthful reporting. Similarly, $-(1+\lambda)(C_\epsilon(q, \bar{\epsilon}, T, \theta) + K_\epsilon(q, \bar{\epsilon}, T, \theta))$ represents the net welfare gains from inducing higher effort while $(\alpha-1-\lambda)(F(\theta)/f(\theta))(\partial/\partial \epsilon)(-K_\epsilon E_\theta - K_\theta)$ equals the marginal effect on information rents of inducing greater effort.

Let the first-best level of aggregate effort (conditional on q and θ) be the solution to $C_\epsilon(q, \bar{\epsilon}, \theta) + K_\epsilon(q, \bar{\epsilon}, \theta) = 0$. As long as $-(\partial/\partial \epsilon)(K_\epsilon E_\theta + K_\theta)$ is not zero at this first-best level, the second-best mechanism will distort the firm's effort choices for all but the best-type firm. If, in addition to Assumptions 1 and 2, $K_{\epsilon\epsilon}$, $K_{\epsilon\theta}$, and $C_{\epsilon\epsilon}$ are positive and $C_{\epsilon\theta}$ is negative, then $-(\partial/\partial \epsilon)(K_\epsilon E_\theta + K_\theta)$ is strictly negative and all types greater than $\underline{\theta}$ will invest in less than the first-best level of aggregate effort.

Assuming strictly positive output and effort choices, the mechanism defined by (10) and (11) with $T = 0$ defines a second-best level of welfare for the union under Assumptions 1-2. Denote this mechanism by $\{q^*(\theta), \bar{\epsilon}^*(\theta), \Pi^*(\theta)\}$ where $\Pi^*(\theta)$ is defined by (6) and $\Pi^*(\bar{\theta}) = 0$. No mechanism can do better. It represents the optimal union tax policy if the member nations were willing to adopt a single harmonized policy that completely replaces existing national tax rules *including tax rates* - what Devereux (2004) refers to as the "single compulsory 'harmonized' tax base" proposal. All accounts of EC

deliberations suggest this option is unlikely to be adopted in the foreseeable future.

SA and FA systems respect national tax rates but as a result impose additional structure on tax rules which may introduce additional welfare distortions. Denote the optimal SA mechanism by $\{q^{s*}(\theta), \bar{e}^{s*}(\theta), \Pi^{s*}(\theta)\}$ and denote the optimal FA mechanism by $\{q^{a*}(\theta), \bar{e}^{a*}(\theta), \Pi^{a*}(\theta)\}$.

Recall from Proposition 1 that if either \hat{C} or \hat{K} depend on effort only via aggregate effort, then the effort choices from a general direct mechanism can also be generated by an SA and an FA mechanism. As long as $t_1 \neq t_2$, the optimal SA and FA mechanisms will also be second-best. (If $t_1 = t_2$, neither the profit-shifting function or the apportionment formula influence any of the firm's choices so the optimal SA and FA mechanisms will be identical, but not necessarily second-best.) Because the choice between SA and FA has no effect on firm incentives under the conditions of Proposition 1, a *tax-incentive dichotomy* is said to exist. This result is summarized as Theorem 2.

Theorem 2. *A tax-incentive dichotomy exists with respect to the choice between the optimal SA and FA mechanisms when the conditions of Proposition 1 are satisfied. In addition, if $t_1 \neq t_2$, then the optimal SA and FA mechanisms are equivalent to the second-best mechanism.*

4. Comparing SA to FA when no tax-incentive dichotomy exists.

Since the conditions of Proposition 1 are only satisfied by special cost functions, the general case will imply that no tax-incentive dichotomy exists and the optimal SA and FA mechanisms generate different economic effects that are not second-best. Throughout this section, assume that $t_1 \neq t_2$.

4.1 Key SA distortions: Cost efficiency.

Recall from (4) that

$$(1-T)C(q, \epsilon, T, \theta) + K(q, \epsilon, T, \theta) = \min_{e_1} (1-T)\hat{C}(q, e_1, \epsilon - e_1, \theta) + \hat{K}(q, e_1, \epsilon - e_1, \theta). \quad (12)$$

Thus by revealed preference, it must be that

$$C(q, \epsilon, 0, \theta) + K(q, \epsilon, 0, \theta) \leq C(q, \epsilon, t_1, \theta) + K(q, \epsilon, t_1, \theta) \quad (13)$$

which implies that pre-tax indirect costs, conditional on aggregate effort ϵ , will be larger under SA than

under the second-best mechanism when $t_1 > 0$. Moreover, if we denote the solution to (4) under SA by $e_1(q, \epsilon, t_1, \theta)$, then $e_1(q, \epsilon, 0, \theta)$ corresponds to the solution to (4) for the second-best mechanism and

$$\partial C / \partial t_1 = (\hat{C}_{e_1} - \hat{C}_{e_2})^2 / [(1-t_1)(\hat{C}_{e_1 e_1} - 2\hat{C}_{e_1 e_2} + \hat{C}_{e_2 e_2}) + \hat{K}_{e_1 e_1} - 2\hat{K}_{e_1 e_2} + \hat{K}_{e_2 e_2}] > 0. \quad (14)$$

Thus, $C(q, \epsilon, t_1, \theta) > C(q, \epsilon, 0, \theta)$ and $K(q, \epsilon, t_1, \theta) < K(q, \epsilon, 0, \theta)$. Because the profit tax rate in the upstream country reduces the marginal after-tax benefit of effort investments in observable costs by $1 - t_1$, SA mechanisms distort effort choices away from reductions in observable costs and towards unobservable costs. By effectively subsidizing observable (deductible) costs, increases in country 1's tax rate reduce the multinational's incentive to invest in reducing observable costs which results in lower unobservable costs and higher total costs.

Proposition 3. *Assume $t_1 > 0$. When no tax-incentive dichotomy exists, SA mechanisms create an incentive for the multinational to reallocate effort (relative to the second-best mechanism) so that observable costs increase, unobservable costs decrease, and the sum of observable and unobservable costs increases.*

Proposition 3 documents the key source of welfare losses caused by a shift from the second-best mechanism to the optimal SA mechanism: higher total production costs due to a reallocation of effort away from reducing observable costs. Related to this change in the cost structure is also a change in the information rents the firm must earn. To the extent that $K_\epsilon E_\theta + K_\theta$ and/or its quantity and effort margins are reduced, the welfare losses due to higher total costs can be moderated. (Since SA mechanisms are themselves direct mechanisms, the optimal SA mechanism can never generate higher expected welfare than the second-best mechanism.)

One way in which the optimal SA mechanism resembles the second-best mechanism is that the highest type firm earns zero rent, $\Pi^s(\bar{\theta}) = 0$. Integrating up the incentive compatibility condition, $d\Pi^s(\theta)/d\theta = -K_\epsilon E_\theta - K_\theta$, and substituting Π^s out of (8) then implies the optimal SA mechanism can be found by choosing q and ϵ pointwise to

$$\max_{\theta} \mathcal{E}_{\theta} \{v(q) + (1+\lambda)(R_1 + R_2 - C - K) - (\alpha-1-\lambda)(F(\theta)/f(\theta))\gamma\} \quad (15)$$

subject to a second-order incentive compatibility constraint where $\gamma(\theta) \equiv -K_{\epsilon}E_{\theta} - K_{\theta} < 0$ equals the multinational's marginal rent evaluated at $T = t_1$. The optimal SA mechanism, $\{q^{s*}(\theta), \bar{\epsilon}^{s*}(t_1, \theta), \Pi^{s*}(\theta)\}$ where $q^{s*}(\theta) = (q_1^{s*}(\theta), q_2^{s*}(\theta))$, can then be used to recover the optimal profit-shifting function, $s(\theta)$, as (5) implies

$$\begin{aligned} \Pi^{s*}(\theta) \equiv & (1-t_1)[R_1(q_1^{s*}(\theta)) - C(q^{s*}(\theta), \bar{\epsilon}^{s*}(t_1, \theta), t_1, \theta)] \\ & + (1-t_2)R_2(q_2^{s*}(\theta)) - K(q^{s*}(\theta), \bar{\epsilon}^{s*}(t_1, \theta), t_1, \theta) + (t_2 - t_1)s(\theta). \end{aligned} \quad (16)$$

Because only the tax rate t_1 affects the firm's effort choices, the profit-shifting function associated with the optimal SA mechanism serves only to control the rents the firm earns.

4.2 Key FA distortions: Firm rents.

FA mechanisms also create effort distortions but the direction and magnitude may differ relative to SA mechanisms. For $a(\theta) < 1$, an apportionment formula will generate smaller effort distortions (relative to a SA mechanism) if $t_2 < t_1$ and larger distortions if $t_2 > t_1$. However, apportionment formulas create an additional source of welfare distortions because the apportionment formula, $a(\theta)$, also affects the firm's effort choices and its rent.

Given $\{q^{a*}(\theta), \bar{\epsilon}^{a*}(T, \theta), \Pi^{a*}(\theta)\}$, the formula $a(\theta)$ controls multinational profit through the effective tax rate $T(\theta)$ via the equation

$$\Pi^{a*}(\theta) \equiv [1 - T(\theta)][R_1(q_1^{a*}(\theta)) + R_2(q_2^{a*}(\theta)) - C(q^{a*}(\theta), \bar{\epsilon}^{a*}(T, \theta), T, \theta)] - K(q^{a*}(\theta), \bar{\epsilon}^{a*}(T, \theta), T, \theta) \quad (17)$$

where $q^{a*}(\theta) = (q_1^{a*}(\theta), q_2^{a*}(\theta))$. In the absence of a tax-incentive dichotomy, this same formula, $a(\cdot)$, also distorts the indirect cost functions C and K through the effort allocation decision. A simple example helps illustrate this point. Suppose $a(\theta) \equiv 1$. Then the indirect cost functions under FA are $C(q, \epsilon, t_1, \theta)$ and $K(q, \epsilon, t_1, \theta)$ which are precisely the indirect cost functions under SA so the FA mechanism with

$a(\cdot) \equiv 1$ creates the same effort allocation incentives as does a SA mechanism. But now $a(\cdot)$ can no longer be used to control the level of firm profit. In equalizing effort incentives, the FA mechanism uses up a degree of freedom that otherwise would have been used to ensure the desired level of firm profit.

This discussion points to a key information-based distinction between SA and FA mechanisms. SA mechanisms introduce an effort distortion through country 1's tax rate while the profit-shifting function controls the overall level of firm profit. FA mechanisms introduce an effort distortion through the apportionment formula, $a(\cdot)$, but this formula simultaneously determines the multinational's rent. Since the optimal SA mechanism induces effort levels that are not second best, the optimal FA mechanism can still yield higher expected welfare by inducing effort choices closer to second-best levels. The key issue is whether improving effort (or productive) efficiency comes at a welfare cost of too much additional firm rent.

With FA, firm profit equals $(1-T)(R_1 + R_2 - C) - K$ which means the union's expected welfare from a mechanism $\{q, \bar{e}^T, T\}$ can be written as

$$\mathcal{E}_\theta \{v(q) + (1+\lambda)T(R_1 + R_2 - C) + \alpha((1-T)(R_1 + R_2 - C) - K)\} \quad (18)$$

where the choice of $T(\cdot)$ via the choice of $a(\cdot)$ affects the firm's indirect costs, C and K , and defines firm rents as

$$(1-T)(R_1 + R_2 - C) - K = \Pi^a(\bar{\theta}) - \int_{x=\theta}^{\bar{\theta}} \gamma(x) dx \quad (19)$$

where again $\gamma(\theta) = -K_\epsilon E_\theta - K_\theta < 0$ is the multinational's marginal information rent.

Absent the second-order incentive compatibility constraint, the problem of deriving the optimal FA mechanism is equivalent to maximizing (18) subject to incentive compatibility constraint, (19), and $\Pi^a(\bar{\theta}) \geq 0$. The union's problem can be viewed as an optimal control problem in which the state variable is the difference between the rent earned by a multinational of type θ and the rent earned by the highest type multinational, $\bar{\theta}$. Thus define $\Gamma(\theta) \equiv \Pi^a(\theta) - \Pi^a(\bar{\theta}) = - \int_{x=\theta}^{\bar{\theta}} \gamma(x) dx$. $\Gamma(\theta)$ will be non-negative and $\Gamma(\bar{\theta}) = 0$. Substituting (19) into the second and third terms of (18) and integrating by parts then implies

$$\mathcal{W}(q, \epsilon, T, \Pi(\bar{\theta}), \theta) = [\nu + (1+\lambda)(R_1 + R_2 - C - K) + (\alpha-1-\lambda)(\Pi(\bar{\theta}) - (F(\theta)/f(\theta))\gamma)]f(\theta) \quad (20)$$

where $\Gamma(\cdot)$ is the state variable, (q, ϵ, T) are the controls, $\Gamma'(\theta) = \gamma(\theta)$, $\Gamma(\bar{\theta}) = 0$, and $\Pi^a(\bar{\theta}) \geq 0$. Denote the Hamiltonian given this formulation by

$$\mathcal{H} = \mathcal{W} + \mu((1-T)(R_1 + R_2 - C) - K - \Pi^a(\bar{\theta}) - \Gamma) + \eta\gamma \quad (21)$$

where $\mu(\theta)$ is the multiplier associated with (19) and $\eta(\theta)$ is the co-state variable. The formal derivation of the optimal FA mechanism is presented in Appendix B.

Remember that the indirect cost functions C and K will be different functions under SA and FA unless FA implies an effective tax rate of t_1 . Moreover, since $\Pi^a(\bar{\theta})$ shows up in constraint (19), we cannot conclude from the monotonic rent structure that minimal firm profit should be zero. Leaving the worst-type firm with positive profit could improve expected union welfare by inducing a more efficient level of effort among all firm types. To understand why it could be welfare optimal for all firm types to earn positive rent, suppose the union were to increase the least productive type firm's rent by a dollar thereby increasing $\Pi^a(\bar{\theta})$ from 0 to 1. To preserve incentive compatibility, rent for every other type would have to increase by 1. This would decrease expected union welfare by $1 + \lambda - \alpha$. At the same time, the only way each type can earn an extra dollar of rent, is for after-tax profits, $(1-T)(R_1 + R_2 - C) - K$, to increase through more efficient levels of q and ϵ or a lower tax rate, T . The shadow price of these efficiency enhancements as captured via (19) is $-\eta(\bar{\theta})$ which under general conditions described in Appendix B will be strictly between 0 and 1. If at $\Pi^a(\bar{\theta}) = 0$, $-\eta(\bar{\theta}) < 1 + \lambda - \alpha$, then the optimal value of $\Pi^a(\bar{\theta})$ must be zero because the marginal social benefit of improving the productive efficiency of the least productive type firm is less than the attendant welfare costs of leaving all types with more rent. If $\Pi^a(\bar{\theta}) > 0$, then $-\eta(\bar{\theta}) = 1 + \lambda - \alpha$ which can only occur when λ is sufficiently smaller than α . $\Pi^a(\bar{\theta})$ is always zero when $\alpha < \lambda$.

Proposition 4. Assume $t_1 \neq t_2$, $-(R_1 + R_2 - C) < \gamma_T \leq 0$, and $(1+\lambda)C_T < R_1 + R_2 - C$. If the social marginal cost of funds is sufficiently small and the welfare weight on firm profit is sufficiently large, then the worst-

type multinational earns strictly positive rent from the optimal FA mechanism.

The bounds on γ_T require that an increase in the effective tax rate increase marginal firm rents (by making γ more negative) but that this increase in marginal rents be less than the firm's taxable income. The last assumption requires that taxable income be large enough to cover the change in social costs due to a change in T . Under these assumptions, Proposition 4 reveals that the optimal FA mechanism can be qualitatively very different from the second-best or optimal SA mechanisms. An immediate implication of Proposition 4 is that as the rent earned by the least productive type multinational increases, the effective tax rate charged this firm must decrease. And, as long as the optimal FA mechanism does not allow any type firm to earn rent equal to its gross profit, $R_1 + R_2 - C - K$, the optimal value of $T(\cdot)$ will be strictly bounded away from zero for all θ . This would be the case if the union's objective was to maximize expected tax revenues, i.e. $v(\cdot, \cdot) \equiv 0$ and $\alpha = 0$. As a result, the optimal FA mechanism does not fully eliminate the cost distortions created when $t_1 > 0$.

4.3 SA vs. FA distortions.

This section compares the optimal SA and FA mechanisms. Recall from the earlier comparison of the optimal SA mechanism and the second-best mechanism that the two are equivalent when $t_1 = 0$ and that the effective tax rate in the optimal FA mechanism is bounded away from zero.¹¹ Thus, under the assumptions of Proposition 4, there will exist statutory tax rates for which the optimal SA mechanism is preferred by the union to the optimal FA mechanism. One such case is when the union is solely interested in maximizing expected tax revenues.

Proposition 5. *Assume $t_1 \neq t_2$, assume the union's objective is to maximize expected tax revenues, and*

¹¹Country 2's tax rate plays no role in the welfare effects of either mechanism because it does not distort the firm's effort choices. If some of the observable costs were generated in country 2, t_2 as well as t_1 would have to be 0 in order for the optimal SA mechanism and the optimal direct mechanism to be equivalent.

assume no tax-incentive dichotomy exists. Then the assumptions of Proposition 4 imply that there exists $t^* > 0$ such that, if $t_1 < t^*$, the optimal SA mechanism yields larger expected tax revenues than the optimal FA mechanism.

Later in this section, Example 1 will show that t^* can be strictly less than one leading to the case in which the optimal FA mechanism is preferred to the optimal SA mechanism.

Returning to the general model in which the union may value spillovers and/or firm profit, we can compare the optimal output and effort levels under SA and FA. Proposition 6 anchors these comparisons by focusing on how the optimal SA and FA mechanisms affect the most productive type multinational.¹²

Proposition 6. Assume $t_1 \neq t_2$ and assume for all q, ϵ, T , and θ that conditions (a)-(c) from Lemma B1 are satisfied. Further suppose that the optimal FA mechanism imposes an effective tax rate on the most productive type multinational no larger than t_1 , that is, $T^{a*}(\theta) \leq t_1$. Then the optimal FA mechanism will induce more production in each country and greater aggregate effort than the optimal SA mechanism if for each $i \in \{q_1, q_2, \epsilon\}$ and for all q, ϵ , and T , $K_i > 0$, $C_{iT} \geq 0$, and $\gamma_i \leq 0$ at $\underline{\theta}$.

Proposition 6 lists three sufficient conditions under which the optimal FA mechanism generates smaller cost distortions, and hence greater production in each country and more aggregate effort, while allowing the firm to earn higher rents relative to the optimal SA mechanism. First, positive marginal unobservable costs, K_i , (with respect to q_1, q_2 , or ϵ depending on the variable under discussion) allow us to sign marginal after-tax profits and determine the direction in which firm rents will fall. Second, a lower effective tax implies lower marginal observable costs so that $T^{a*}(\theta) < t_1$ implies a stronger cost reduction incentive than when $T^{a*}(\theta) = t_1$. And third, larger production or effort values increase the magnitude of the information rents earned by this firm (recall that γ is negative). Because Lemma B1

¹²The comparisons reported in Proposition 6 rely on the same technical restrictions used in Proposition 4 that bound the shadow value of reducing efficiency distortions, the details of which are presented in Lemma B1 in Appendix B.

implies the shadow price of increased rents, μ , is smaller in magnitude than the weight put on lower costs, f , this third effect moderates the cost efficiency effect but does not reverse it. Finally, if $T^{a*}(\theta) > t_1$, then the this third (information rent) effect can dominate and thus result in lower output in at least one country and/or less aggregate effort.

To extend this analysis for $\theta > \underline{\theta}$, two additional factors arise. First, while the marginal rent effects reflected by γ_i are zero at $\underline{\theta}$, they will be non-zero for $\theta > \underline{\theta}$. Thus, for higher-type firms, the incentive to induce higher production and aggregate effort under the optimal FA mechanism will be reduced and can be reversed. Second, the proof of Proposition 6 relies on the fact that near $\underline{\theta}$ the optimal SA and FA mechanisms will be similar. As one considers higher-type firms, the local similarities need no longer persist. As the optimal SA and FA mechanisms diverge for $\theta > \underline{\theta}$, it can become possible for the optimal FA mechanism to result in lower production or effort. However, it is not possible for the optimal FA mechanism to result in lower production in every country and lower aggregate effort under the conditions of Proposition 6. Example 1 which follows provides evidence that the tax rate and marginal rent terms can be large enough to reverse the predictions of Proposition 6 when $t_1 < T^{a*}(\theta)$ or when $\theta > \underline{\theta}$. In so doing, Example 1 illustrates the competing effects of efficient effort investment and rent extraction. The example also provides evidence that the difference in production and effort levels between the optimal SA and the optimal FA mechanisms can be consistent with the local effects identified in Proposition 6.

Example 1. Let $R_i(q_i) = (10 - q_i)q_i$ and let $\hat{C} = (2 - e_1)q_1 + (\theta - e_2)q_2$ and let $\hat{K} = e_1^2 q_1 / 2 + e_2^2 q_2 / 2 + 16$. The fixed cost term in \hat{K} is included for calibration reasons discussed below. The simplest aggregation function to use is $\phi(e) = e_1$ (recall note 9) which implies that $e_2(T, \theta) = 1 - T$,

$$C(q, \epsilon, T, \theta) = (2 - \epsilon)q_1 + (\theta + T - 1)q_2, \quad E(q, \bar{c}, \theta) = 2 - \bar{c}/q_1 + (\theta + T - 1)(q_2/q_1),$$

$$K(q, \epsilon, T, \theta) = \epsilon^2 q_1 / 2 + (1 - T)^2 q_2 / 2 + 16, \quad \text{and} \quad \gamma(\theta) = -\epsilon(\theta)q_2(\theta).$$

$$\int_{w=\theta}^{\bar{\theta}} \epsilon(w)q_2(w)dw.$$

θ is uniformly distributed on $[3/2, 5/2]$ so $f(\theta) = 1$ and $F(\theta) = \theta - 3/2$. This example considers the case of tax revenue maximization, $\alpha=0$ and $v(q)=0$, so $\Pi^a(\bar{\theta}) = 0$. λ is set to zero. It turns out that $W+\eta\gamma$ (the Hamiltonian without constraint (19)) is not globally concave in (q, ϵ, T, Γ) . For q_1 small or q_2 small, the Hessian conditions associated with the second-order and higher principal minors will not be satisfied. Fortunately, Arrow's sufficiency condition (see Theorem 2.5 in Seierstad and Sydsæter (1987)) can be verified. This is done in Appendix D. As a result, conditions (B.1)-(B.9) in Appendix B define the optimal FA mechanism.

The graphs in Figure 1 describe the optimal FA mechanism. Production levels, country 1 effort, multinational rent, and union welfare are all decreasing in type. Because country 2 production and country 1 effort are decreasing in θ , second-order incentive compatibility constraint (7) is satisfied. Country 2 effort and the effective tax rate are not monotonic in this example although they can be strictly decreasing if country revenues are sufficiently large. The optimal SA mechanism is qualitatively similar.

Three features of the effective tax rate under the optimal FA mechanism in Figure 1d are important to note. First, at $\theta = \bar{\theta}$, the after-tax return is defined by the zero profit condition. Hence, $1-T = K/(R_1 + R_2 - C)$. Without a fixed cost term in \hat{K} , $T(\bar{\theta})$ would be very close to one simply because unobservable indirect costs are small relative to observable operating profit. By including a fixed cost term in unobservable costs, a more realistic range for $T(\cdot)$ arises without increasing the analytic complexity of the example. In this example, the range of effective tax rates is .48 to .51. Second, the hump-shaped curve in Figure 1d reflects the tradeoff between rent extraction and effort incentives. For low types, the dominant effect is the rent extraction effect. As firm type increases above $\underline{\theta}$, the union can extract more rent from the multinational by imposing a higher effective tax rate which distorts effort in both countries down. The lower investment in country 1 effort in turn reduces the marginal information rent the multinational earns. For high types, the dominant effect is the effort incentive effect. Now the union will offer higher types a lower tax rate in order to encourage more investment in country 2 effort.

This tradeoff is offered to partially offset the lower productivity associated with higher type firms. Overall marginal rents still decline with θ although at a slower rate than if a lower tax rate was not offered.¹³ Third, the value of $T(\cdot)$ is independent of the statutory tax rates, t_1 and t_2 , because the formulation of the optimal FA mechanism does not require *a priori* that T fall between t_1 and t_2 . Imposing this restriction would add a bias against FA mechanisms. Thus, the solution to (21) represents the best chance for the optimal FA mechanism to dominate the optimal SA mechanism. What will depend on both t_1 and t_2 is the apportionment formula, $a(\theta) = (T(\theta) - t_2)/(t_2 - t_1)$.

As noted before, the optimal SA mechanism does depend on t_1 via the indirect cost functions. To compare the optimal FA mechanism with the optimal SA mechanism in a way that reflects this sensitivity, the graphs in Figure 2 compare the optimal FA mechanism to each of three optimal SA mechanisms corresponding to three different values of t_1 : a lower rate than that implied by the optimal FA mechanism, $t_1 = .3$ (thin solid lines), the minimum rate from the optimal FA mechanism, $t_1 = .48$ (dashed lines), and a rate that exceeds $T(\theta)$ for all θ from the optimal FA mechanism, $t_1 = .6$ (thick solid lines). Proposition 6 can be invoked for the last two cases in which t_1 is equal to .48 or .6. The relationship between the optimal FA mechanism and the optimal direct mechanism will be qualitatively similar to the relationship between the optimal FA mechanism and the optimal SA mechanism when $t_1 = .3$. For each component, the lines describe the difference between the optimal FA mechanism value minus the optimal SA mechanism value.

Figures 2a, c, and d reveal that for this example the optimal FA mechanism induces more production in country 1, more cost-reducing effort in country 1, and larger rents than the optimal SA mechanism. The sign of the differences in country 2 production (Figure 2b), cost-reducing effort in country 2 (inferred from Figure 1d), and union welfare (Figure 2e) can depend both on country 1's tax

¹³For other examples, this trade-off can manifest itself in the form of non-monotonic production quantities or effort levels while still satisfying the second-order incentive constraint.

rate, t_1 , and the value of θ . Consistent with Proposition 6, country 2 production and effort are higher under the optimal FA mechanism for the best-type firm when $t_1 \geq .48$. As θ increases, country 2 production remains larger under the optimal FA mechanism when $t_1 = .6$ while the reversal possibilities described in the discussion following Proposition 6 arise when $t_1 = .48$ for $\theta > 1.55$ and they arise for all θ when $t_1 = .3$. Country 2 effort is also larger under the optimal FA mechanism for some of the lowest and highest productivity types as long as $t_1 > T^{a*}(\theta) = .48$ since $e_2 = 1 - T$.

Figure 2e shows that union welfare from the optimal FA mechanism minus union welfare from the optimal SA mechanism is increasing in θ . For θ close to $\underline{\theta}$, the incentive under the optimal FA mechanism to induce more efficient cost-reducing effort in return for larger rents results in larger tax revenues under the optimal FA mechanism if the optimal SA distortions are large enough. This is the case when $t_1 = .6$ but not for t_1 equal to $.48$ or $.3$. For θ close to $\bar{\theta}$, the optimal FA mechanism has the largest advantage over the optimal SA mechanism due to the reduction in the effective tax rate reflected in Figure 1d which induces more cost-reducing effort via higher marginal rents. Since the least productive type earns zero rent under both the optimal FA and SA mechanisms, the first-order effect from higher rents is close to zero for θ close to $\bar{\theta}$ so the cost-reduction effect dominates. Figure 2f shows the net effect on the union's expected welfare. Consistent with Proposition 5, the optimal SA mechanism yields higher expected tax revenues when t_1 is small because the cost distortions generated under the optimal SA mechanism are also small. In order for the cost efficiency/rent trade-off inherent under the optimal FA mechanism to yield larger expected tax revenues, the cost distortions under the optimal SA mechanism must be sufficiently large. In this example, the optimal FA mechanism generates a higher level of expected union welfare for t_1 above approximately $.5$.

Finally, suppose we modify the union's objective by putting positive weight on firm profit. This would be consistent with an argument for FA based on union preferences that weight productive efficiency high relative to tax revenues. For values of α close to zero, increases in α do increase the

range of upstream tax rates for which the optimal FA mechanism yields larger expected welfare than the optimal SA mechanism. For values of α closer to one, Figure 3 plots the profit paid to a multinational of type $\bar{\theta}$ under the optimal FA mechanism as a function of α . Consistent with Proposition 5, this profit will be strictly positive for α greater than approximately .95 and it increases with α . For instance, when $\alpha = .978$, the least productive type firm earns a profit of 10 under the optimal FA mechanism while the most productive type firm earns a profit of 4.18 under the optimal SA mechanism. Thus, all types can earn significantly higher profits under the optimal FA mechanism and generate much lower expected union welfare when α is large enough. For increases in α close to one, the range of upstream tax rates that favor the optimal FA mechanism now decreases.

5. Concluding remarks.

Separate accounting and formula apportionment are the two main approaches governments use to tax multijurisdictional firms. Recent interest by the EU in formula apportionment has prompted comparative economic analyses of various apportionment formulas to separate accounting rules. This is the first paper to model explicitly the informational disadvantages that prompt national and state governments to use either transfer prices or apportionment formulas and to derive the optimal separate accounting and formula apportionment mechanisms under these informational constraints. Comparing these optimal mechanisms reveals that a shift from separate accounting to formula apportionment creates a tension between the use of formula apportionment to encourage more efficient investment within the firm and the use of separate accounting to extract more rents. The optimal formula apportionment mechanism will be preferred by the union to the optimal separate accounting mechanism only if the cost distortions associated with the optimal separate accounting mechanism are sufficiently large. This will be the case when national tax rates are large in the upstream countries. One key indication of the limited rent extraction capability of apportionment formulas is the result that the union may have to leave even the worst-type firm with positive rent.

The current model can be extended to incorporate several other aspects of multinational taxation: the introduction of multiple factors and double taxation. The first extension would be of interest in a general equilibrium model in which factor prices are endogenous. In the current model, only output prices are endogenous as reduced form cost functions are used. Depending on how a multinational's home country handles repatriated earnings, double taxation policies could also influence the extent to which apportionment formulas and separate accounting regulations distort unobservable effort decisions. The current model assumes no repatriation motive for the multinational and that each government exempts foreign-source profits from taxation (consistent with the double taxation policies of most EU countries). If a multinational's home country used a credit rule for double taxation and the multinational was in an excess credit position, income repatriation issues could confound effort incentives under both formula apportionment and separate accounting.

At several points in the paper, it was noted that country 2's statutory tax rate plays no role in comparing the optimal separate accounting mechanism to the optimal direct mechanism. This is due to the fact that all observable (upstream) costs affected by the multinational's effort choices are realized in country 1. If these costs were distributed between both countries, optimal effort choices under separate accounting would depend on both tax rates. The decision to locate all observable costs in country 1 was made to simplify the presentation of the tax-incentive dichotomy arguments. Distributing observable costs across both countries would not alter the qualitative properties described in Propositions 4-6 nor would it change the basic methodology.

The model also assumes that the union welfare function weights tax revenues collected in each country equally. Distributional concerns about the location of production and revenues can be reflected in the $v(q)$ as long as v is concave. Introducing welfare weights to reflect distributional preferences for tax revenues as well as a more explicit modelling of revenue and production distribution preferences would not change the basic form of the analysis nor the qualitative trade-offs between rent extraction and

efficiency identified in this paper. However, their introduction could identify new trade-offs not captured in this model as well as add substantially to the paper's length. As a result, I leave the study of these distributional questions to future research.

There are two issues studied in the international tax literature which this paper does not address. The first is the effect of tax competition. In order to focus on the informational differences between separate accounting and apportionment formulas, it was assumed that the countries' statutory tax rates were exogenous. The results of this paper can be viewed as a first step in understanding the broader tax competition implications of these two types of tax systems. The second issue concerns the possible adoption of a hybrid system in which a union allows multinationals to choose between separate accounting and formula apportionment as is currently done in Canada. (See Mintz and Smart (2004)). Both issues are important topics for future research.

References

- Baron, D. and R. Myerson, 1982, Regulating a monopolist with unknown costs. *Econometrica* 50:911-930.
- Burbidge, J., K. Cuff, and J. Leach, 2006, Tax competition with heterogeneous firms. *Journal of Public Economics* 15:447-470.
- Devereux, M., 2004, Debating proposed reforms of the taxation of corporate income in the European Union. *International Tax and Public Finance* 11:71-89.
- Eichner, T. and M. Runkel, 2011, Corporate income taxation of multinationals in a general equilibrium model. *Journal of Public Economics* 95:723-33.
- European Commission, 2001, Towards an internal market without tax obstacles: A strategy for providing companies with a consolidated corporate tax base for their EU-wide activities, Communication from the Commission to the Council, the European Parliament and the Economic and Social Committee, COM (2001) 582 final.
- _____, 2002, Company taxation in the internal market, COM(2001) 582 Final, Brussels.
- _____, 2011, Proposal for a Directive on a Common Consolidated Corporate Tax Base (CCCTB), COM (2011) 121/3, Brussels.
- Gérard, M., 2005, Multijurisdictional firms and governments' strategies under alternative tax designs, mimeo.
- Gordon, R. and J. Wilson, 1986, An examination of multijurisdictional corporate income taxation under formula apportionment. *Econometrica* 54: 1357-1374.
- Gresik, T., 2001, The taxing task of taxing transnationals. *Journal of Economic Literature* 39: 800-38.
- _____, 2010, Separate accounting vs. formula apportionment: A private information perspective. *European Economic Review* 54:133-149.
- Hellerstein, W. and C. McLure Jr., 2004, The European Commission's report on company income

taxation: What the EU can learn from the experience of the US states. *International Tax and Public Finance* 11: 199-220.

Kind, H., K. Midelfart, and G. Schjelderup, 2005, Corporate tax systems, multinational enterprises, and economic integration. *Journal of International Economics* 65:507-521.

Laffont, J.-J and J. Tirole, 1994, *A Theory of Incentives in Procurement and Regulation*. MIT Press.

Mintz, J., 2004, Corporate tax harmonization in Europe: It's all about compliance. *International Tax and Public Finance* 11: 221-234.

Mintz, J. and J. Martens-Weiner, 2003, Exploring formula apportionment for the European Union. *International Tax and Public Finance* 10: 695-711.

Mintz, J. and M. Smart, 2004, Income shifting, investment, and tax competition: theory and evidence from provincial taxation in Canada. *Journal of Public Economics* 88: 1149-1168.

Mirrlees, J.M., 1971. An exploration in the theory of optimum income taxation. *Review of Economic Studies* 38: 175–208.

Nielsen, S., P. Raimondos-Møller, G. Schjelderup, 2003, Formula apportionment and transfer pricing under oligopolistic competition. *Journal of Public Economic Theory* 5: 417-436.

_____, 2010, Company taxation and tax spillovers: Separate accounting versus formula apportionment. *European Economic Review* 54:121-132.

OECD, 2010, OECD transfer price guidelines for multinational enterprises and tax administrations, Paris.

Seade, J., 1977, On the shape of optimal tax schedules. *Journal of Public Economics* 7: 203–236.

Seierstad, A. and K. Sydsæter, 1987, *Optimal Control Theory with Economic Applications*. North-Holland.

Sørensen, P., 2003, Company tax reform in the European Union, EPRU working paper #03-08.

_____, 2004, Company tax reform in the European Union. *International Tax and Public Finance* 11: 91-115.

Stiglitz, J.E., 1982, Self-selection and Pareto efficient taxation. *Journal of Public Economics* 17: 213–240.

Zodrow, G., 2004, Tax competition and tax coordination in the European Union. *International Tax and Public Finance* 11: 651-671.

Appendix A: Proof of Proposition 1

A more general version of Proposition 1 is presented here to allow for a wider range of aggregate effort functions.

Proposition 1. *Suppose there exists a smooth, weakly increasing aggregate effort function $\phi(e)$ such that the firm's observable costs depend only on aggregate effort, $\phi(e)$, or if $T < 1$ and unobservable costs depend only on aggregate effort, $\phi(e)$, then the firm's indirect observable costs and its indirect unobservable costs do not depend on the effective tax rate, and as long as a finite solution to (4) exists, indirect observable costs are strictly decreasing in ϵ for all $T < 1$.*

Proof. First, suppose that $\hat{C}(q, e, \theta) = \hat{C}(q, \phi(e), \theta)$. Then (4) is equivalent to minimizing \hat{K} subject to $\phi(e) = \epsilon$ for all T and the solutions to (4) is independent of T . C_ϵ must be strictly negative because by assumption $\hat{C}_{e_i} < 0$ and $\phi_{e_i} > 0$.

Second, suppose that $\hat{K}(q, e, \theta) = \hat{K}(q, \phi(e), \theta)$. Now (4) is equivalent to minimizing $(1-T)\hat{C}$ subject to $\phi(e) = \epsilon$. As long as $T < 1$, it will not distort the effort choices that minimize \hat{C} . Minimizing $(1-T)\hat{C}$ subject to $\phi(e) = \epsilon$ implies the Lagrangian $\mathcal{L} = (1-T)\hat{C} + \delta(\phi(e) - \epsilon)$. The first-order conditions then imply $\delta = -(1-T)\hat{C}_{e_i}/\phi_{e_i} > 0$. By the Envelope Theorem, if a finite solution to (4) exists, then $C_\epsilon = -\delta < 0$. *Q.E.D.*

Appendix B: Derivation of the optimal FA mechanism and proof of Proposition 4.

The optimal FA mechanism will be the solution to the following set of Euler and transversality conditions:

$$[v_{q_1} + (1+\lambda)(R_1' - C_{q_1} - K_{q_1})]f + \mu[(1-T)(R_1' - C_{q_1}) - K_{q_1}] = [-\eta + (\alpha-1-\lambda)F]\gamma_{q_1}, \quad (\text{B.1})$$

$$[v_{q_2} + (1+\lambda)(R_2' - C_{q_2} - K_{q_2})]f + \mu[(1-T)(R_2' - C_{q_2}) - K_{q_2}] = [-\eta + (\alpha-1-\lambda)F]\gamma_{q_2}, \quad (\text{B.2})$$

$$-(1+\lambda)[C_\epsilon + K_\epsilon]f - \mu[(1-T)C_\epsilon + K_\epsilon] = [-\eta + (\alpha-1-\lambda)F]\gamma_\epsilon, \quad (\text{B.3})$$

$$-(1+\lambda)[C_T + K_T]f - \mu[R_1 + R_2 - C] = [-\eta + (\alpha-1-\lambda)F]\gamma_T, \quad (\text{B.4})$$

$$\partial\mathcal{H}/\partial\Gamma = -\mu = -\eta', \quad (\text{B.5})$$

$$\Gamma'(\theta) = \gamma(\theta), \quad (\text{B.6})$$

$$(19), \eta(\underline{\theta}) = 0, \Pi^a(\bar{\theta}) \geq 0, \text{ and } \Gamma(\bar{\theta}) = 0, \quad (\text{B.7})$$

and

$$\alpha - 1 - \lambda - \int_{\theta=\underline{\theta}}^{\bar{\theta}} \mu(\theta) d\theta \leq 0 \quad (\text{B.8})$$

where (B.8) is the first-order condition with respect to $\Pi^a(\bar{\theta})$. Given (B.5-B.6), (B.8) can be rewritten as

$$-\eta(\bar{\theta}) \leq 1 + \lambda - \alpha. \quad (\text{B.9})$$

The following lemma provides sufficient conditions under which we can bound $\eta(\cdot)$ and $\mu(\cdot)$.

Lemma B1. Assume $t_1 \neq t_2$. If (a) $-(R_1 + R_2 - C) < \gamma_T \leq 0$ and (b) $(1 + \lambda)C_T < R_1 + R_2 - C$ for all q, ϵ, T , and θ , then $0 > \eta(\theta) \geq -F(\theta)$. In addition, if $\alpha \geq \lambda$ or (c) $\gamma_T > (R_1 + R_2 - C - (1 + \lambda)TC_T) / ((\alpha - \lambda)F/f)$, then $0 > \mu(\theta) > -f(\theta)$.

Proof of Lemma B1.

Rewrite (B.4) as $(R_1 + R_2 - C)\eta' - \gamma_T \eta = \beta$ where $\beta = -(1 + \lambda)(C_T + K_T)f - (\alpha - 1 - \lambda)F\gamma_T$. Since $(1 - T)C_T + K_T = 0$, $C_T + K_T = TC_T \geq 0$. Thus, $\gamma_T \leq 0$ implies $\beta \leq 0$. Given $\eta(\underline{\theta}) = 0$, the solution to (B.4) is

$$\eta(\theta) = e^{\int_{w=\underline{\theta}}^{\theta} \gamma_T / (R_1 + R_2 - C) dw} \cdot \int_{z=\underline{\theta}}^{\theta} e^{-\int_{w=\underline{\theta}}^z \gamma_T / (R_1 + R_2 - C) dw} \cdot \beta(z) / (R_1 + R_2 - C) dz. \quad (\text{B.10})$$

Since $\beta(\theta) \leq 0$ for all θ , (B.10) implies $\eta(\theta) \leq 0$ for all θ . Moreover, $\gamma_T \leq 0$ implies

$$\eta(\theta) \geq e^{\int_{w=\underline{\theta}}^{\theta} \gamma_T / (R_1 + R_2 - C) dw} \cdot \int_{z=\underline{\theta}}^{\theta} e^{-\int_{w=\underline{\theta}}^z \gamma_T / (R_1 + R_2 - C) dw} \cdot \beta(z) / (R_1 + R_2 - C) dz \geq \int_{z=\underline{\theta}}^{\theta} \beta(z) / (R_1 + R_2 - C) dz. \quad (\text{B.11})$$

The assumptions, $|\gamma_T| < R_1 + R_2 - C$ and $(1 + \lambda)C_T < R_1 + R_2 - C$, mean $\beta / (R_1 + R_2 - C) > -1$ and $\eta(\theta) \geq -F(\theta)$.

Differentiating $\eta(\theta)$ from (B.10) implies

$$\mu(\theta) = \gamma_T \eta / (R_1 + R_2 - C) + \beta(\theta) / (R_1 + R_2 - C) \geq -(1 + \lambda)TC_T f / (R_1 + R_2 - C) - (\alpha - \lambda)F\gamma_T / (R_1 + R_2 - C). \quad (\text{B.12})$$

If $\alpha \geq \lambda$, $(1+\lambda)C_T < R_1 + R_2 - C$ and $\gamma_T \leq 0$ implies $\mu(\theta) > -f'(\theta)$. If $\alpha < \lambda$ and

$\gamma_T > (R_1 + R_2 - C - (1+\lambda)TC_T)/((\alpha-\lambda)F/f)$, then again (B.12) implies $\mu(\theta) > -f'(\theta)$. *Q.E.D.*

Conditions (a) and (c) in Lemma B1 require that an increase in the effective tax rate increase firm rents (by making γ more negative) and that this marginal rent effect is not too large relative to taxable income, $R_1 + R_2 - C$. Condition (b) requires that taxable income be large enough to cover the change in observable social costs due to a change in T .

Proposition 4 then follows directly from inequality (B.9) and Lemma B.1.

Appendix C: Proof of Proposition 6.

The proof of Proposition 6 will rely on Lemma B.1. Consider the choice of country 2 output. The Euler condition for q_2 , (B.2), can be rewritten as

$$\begin{aligned} \mathcal{H}_{q_2}(q, \epsilon, T, \theta) &= [v_{q_2} + (1+\lambda)(R_2' - C_{q_2}(T) - K_{q_2}(T))]f \\ &+ \mu[(1-T)(R_2' - C_{q_2}(T)) - K_{q_2}(T)] + [\eta - (\alpha-1-\lambda)F]\gamma_{q_2}(T) = 0 \end{aligned} \quad (\text{C.1})$$

where $C_{q_2}(T)$, $K_{q_2}(T)$, and $\gamma_{q_2}(T)$ is shorthand notation for $C_{q_2}(q, \epsilon, T, \theta)$, $K_{q_2}(q, \epsilon, T, \theta)$, and $\gamma_{q_2}(q, \epsilon, T, \theta)$ and is used to emphasize the dependence of costs and marginal rents on the effective tax rate. In order to decompose the effect of T on rent extraction versus the effect on efficiency, it is possible to rewrite \mathcal{H}_{q_2} from (C.1) as

$$\begin{aligned} \mathcal{H}_{q_2}(q, \epsilon, T, \theta) &= [v_{q_2} + (1+\lambda)(R_2' - C_{q_2}(t_1) - K_{q_2}(t_1) + C_{q_2}(t_1) - C_{q_2}(T) + K_{q_2}(t_1) - K_{q_2}(T))]f \\ &+ \mu[(1-T)(R_2' - C_{q_2}(t_1)) - K_{q_2}(t_1) + (1-T)(C_{q_2}(t_1) - C_{q_2}(T)) + K_{q_2}(t_1) - K_{q_2}(T)] \\ &+ \eta\gamma_{q_2}(T) - (\alpha-1-\lambda)F[\gamma_{q_2}(t_1) + \gamma_{q_2}(T) - \gamma_{q_2}(t_1)] \end{aligned} \quad (\text{C.2})$$

where terms such as $C_{q_2}(t_1)$ mean $C_{q_2}(q, \epsilon, t_1, \theta)$. For the optimal SA mechanism,

$$[v_{q_2} + (1+\lambda)(R_2' - C_{q_2}(t_1) - K_{q_2}(t_1))]f - (\alpha-1-\lambda)F\gamma_{q_2}(t_1) = 0 \quad (\text{C.3})$$

evaluated at $(q^{s*}, \bar{\epsilon}^{s*}, t_1, \theta)$. Combining (C.2) and (C.3) implies that

$$\begin{aligned}
\mathcal{H}_{q_2}(q^{s*}, \bar{\epsilon}^{s*}, T, \theta) &= (1+\lambda)[C_{q_2}(t_1) - C_{q_2}(T) + K_{q_2}(t_1) - K_{q_2}(T)]f \\
&+ \mu[-T \cdot K_{q_2}^T(t_1) + (1-T)(-v_2 + (\alpha-1-\lambda)F\gamma_{q_2}(t_1)/f)/(1+\lambda) \\
&+ (1-T)(C_{q_2}(t_1) - C_{q_2}(T)) + K_{q_2}(t_1) - K_{q_2}(T)] \\
&+ \eta\gamma_{q_2}(T) - (\alpha-1-\lambda)F[\gamma_{q_2}(T) - \gamma_{q_2}(t_1)].
\end{aligned} \tag{C.4}$$

Let $T^{a*}(\theta)$ denote the optimal effective tax rate under formula apportionment and suppose $T^{a*}(\theta) = t_1$. At $\theta = \underline{\theta}$, (C.4) simplifies down to $\mathcal{H}_{q_2}(q^{s*}, \bar{\epsilon}^{s*}, t_1, \underline{\theta}) = -\mu[t_1 K_{q_2}(t_1) + (1-t_1)v_2/(1+\lambda)] > 0$ as $\eta(\underline{\theta})=0$. The term in brackets is equal to $-[(1-t_1)(R_2' - C_{q_2}'(t_1)) - K_{q_2}(t_1)]$, the negative of marginal after-tax profit at t_1 , and is positive. With marginal after-tax profits less than zero under SA, FA gives the union the incentive to reduce the rent earned by the lowest-type multinational by increasing country 2 output beyond the optimal SA level when t_1 is equal to the optimal effective tax rate levied on the lowest-type multinational.

Note also that

$$\partial \mathcal{H}_{q_2}(q^{s*}, \bar{\epsilon}^{s*}, T, \theta) / \partial t_1 = (1+\lambda)[C_{q_2 T}(t_1) + K_{q_2 T}(t_1)]f - \mu T \cdot K_{q_2 T}(t_1) + \mu[(1-T)C_{q_2 T}(t_1) + K_{q_2 T}(t_1)] \tag{C.5}$$

Since $(1-T)C_T(q, \epsilon, T, \theta) + K_T(q, \epsilon, T, \theta) = 0$ for all q, ϵ, T , and θ , it will also be true that

$(1-T)C_{q_2 T}(q, \epsilon, T, \theta) + K_{q_2 T}(q, \epsilon, T, \theta) = 0$ for all q, ϵ, T , and θ and (C.5) is equivalent to

$$\partial \mathcal{H}_{q_2}(q^{s*}, \bar{\epsilon}^{s*}, T, \theta) / \partial t_1 = C_{q_2 T}(t_1)[(1+\lambda)t_1 f + \mu T(1-t_1) + \mu(t_1 - T)]. \tag{C.6}$$

Lemma B.1 implies the bracketed term in (C.6) is positive for all $t_1 \geq T$ (because $(1+\lambda)f + \mu > 0$) which in turn implies that $\partial \mathcal{H}_{q_2}(q^{s*}, \bar{\epsilon}^{s*}, T^{a*}, \theta) / \partial t_1$ is positive for all $t_1 \geq T^{a*}(\theta)$ if $C_{q_2 T} \geq 0$. While the above analysis focused on country 2 production for the sake of concreteness, the same analysis can be done with respect to q_1 and ϵ . Q.E.D.

Appendix D: Arrow sufficiency condition

Define $\tilde{\mathcal{H}}(q, \epsilon, T, \Gamma, \theta, \eta) = w + \eta\gamma$ (which by construction is independent of Γ) and define $T^*(q, \epsilon, \Gamma, \theta)$ to be the solution to (19). Then define $\hat{\mathcal{H}}(\Gamma, \theta, \eta) = \max_{(q, \epsilon)} \tilde{\mathcal{H}}(q, \epsilon, T^*, \Gamma, \theta, \eta)$. Arrow's sufficiency condition requires that, for some η , $\hat{\mathcal{H}}$ be concave in Γ for all θ . By the envelope theorem,

$\hat{\mathcal{H}}_\Gamma(\Gamma, \theta, \eta) = \tilde{\mathcal{H}}_\Gamma(q, \epsilon, T^*, \Gamma, \theta, \eta) = \tilde{\mathcal{H}}_\Gamma(q, \epsilon, T, \Gamma, \theta, \eta) T_\Gamma^*(q, \epsilon, \Gamma, \theta)$ where the last equality arises because $\tilde{\mathcal{H}}$ is

independent of Γ . For Example 1, $\hat{\mathfrak{H}}_{\Gamma} = T^* C_T f(R_1 + R_2 - C)$ evaluated at T^* and at the values of q and ϵ that maximize $\tilde{\mathfrak{H}}$ given the solution to $\eta(\theta)$ from (B.1)-(B.8). We need to show that $\hat{\mathfrak{H}}_{\Gamma}$ is decreasing in Γ . This was done in *Mathematica* by numerically maximizing $\tilde{\mathfrak{H}}$ over $q_i \in [0,10]$ and $\epsilon \in [0,2]$. Outside these bounds $\tilde{\mathfrak{H}}$ is clearly strictly concave. Then $\hat{\mathfrak{H}}_{\Gamma}$ was evaluated at these optimal values. The results are presented in Figure D1 in which Γ ranges from 0 to 20. The graph shows $\hat{\mathfrak{H}}$ is clearly concave in Γ .

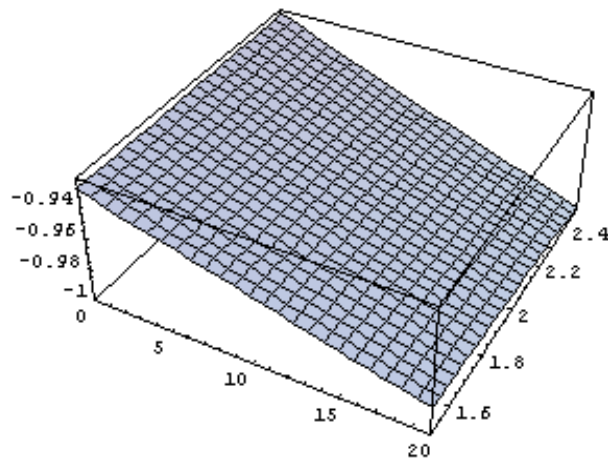


Figure D1: $\hat{\mathfrak{H}}_{\Gamma}$

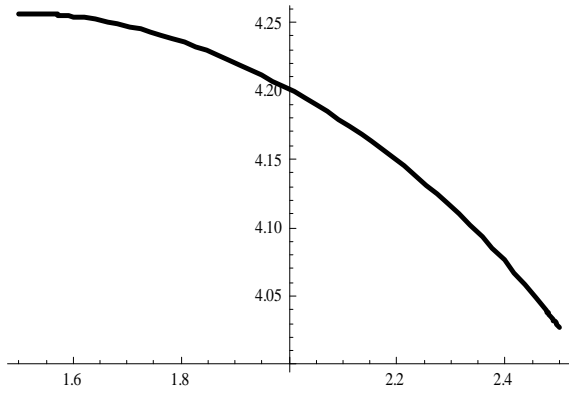


Figure 1a: Country 1 production, $q_1(\theta)$

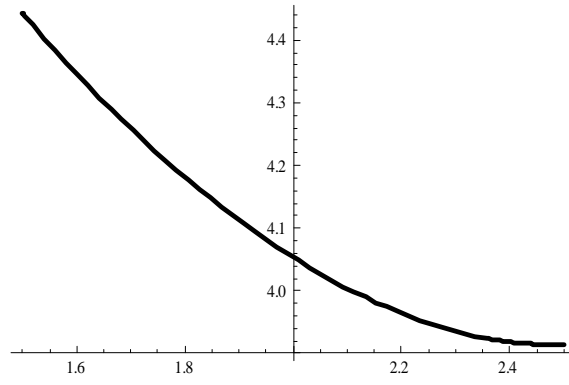


Figure 1b: Country 2 production, $q_2(\theta)$

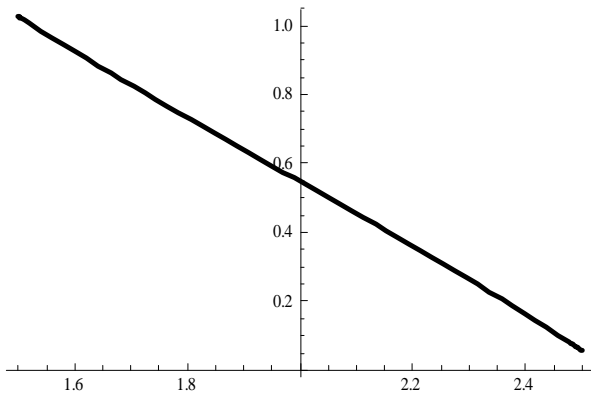


Figure 1c: Country 1 cost-reducing effort, $e_1(\theta)$

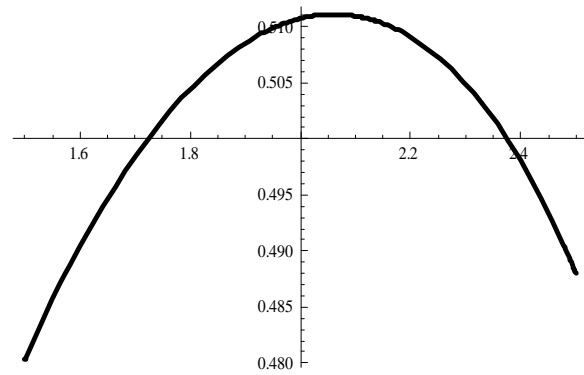


Figure 1d: Effective FA tax rate, $T(\theta)$, and $1-e_2(\theta)$

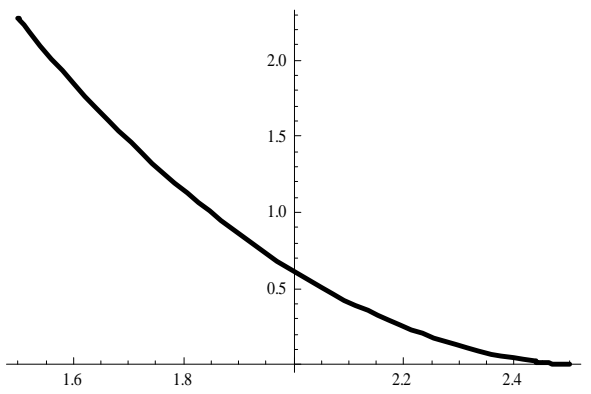


Figure 1e: Multinational rent, $\Gamma(\theta)$

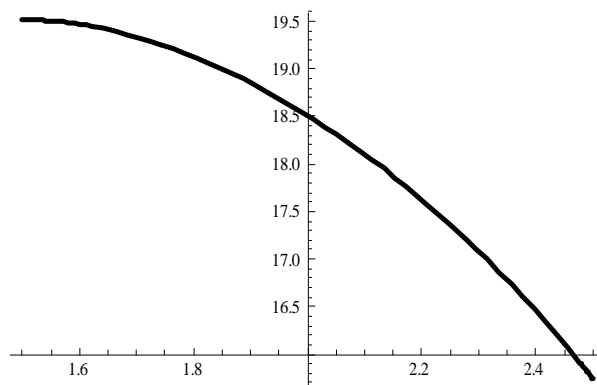


Figure 1f: Union welfare, $W(\theta)$

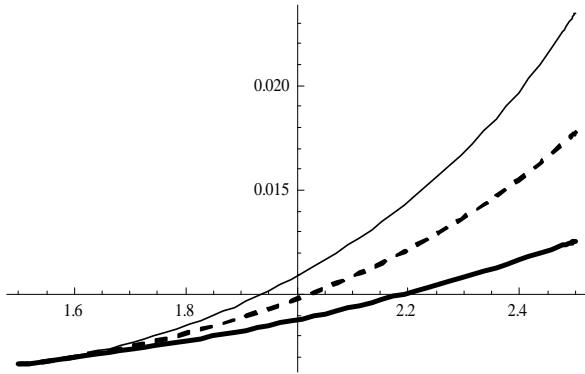


Figure 2a: Differences in country 1 production (FA-SA)

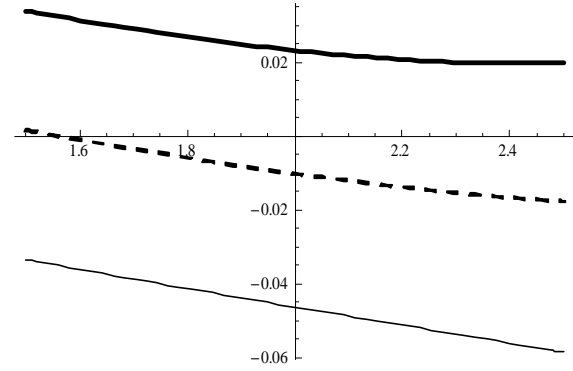


Figure 2b: Differences in country 2 production (FA-SA)

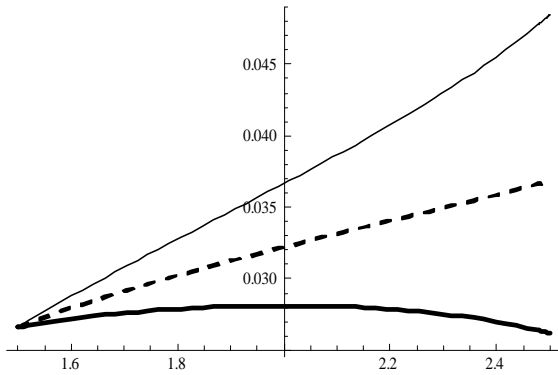


Figure 2c: Differences in country 1 effort (FA-SA)

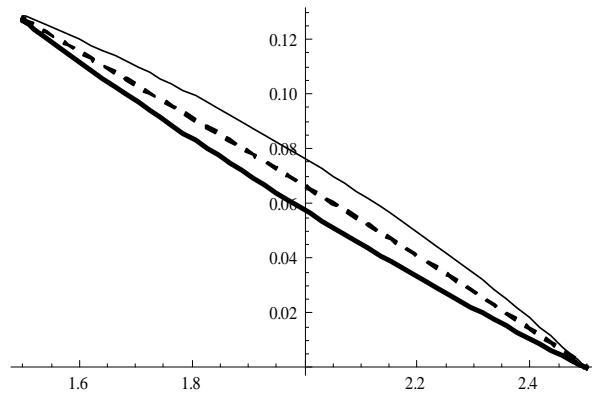


Figure 2d: Differences in multinational rent (FA-SA)

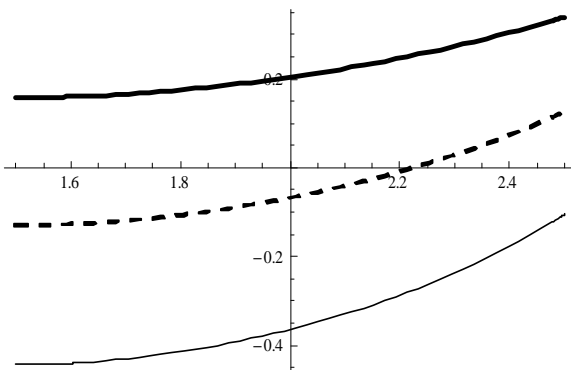


Figure 2e: Differences in union welfare (FA - SA)

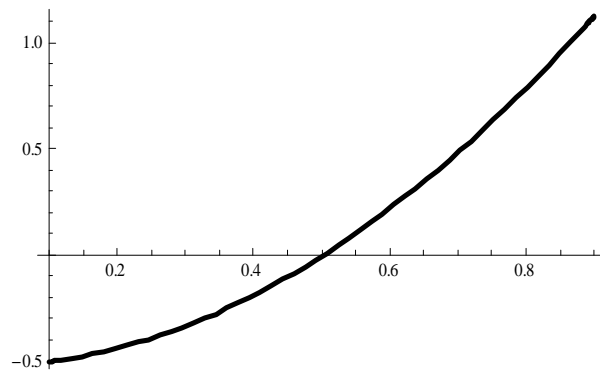


Figure 2f: Differences in expected union welfare as a function of t_1 (FA-SA)

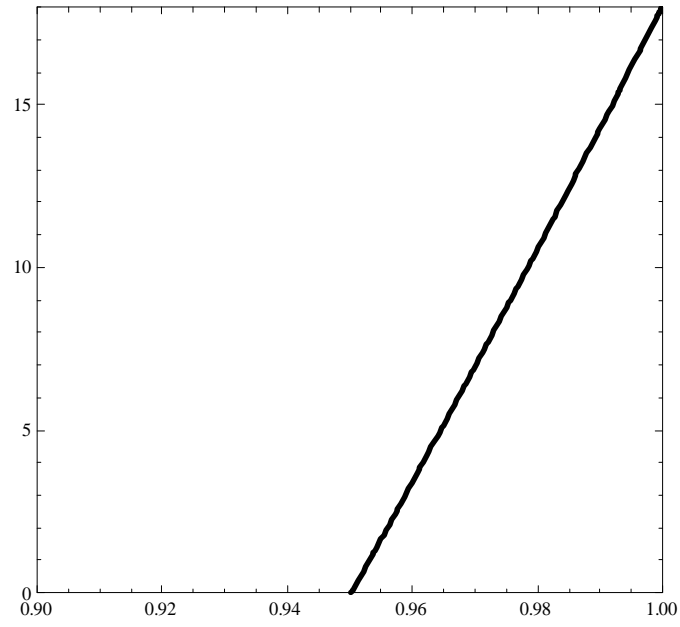


Figure 3: The rent earned by the worst-type firm in the optimal FA mechanism as a function of α .