

Efficient Delegation by an Informed Principal¹

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Abstract: Motivated by examples from the insurance industry, the automobile industry, retailing, and multinational strategy, we study an organizational structure we refer to as "partial delegation." In a bargaining problem between an informed party and an uninformed party, partial delegation involves the informed party delegating control of bargaining responsibilities to an agent while retaining control of its private information. We show that partial delegation, with or without an observable agency contract, enables the informed party to earn information rents without creating quantity distortions. That is, first-best quantities are traded in equilibrium. In general, we argue that partial delegation allows an informed principal to promote more efficient agreements with outside parties while also endogenously improving its bargaining power.

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1. Introduction.

Delegated authority is ubiquitous. Examples range from democratic government to representative bargaining to divisionalization in large corporations. When a subordinate has superior information, delegating decision-making authority can result in more efficient decisions but it also introduces agency costs. Much of the incentive-based literature on delegation seeks to identify organizational and informational conditions under which this tradeoff supports delegation over centralization.⁴ In these cases, the focus is on what organizational form better serves an uninformed principal. We seek instead to focus on the benefits of the commitment role of delegation to a privately informed principal in a bargaining problem. While there is a large and growing literature that studies delegation under complete information, we are aware of very few papers that focus on delegation in private information settings. Consider the following four examples.

Faced with mounting liabilities from asbestos litigation Lloyd's of London formed Equitas Ltd. as a reinsurance company in September 1996.⁵ Lloyd's capitalized Equitas with £11.2 billion in return for which Equitas reinsured all of the syndicate's pre-1993 asbestos liabilities. The contract between Lloyd's and Equitas further authorized Equitas to negotiate and settle any asbestos claims as Lloyd's agent (Hall, 2002). According to Standard and Poor's (2003), if Equitas were to become insolvent, Lloyd's would not be liable for any unmet liabilities. However, the contract creating Equitas also includes a formula for returning funds to Lloyd's after all claims have been settled and provisions for increasing Lloyd's payments to Equitas (American Names Association, 2000). The motivation for this type of delegation was to address two problems: A number of court cases had weakened Lloyd's legal bargaining position in settling claims and the increased liability made it increasingly difficult to attract new capital to the syndicate.

Automobile manufacturers do not sell cars directly to consumers but instead work through dealerships. A dealership buys each car at what is referred to as the invoice price and then bargains with customers over a final sale price. The price paid by a customer can be below the dealership's invoice price as manufacturers offer dealerships holdback agreements which amount to a non-linear quantity bonus. If a dealership becomes bankrupt, the manufacturer bears no liability beyond the warranty.

⁴E.g., Baron and Besanko (1992, 1999), Melamud, Mookherjee, and Reichelstein (1992, 1995, 1997), Baliga and Sjoström (1998), Laffont and Martimort (1998), Severinov (1999), and Mookherjee and Tsumagari (2001).

⁵We thank Mort Kamien for bringing this example to our attention.

Large retailers like WalMart, purchase over 1500 different categories of products. Much of its purchases are done at the regional level using a practice called category management. A manager is assigned to each product category in the store. This manager is responsible for negotiating with outside suppliers and is compensated on the basis of category profits. Part of the category's costs include shelf space costs as well as overhead charges that are paid to the retailer.⁶ Rather than giving the category manager discretion over how to use information such as the opportunity cost of shelf space, some retailers will "tie the hands" of a category manager by fixing its shelf space charge even though the manager's compensation includes other (possibly non-linear) components.

Finally, consider the problem of regulating a monopolist that is a subsidiary of a foreign-owned multinational (e.g., Gresik and Nelson (1994)). The subsidiary operates as a separate economic unit but is required to conduct business, such as trade in intermediate goods, with its parent on terms (i.e. transfer prices) usually set by the parent. In deciding how much to produce, the subsidiary is influenced by the taxes and regulations of its host government but also by the financial incentives provided by the parent. These financial incentives are determined by a combination of the transfer price set by the parent and by (non-linear) dividend and profit repatriation plans.

All four examples have several features in common. First, they all involve a bargaining problem between a well-informed party and a poorly-informed party. Lloyd's has private information about its aggregate insurance exposure and its available capital. Most insureds do not. Manufacturers and retailers have private information about production costs of intermediate and final goods and about opportunity costs of shelf space. Second, in delegating bargaining authority to an agent, all four examples involve a public and verifiable, payoff-relevant message from the informed party to its agent: the initial Equitas capitalization, the invoice price of the car, the shelf space charges, and the transfer price. These messages are public in the sense that they are observable by the other principal bargaining party (the insureds, the automobile customers, and the host country).⁷ Unlike the sticker price or the manufacturer's suggested retail price, which may be based on factors unrelated to agent compensation, the above "messages" generate an actual transfer of resources between the agent and its principal and hence are clearly and directly payoff relevant. This is not to say that these transfers cannot be offset through other transactions. They can as will be discussed in the next point. Nonetheless, the examples all include real transfers

⁶See Blattberg and Fox (1995-6) for details on how category management is being implemented.

⁷While less common in Europe, the public nature of the dealer's invoice price is very common in the United States.

between the informed party and its agent that are public and verifiable. Third, there exists a second non-linear component to the contract between the informed party and its agent: the additional capitalization and refund provisions of the Equitas contract, holdback agreements, profit-sharing compensation for the category manager, and the dividend/profit repatriation policies. This part of the contract is generally not observable to the uninformed party. It can be used to both supplement or offset the observable transfer. Fourth, the agent faces an individual rationality constraint independent of its principal.

An intriguing feature of the principal-agent contract in all the examples is the combination of a public component and a less-public non-linear component. What is the significance of the initial capitalization of Equitas if Lloyd's is prepared to provide additional financing? Why compensate car dealers with a combination of a piece rate (invoice price) and a nonlinear holdback? Why charge category managers a linear rate for shelf space and offer a non-linear profit sharing contract? After all, a well-designed non-linear contract can always incorporate the economic incentives of both the linear and non-linear compensation.⁸ In this paper, we will argue that this type of delegation contract, combining a public compensation component with a supplemental non-linear contract, allows for competition via non-linear contracts by the informed party *and the uninformed party* that has very strong efficiency and surplus extraction properties.

In a complete information setting, the significance of a commitment effect has been well-established in papers by Schelling (1956), Fershtman and Judd (1987), and Katz (1991) as well as many others.⁹ All of these papers demonstrate the potential benefit arising from the separation of ownership and control. We extend this literature by endowing a principal in a bargaining problem with private information. In so doing we identify a second important aspect of delegation: the separation of control of economic decision-making from control of private information. We refer to the practice of delegating decision-making control while retaining control of private information as “partial delegation” and we show that partial delegation allows a privately informed principal to trade a first-best quantity and to extract all of the surplus minus a constant from a bargaining game.

⁸The transfer price example is more complex because, in addition to the piece rate/non-linear aspects of multinational finance, a transfer price shifts pre-tax dollars within the multinational while dividend payments shift after-tax dollars.

⁹More recently, Koçkeson and Ok (2004) apply forward induction arguments to complete information bargaining games and show that delegation by a weak bargaining party can yield strategic benefits.

Consider the case of a firm with private valuation information bargaining with a customer over the price and quantity of a good.¹⁰ If the firm and its customer bargain directly over the quantity to be transacted, the bargaining outcome may be second-best due to a misalignment of private information and bargaining power. One well-known misalignment leading to inefficient quantities occurs when the customer can make a take-it-or-leave-it offer to the firm. In an effort to limit the information rents earned by the firm, the customer will find it profitable to use its offer to induce transaction quantities below first-best levels.¹¹

Suppose however the informed seller contracts with an agent to bargain on its behalf. The seller has essentially two options concerning the control of its private information. The first option involves the seller whispering a, possibly incorrect, private information report to the agent. Formally, this communication amounts to the firm offering its agent a set of non-linear contracts, one non-linear contract for each possible report. The agent in turn is responsible for accepting or rejecting any customer offers. The customer remains in a position to make a take-it-or-leave-it offer which is equivalent to presenting the agent a single non-linear sale contract. We refer to this option as "full delegation." From the customer's perspective, the agent is privately informed. Therefore, the customer still faces an incentive to limit the information rents it pays the agent as the agent controls how the information conveyed by the selling firm influences its acceptance/rejection decision. Not surprisingly, the details of the actual contract between the selling firm and its agent constrain how successfully the customer can limit the agent's information rents. The selling firm's contract creates a classic commitment effect augmented to account for the role of private information. We show in section 4 that with private information, multiple equilibria with distinct quantity schedules exist making the equilibrium effect of commitment indeterminate under full delegation. To argue that full delegation is beneficial for the firm relative to direct bargaining requires invoking some sort of equilibrium selection criteria as in Koçkeson and Ok (2004). Rather than following this possibility, we consider a second option.

With the second option, the seller offers the agent a set of non-linear contracts. The specific contract the seller uses to compensate the agent is based on a public, payoff-relevant type announcement made before the agent and the customer bargain. Because the type report is payoff relevant, the

¹⁰In this regard, the information structure of our bargaining game is the same as in Ausubel and Deneckere (1989).

¹¹What is important for our results is the mismatch between exogenous bargaining power and the distribution of private information. The privately informed party can either be the buyer or the seller.

bargaining game between the agent and the customer is now one of complete but imperfect information because the customer can now condition its offer on the public report. Formally, this is equivalent to the customer also offering the agent a set of non-linear contracts. At a best-response level, for each possible public report, both the customer and the firm use their offers/contracts to the agent to offset the pecuniary externality created by the other's contract/offer and give the agent zero profit in any agent-customer subgame. Only the first-best quantity schedule satisfies the best-response conditions of the seller, its agent, and the customer. Furthermore, the seller collects all the surplus up to a constant with the remainder accruing to the customer. This gives all but the worst type of the informed seller a strict incentive to delegate in this manner. Since the agent in our model does not possess better information than its parent firm and since we do not allow the act of delegation to weaken the customer's ability to make take-it-or-leave-it offers, this result is due to the strategic value of partial delegation and not because of any contractual or informational incompleteness experienced by the informed party nor because of any strategic signalling associated with the act of delegation.

The nature of partial delegation equilibria raise two important questions. Can an informed party credibly commit to only use a public channel to communicate type information to an agent? What is the role of private information? The first question focuses on the critical difference between our full delegation and partial delegation settings. Partial delegation requires that the informed seller be able to commit to communicating private information to its agent only through a public channel. If private communication is possible, even with public communication, then we are back in a full delegation environment. We argue that because of the strong implementation feature of our partial delegation game, the informed seller has no strict incentive to engage in private communication. Any agent incentive the seller would want to create through private communication, can also be created through the non-linear component of its agent contract. By avoiding private type communication, the seller eliminates any equilibrium coordination issues present under full delegation.

The second question focuses on the role of private information. On the surface it appears that our main result does not rely on private information and is essentially a restatement of the ideas of Fershtman and Judd (1987). In fact, private information is essential for our results. Without private information, there is no information control issue. It is true that, in the complete information version of our bargaining game with delegation, among all differentiable equilibria, the quantity is unique and first-best. But this is also true for the complete information version of our direct bargaining game (i.e., between the firm and the customer). Our results emphasize the importance of separating the control of economic decision-making from the control of private information as a response to the distortions private information

creates. The extant wisdom about the impact of private information is that it introduces distortions in response to information rent effects. Our analysis shows that, under partial delegation, an informed party will earn higher information rents than it could under a direct bargaining scenario in which it has a weak bargaining position, and that it can do so without reliance on the very subtle strategic communications needed to resolve equilibrium coordination issues.

In the next section, we set up a simple model of principal to principal (seller to customer) bargaining to establish a baseline. In Section 3, we then permit the informed principal to partially delegate bargaining responsibilities to an agent and characterize the equilibria of the resulting common agency game. In this section, we consider three scenarios: partial delegation with observable contracts, partial delegation with unobservable contracts, and delegation by both the seller and the customer. The first scenario presents our main result in the simplest context. The latter two demonstrate the robustness of the first-best quantity result. In Section 4, we analyze a game with fully delegated bargaining. We offer concluding comments in Section 5.

2. A Model of Bargaining Between Two Principals.

To understand the role of delegation, we first consider the strategic interaction between an informed seller and an uninformed buyer negotiating the terms of trade for q units of a product in the context of a standard and well-known model. The seller has production costs $c(q, \theta)$. The parameter, θ , is the seller's private information. Throughout the paper, it will be common knowledge that θ is drawn from the distribution $F(\cdot)$ on $[\underline{\theta}, \bar{\theta}]$. The buyer's gross benefit from trade is $B(q)$.

Assumption 1. *a. $c(0, \theta) = 0$.*

b. $c(q, \theta)$ is convex and strictly increasing in q .

c. $c(\cdot, \theta)$ and $c_q(\cdot, \theta)$ are strictly increasing in θ .

d. $b(0) = 0$.

e. $b(q)$ is concave and strictly increasing in q .

f. $F(\theta)/f(\theta)$ is strictly increasing in θ .

Assumptions (1a)-(1f) are all standard. Requiring marginal production costs to be increasing in θ ensures that our model satisfies the usual single-crossing property. Assumption (1f) is a common technical assumption.

Let $w(q, \theta) \equiv b(q) - c(q, \theta)$ denote the total surplus from any transaction. Then the first best quantity, $q^F(\theta)$, is the solution to $w_q(q, \theta) = 0$. From Assumptions (1b) and (1e), total surplus is strictly concave and the first-best quantity is unique. Assumption (1c) implies that the first-best quantity and $w(\cdot, \theta)$ are strictly decreasing in θ .

For any Bayes-Nash equilibrium of any bargaining game, the resulting allocation can be described by a pair $(q(\theta), t(\theta))$ where $q(\theta)$ denotes the quantity traded and $t(\theta)$ is the amount the buyer pays the seller when the seller's type is θ . At this point it would be natural to invoke the Revelation Principle and derive the set of interim incentive efficient allocations, that is, those allocations that maximize an interim weighted social welfare function subject to incentive constraints (interim incentive compatible, interim individually rational, and *ex post* budget balancing). Alternatively, Rochet's (1985) Taxation Principle allows one to represent any equilibrium allocation by a non-linear price contract $T(q)$. It represents the payment the buyer makes to the seller in order to buy q units. The implied bargaining game is one in which the parties negotiate over $T(\cdot)$ and then the seller chooses q . For any direct revelation allocation $(q(\theta), t(\theta))$ there exists a non-linear contract such that in equilibrium the seller chooses $q(\theta)$ and $T(q(\theta)) = t(\theta)$. Thus, deriving the set of non-linear contracts that maximize an interim weighted social welfare function subject to the same incentive constraints is equivalent to the standard direct revelation approach. We present the non-linear contract approach for consistency with the analysis in forthcoming section where it will be necessary to focus on non-linear contracts.

For any contract $T(q)$, total profit for the buyer and for the seller is

$$B(q) = b(q) - T(q) \text{ and } s(q, \theta) = T(q) - c(q, \theta). \quad (1)$$

Assuming that $T(q)$ is differentiable, any strictly positive profit-maximizing quantity for the seller, $q(\theta)$, will satisfy

$$T'(q(\theta)) = c_q(q(\theta), \theta) \text{ and } T''(q(\theta)) \leq c_{qq}(q(\theta), \theta). \quad (2)$$

Denote the seller profit generated at its optimal quantity choice by $S(\theta) \equiv s(q(\theta), \theta)$. Differentiation of $S(\theta)$ yields the incentive compatibility conditions that $q(\theta)$ must satisfy:

$$dS(\theta)/d\theta = -c_\theta(q(\theta), \theta) = w_\theta(q(\theta), \theta) < 0 \quad (3)$$

and

$$q'(\theta) \leq 0. \quad (4)$$

By integrating (3), seller profit must equal

$$S(\theta) = S(\bar{\theta}) - \int_{\theta}^{\bar{\theta}} w_{\theta}(q(t), t) dt. \quad (5)$$

In order for the buyer and the seller to be willing to participate in a bargaining game, it must yield a non-negative payoff to each party. For the seller, it follows from (5) that participation is guaranteed for all values of the private information if $S(\bar{\theta}) \geq 0$. Similarly, participation by the supplier yields a non-negative expected return if

$$\mathcal{E}_{\theta}[w(q(\theta), \theta) - S(\theta)] \geq 0. \quad (6)$$

Incentive efficiency, as defined by Holmström and Myerson (1983), describes the set of quantity-money allocations $(q(\theta), T(q(\theta)))$ for which there exists no other incentive compatible allocations $(q^1(\theta), T^1(q^1(\theta)))$ that yields higher expected profit for the buyer without giving any seller type lower profit or higher profit for any seller type without giving the buyer lower expected profit. Myerson (1985) and Wilson (1985) show that this incentive constrained Pareto frontier can be described by maximizing the weighted welfare function

$$\hat{W} = \int_{\underline{\theta}}^{\bar{\theta}} [\alpha(\theta)S(\theta) + \gamma B(q(\theta))] f(\theta) d\theta \quad (7)$$

subject to incentive compatibility constraints, (3) and (4), and participation constraints, (6) and $S(\bar{\theta}) \geq 0$. The non-negative weights, $\alpha(\cdot)$ and γ , are normalized so that

$$\gamma + \int_{\underline{\theta}}^{\bar{\theta}} \alpha(\theta) f(\theta) d\theta = 1. \quad (8)$$

Intuitively, these welfare weights reflect the relative bargaining power of the buyer and seller. We provide a complete characterization of all incentive efficient allocations with strictly decreasing quantity schedules in the Appendix in conjunction with our analysis in section 4. However, to establish the main point of this paper, it is sufficient to focus on two specific welfare weight distributions and hence on two specific constrained Pareto efficient allocations: (i) $\gamma = 1$ and $\alpha(\cdot) \equiv 0$ and (ii) $\gamma = 0$ and $\alpha(\cdot) \equiv 1$. We describe the associated allocations in the following proposition.

Proposition 1. *If $\gamma = 1$ and $\alpha(\cdot) \equiv 0$, the incentive efficient allocation implies $S(\bar{\theta}) = 0$ and $w_q(q(\theta), \theta) = (F(\theta)/f(\theta))w_{q\theta}(q(\theta), \theta)$. If $\gamma = 0$ and $\alpha(\cdot) \equiv 1$, the incentive efficient allocation implies*

$$S(\bar{\theta}) = w(q^F(\bar{\theta}), \bar{\theta}) \text{ and } q(\bar{\theta}) = q^F(\bar{\theta}).$$

The first result in Proposition 1 is the solution to the standard principal-agent type problem in which the informed party with the highest possible type earns zero profit and all other types earn positive profit. Only the lowest type trades a first-best quantity. All other types trade an inefficiently low quantity. This allocation is the unique equilibrium allocation that would arise in a game in which the buyer has the power to credibly make a take-it-or-leave-it offer to the seller. For future reference, we denote this quantity schedule by $q^0(\theta)$. The properties of this allocation are robust in the sense that $S(\bar{\theta}) = 0$ and $q(\theta) < q^F(\theta)$ for all $\theta > \underline{\theta}$ as long as the average seller weight is less than the buyer weight, i.e., $\gamma > 1/2$, and low seller type profit is weighted more than high seller type profit, i.e., $\alpha'(\cdot) < 0$. The important feature driving this result is the fact that the privately informed party is in the weaker bargaining position due to exogenous factors. The seller's bargaining power is weakest when $\gamma = 1$.

The second result in Proposition 1 is included to remind the reader that first-best quantities are incentive efficient for some welfare weights. The extreme weights in Proposition 1 correspond to a bargaining game in which the informed seller has the ability to make a take-it-or-leave-it offer to the buyer. Since the seller can always extract all of the surplus generated by trade, it is incentive compatible for the seller to offer a contract that induces first-best quantities.¹² This result is also robust to variations in the welfare weights such that $\alpha(\theta)$ is a constant greater than $1/2$ (and $\gamma < 1/2$.) For $\alpha(\theta) < 1$, $S(\bar{\theta})$ is less than $w(q^F(\bar{\theta}), \bar{\theta})$ as the seller now earns all of the surplus minus the buyer's constant share. This result is indicative of a bargaining outcome when the ownership of private information and exogenous bargaining power are aligned.

3. Partial Delegation.

We now allow the seller to hire an agent to bargain on its behalf. The seller must make two delegation decisions. The first delegation decision is the standard one which gives the agent authority to negotiate a price and a quantity. The second delegation decision has to do with how the seller shares its private information with its agent. In this section, we consider a partial delegation scenario in which the seller makes a public type report to both the agent and the buyer. We use the term "partial delegation" because the seller delegates the responsibility for deciding how much to purchase to its agent but retains control of how information is released to the buyer. In the next section we consider the "full delegation" case in which the seller communicates type information to the agent privately.

¹²See Spulber (1988) and Ausubel and Deneckere (1989) for more details.

Previous efforts to model delegation with a privately informed principal include Katz (1991) and Caillaud and Hermalin (1993). Katz offers an example that suggests private information can provide a motive for delegation. In his paper, the agent is uninformed when offered a contract by the informed principal but the true value of this private information is truthfully revealed to the agent and the outside principal prior to any output decisions. In contrast, the seller in our model retains the ability to distort its information report to both the agent and the buyer. Caillaud and Hermalin examine the benefits of employing an agent in a signalling model. In their paper, as soon as an agent is hired, the informed principal becomes uninformed and the agent now controls the private information. This creates countervailing incentives due to a tension between the screening motives of the delegating principal to learn its agent's type and its signalling motives. Caillaud and Hermalin show that these countervailing incentives can result in a higher equilibrium payoff to the informed principal. Again, in contrast, the informed seller in our model never becomes uninformed.

We study this idea of partial delegation by deriving the subgame perfect equilibria of the following two-stage game. In the first stage, the seller hires an agent. The agent receives a contract of the form $T^s(q, \kappa)$ where κ denotes a public type report, also made in stage one. It represents a payment from the agent to the seller. For example, κ could be the invoice price of a car and any non-linear component to T^s could correspond to a hold-back. In the second-stage, the agent and the buyer negotiate by having the buyer make the agent a take-it-or-leave-it offer which the agent either accepts or rejects. This last feature maintains the distribution of bargaining power from Proposition 1 that resulted in a second-best level of trade. Because κ is a public report, the buyer can condition its offer on κ which means we can represent the equilibrium in this second stage by a collection of non-linear contracts (indexed by κ), $T^b(q, \kappa)$ and a quantity choice by the agent. With these contracts, the agent's payoff for any q is

$$a(q, \kappa) = T^b(q, \kappa) - T^s(q, \kappa). \quad (9)$$

Denote the agent's quantity choice by $q^a(\kappa)$. It must satisfy

$$T_q^b(q^a(\kappa), \kappa) = T_q^s(q^a(\kappa), \kappa) \text{ and } T_{qq}^b(q^a(\kappa), \kappa) \leq T_{qq}^s(q^a(\kappa), \kappa) \quad (10)$$

as long as $a(q^a(\kappa), \kappa) \geq 0$. The seller's profit now equals

$$s(q, \kappa, \theta) = T^s(q, \kappa) - c(q, \theta), \quad (11)$$

and the buyer's profit now equals

$$B(q, \kappa) = b(q) - T^b(q, \kappa). \quad (12)$$

In the next two subsections, we will consider the possibility that T^s is observable to the buyer and the possibility it is not observable to the buyer.

3.1. Fully observable seller contract.

Suppose T^s is observable to the buyer. Remember that, in the second-stage bargaining game, the buyer has the power to make the agent a take-it-or-leave-it offer. Given (9), the buyer's best-response to any T^s will result in an agent payoff of zero. Using (12), this implies that the buyer will seek to maximize

$$b(q) - T^s(q, \kappa) \quad (13)$$

subject to (13) being non-negative. Let $q^b(\kappa)$ denote the solution to the buyer's problem for each κ . Thus, the buyer's best response will be a non-linear contract that implies $q^a(\kappa) = q^b(\kappa)$ and zero agent profit.

Now consider the seller's first-stage choices. From (11) - (13), the seller's problem is to choose q^a and κ to maximize

$$b(q^a) - c(q^a, \theta) - B(q^a, \kappa) \text{ subject to } B(q^a, \kappa) \geq 0. \quad (14)$$

If $\kappa(\theta)$ denotes the seller's equilibrium reporting strategy, then the solution to (14) must imply $B(q^a(\kappa(\theta)), \kappa(\theta)) = 0$ and $q^a(\kappa(\theta)) = q^F(\theta)$. Since $q^F(\theta)$ is strictly decreasing in θ , the latter condition can be satisfied only if $\kappa(\theta)$ is strictly monotone. Moreover, there will be multiple subgame perfect equilibria because this analysis determines only $q^a \circ \kappa$ and not $q^a(\cdot)$ and $\kappa(\cdot)$ separately.¹³

The last step in our analysis is to construct non-linear contracts that support these equilibrium outcomes. The construction is not unique which implies an additional source of multiple equilibria. Assume the seller's contract is of the form

$$T^s(q, \kappa) = \kappa q + b(q) + \int_{t=q}^{q^F(\bar{c})} q^{F^{-1}}(t) dt + \mu(\kappa), \quad (15)$$

and assume the buyer's contract is of the form

$$T^b(q, \kappa) = b(q) - \phi. \quad (16)$$

A seller contract like (15) makes (13) strictly concave in q for all κ and implies $q^b(\kappa) = q^F(\kappa)$ as long as buyer profit is non-negative at this quantity. This can be guaranteed by setting

$$\mu(\kappa) = -\kappa q^F(\kappa) - \int_{q^F(\kappa)}^{q^F(\bar{c})} q^{F^{-1}}(t) dt. \quad (17)$$

¹³The fact that the equilibrium form of $\kappa(\cdot)$ is indeterminate does not mean the report is cheap talk. It simply means there are many ways to index contracts. Given the choice of an index "language", the choice of a specific value of κ commits the seller to a specific agent contract which is payoff relevant to the seller and to the agent.

Therefore, (16) with $\phi=0$ is a best-response for the buyer. Eqs. (15) and (16) also imply that agent profit is strictly concave and maximized at $q^a(\kappa)=q^F(\kappa)$. Proposition 2 summarizes this analysis.

Proposition 2. *If the seller's type report and its contract to its agent are observable by the buyer, then all subgame perfect equilibria result in first-best trade and full surplus extraction by the seller.*

Proposition 2 presents the main result of this paper in perhaps the simplest and starkest manner. Hiring an agent creates a commitment effect that can offset any exogenous weakness in one's bargaining position *even in the presence of private information*. This idea is well-known in complete information settings and goes back to the work of Schelling (1956) and Fershtman and Judd (1987). In light of the private information, Proposition 2 is surprising because the requirement that the seller's type report be public does not imply that the report must be truthful. Given the huge literature documenting rent extraction distortions due to the existence of private information, it certainly would seem reasonable to expect that the uninformed buyer would seek to reduce the information rents it would indirectly have to pay the seller by reducing the quantity it purchases as a function of the public report. As noted in the introduction, the reason first-best quantities arise in equilibrium is because the public report, truthful or not, eliminates any rent extraction motive for the agent because in equilibrium the buyer can use the report to leave the agent with zero surplus. Proposition 2 suggests that the key to securing the strong complete-information commitment properties of delegation in a private information environment is to have an informed principal retain control of its private information while delegating the quantity choice to an agent.

3.2. Unobservable seller contract.

The insights from Katz (1991) motivate one to investigate what aspects of Proposition 2 are driven by the observability of the seller's contract. Assuming the seller's contract is unobservable to the buyer is formally equivalent to assuming the buyer and seller simultaneously announce non-linear contracts to the agent. Unlike in section 2 where we had the option of restricting attention to direct mechanisms or to non-linear contracts, it is now imperative that we focus on competition in non-linear contracts. Simultaneous contracting implies we are studying a class of common agency games. From a normative perspective, the works of Peters (2002) and Martimort and Stole (2003) show that restricting attention to competition in non-linear contracts involves no loss of generality because of the quasi-linear preferences of the principals and the agents. The same cannot be said about competition in direct mechanisms. Thus, to ensure that our results are not driven by ad hoc contract space restrictions, it is now important that we allow the buyer and the seller to compete in non-linear contracts.

We derive the Bayes-Nash equilibria of the following game. In the first stage, the seller offers the agent a non-linear contract, $T^s(q, \kappa)$, and makes a public type report, κ . Simultaneously, the buyer offers the agent a non-linear contract, $T^b(q, \kappa)$. $T^b(q, \kappa)$ is not a single contract whose specification requires the buyer to correctly anticipate the seller's report but rather it is a set of contracts that are contingent on the seller's report. By admitting these types of contracts, the buyer is effectively able to make "invoice-plus" offers. In the second stage, the agent accepts or rejects both contracts and chooses q . We focus on equilibria of this game for which the contracts are differentiable and we will refer to these equilibria as "differentiable partial delegation" equilibria.¹⁴

With unobservable contracts, there will always exist no-production equilibria in which each principal's contract guarantees that, no matter what quantity the agent chooses, the other principal will earn negative profit. This can be true for all θ or for some θ . For such θ , the buyer and the seller each have the incentive to make sure the agent does not trade. Consistent with the rest of the common agency literature, we ignore these equilibria and focus only on positive production equilibria for all θ .

First, consider the buyer's best response to any seller strategy $(T^s(q, \kappa), \kappa(\theta))$. The analysis is identical to the analysis of the buyer's problem in the previous section. As long as the buyer's best response implies a positive, finite quantity, (13) will imply zero agent profit and

$$b'(q^a(\kappa)) = T_q^s(q^a(\kappa), \kappa). \quad (18)$$

Second, consider the seller's best response to any buyer strategy, $T^b(q, \kappa)$. Agent profit will be zero under any best response that implies $q^a > 0$. Thus, for $q^a > 0$, the seller's payoff equals

$$s(q^a, \kappa, \theta) = T^b(q^a, \kappa) - c(q^a, \theta). \quad (19)$$

From (19), necessary conditions for the seller's best response to imply a positive, finite quantity are

$$T_q^b(q^a, \kappa(\theta)) = c_q(q^a, \theta), \quad (20)$$

$\kappa(\theta)$ strictly monotone, and

¹⁴When both principals are fully informed, this differentiability condition will imply that $q(\theta) = q^F(\theta)$ in any Nash equilibrium with an interior solution for the agent's problem. If we admit non-differentiable schedules, a continuum of equilibrium quantities will exist even under complete information. Such equilibria are typically ruled out in complete information analyses (as are partial or no participation equilibria) sometimes by appealing to Bernheim and Whinston's (1986) notion of "truthful" equilibria. For consistency, we maintain these exclusions in both the complete and incomplete information versions of this game. However, our restriction to differentiable, full-participation equilibria is actually weaker than the Bernheim and Whinston restriction.

$$T_{\kappa}^b(q^a, \kappa(\theta)) = 0 \quad (21)$$

for all θ . Together (10), (18), and (20) imply $b'(q^a) = c_q(q^a, \theta)$.¹⁵ Thus, the only possible equilibrium quantity is $q^F(\theta)$.

To complete our derivation of equilibria, we need to identify non-linear contracts that satisfy (21) and the second-order conditions for the agent, buyer, and seller problems. The contracts in (15) and (16) also work in this context.¹⁶ The one key difference with unobservable contracts is that ϕ , the buyer's equilibrium profit, can be strictly positive in equilibrium. This means that equilibrium payoffs are no longer unique when contracts are unobservable. In fact, if one defines the seller's equilibrium profit as $\Sigma(\theta) = s(q^a(\kappa(\theta)), \kappa(\theta), \theta)$, then by the envelope theorem, $\Sigma'(\theta) = s_{\theta}(q^F(\theta), \kappa(\theta), \theta) = w_{\theta}(q^F(\theta), \theta)$. Similarly, $dw(q^F(\theta), \theta)/d\theta = w_{\theta}(q^F(\theta), \theta)$. Thus, the buyer's equilibrium profit, $w(q^F(\theta), \theta) - \Sigma(\theta)$, must equal a constant.

Proposition 3. *If the buyer can observe the seller's type report but not the contract the seller offers its agent, then all positive production, differentiable equilibria result in first-best trade. The seller earns all of the surplus up to a constant.*

The multiple equilibria identified in Proposition 3 all result in the same level of trade but do differ in the amount of buyer and seller profit. When the seller's contract was observable, this second source of multiplicity was not present and the seller clearly benefitted from that form of partial delegation. Is the same true with unobservable contracts? The answer is "Almost." The difference in the seller's equilibrium profit with and without delegation when the buyer has the power to make take-it-or-leave-it offers is $\Sigma(\theta) - S(\theta)$ or

$$\Sigma(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} [w_{\theta}(q^0(t), t) - w_{\theta}(q^F(t), t)] dt. \quad (22)$$

For $\theta < \bar{\theta}$, the integral is strictly positive and seller individual rationality requires that $\Sigma(\bar{\theta})$ be non-negative. Thus, (22) is strictly positive for all $\theta < \bar{\theta}$.

Proposition 4. *Suppose the buyer has the power to make take-it-or-leave-it offers when bargaining*

¹⁵Also note that the buyer and seller contracts are "locally truthful" in the sense of Bernheim and Whinston (1986), because the marginal payments to and from the agent reflect the buyer's marginal benefit and the seller's marginal cost.

¹⁶Other non-linear contracts will work as well. We use the form of the contracts in (15) and (16) because they parallel the structure of the contracts observed in the automobile and retailing industries.

directly with the seller and when bargaining with a seller agent. For all seller types less than $\bar{\theta}$, all positive-production, differentiable equilibria of the partial delegation result in higher equilibrium seller profit than direct bargaining even if the seller's contract to its agent is not observable.

It is important to recognize that partial delegation does not expand the incentive constrained Pareto frontier. It does not constitute a Pareto improvement over direct seller-to-buyer bargaining. Rather, because Proposition 1 (following on the work in Spulber (1998)) establishes welfare weights for which first-best quantities are incentive efficient, the correct interpretation of Proposition 2 and 3 is one of implementation. When direct bargaining puts an informed trader in a weak bargaining position, partial delegation allows an informed principal to implement a preferred incentive efficient allocation.

3.3. Delegation by the seller and the buyer.

In this subsection, we allow the buyer to also hire an agent. The analysis helps us address whether it is the act of delegation itself that confers advantage on the seller or it is the separation of control of decision-making from the control of information.

Consider a two-stage game in which the seller and the buyer simultaneously hire their own agents and offer their agent a non-linear contract. This gives both the buyer and the seller the opportunity to create a commitment effect. The seller, still the only privately informed party, also announces κ . In the second-stage, the two agents bargain over quantity and price with the buyer's agent maintaining all the bargaining power. Profit for the buyer agent is defined by

$$a^b(q, \kappa) = T^b(q, \kappa) - T(q, \kappa)$$

and profit for the seller agent is

$$a^s(q, \kappa) = T(q, \kappa) - T^s(q, \kappa)$$

where $T^b(\cdot, \cdot)$ and $T^s(\cdot, \cdot)$ are the buyer and seller contracts to their respective agents and $T(\cdot, \cdot)$ is the payment from the buyer agent to the seller agent. We maintain the convention that the seller-agent's contract is a payment to the seller and we adopt the convention that the buyer-agent's contract is a payment from the buyer. Profit for the buyer and the seller are still defined as in (11) and (12).

Given $T(\cdot, \cdot)$ and $T^s(\cdot, \cdot)$, the seller agent will choose $q^a(\kappa)$ such that $T_q(q^a, \kappa) = T_q^s(q^a, \kappa)$. With the buyer's agent having all the bargaining power, any second-stage equilibrium must imply $a^s(q^a, \kappa) = 0$. This means the buyer agent will choose $T(\cdot, \cdot)$ to maximize $T^b - T^s$ by inducing the seller agent to choose $q^a(\cdot)$ such that $T_q^b(q^a, \kappa) = T_q^s(q^a, \kappa)$. In the first stage, the seller's best response will imply $T^b(q^a, \kappa) = T(q^a, \kappa)$. The seller will choose $\kappa(\cdot)$ and $T^s(\cdot, \cdot)$ to maximize $T^b - c$. Thus, the seller's best response must satisfy $T_q^b(q^a, \kappa) = c_q(q^a, \theta)$ as in (20). Similarly, the buyer's best response will imply $T^b(q^a, \kappa) = T^s(q^a, \kappa)$ and $T_q^s(q^a, \kappa) = b'(q^a)$ as in (18). Combining these last three first-order conditions

again requires in equilibrium that $q^a(\kappa(\theta)) = q^F(\theta)$. The equilibrium conditions on the contracts $T^b(\cdot, \cdot)$ and $T^s(\cdot, \cdot)$, especially the seller incentive compatibility condition $\Sigma'(\theta) = w_\theta(q^F(\theta), \theta)$, are identical to those in Section 3.2. This means that Proposition 3 applies to this setting as well.

If the buyer can hire an agent first, then all the analysis in the previous paragraph remains the same except for the buyer's best-response. The buyer's problem implied by solving the agent-to-agent bargaining stage and by the seller's best-response is identical to the buyer's problem under direct bargaining from section 2. Thus, a necessary condition for delegation to result in a first-best quantity is the ability of the seller to delegate simultaneously with the buyer or before the buyer. It is not sufficient. The next section will show that full delegation supports many equilibrium quantities and hence does not facilitate first-best trade.

4. Full Delegation.

A final way to emphasize the importance of separating control of the output-price decisions from control of private information, is to now consider a three-stage game in which the seller delegates both responsibilities to its agent. In the first-stage the seller offers the agent a contract, $T^s(q, \kappa)$, and gives the agent a private type report, κ .¹⁷ In the second stage, the buyer makes the agent a take-it-or-leave-it offer, $T^b(q)$, without observing the seller's contract or the seller's type report. In the third-stage, the agent accepts or rejects the buyer's offer. If the offer is accepted, the agent chooses a quantity. As noted earlier, this formulation of the second-stage is equivalent to the buyer offering the agent an incentive feasible direct contract that depends on a type report from the agent that maximizes the buyer's expected profit. By focusing on the contract that maximizes the buyer's expected profit, we keep the buyer's exogenous bargaining power the same as in the direct bargaining and partial delegation games. We again restrict attention to differentiable contracts and solve for subgame perfect Nash equilibria. We will refer to such equilibria as "full delegation" equilibria. We will show that this set of equilibria is quite large. Thus, in comparison with the last section, we find that the importance of the public report lies in its ability to induce a unique equilibrium quantity.

In the third stage, the agent chooses q to maximize

$$a^f(q, \kappa) \equiv T^b(q) - T^s(q, \kappa) \tag{23}$$

subject to the agent being able to earn non-negative profit. The "f" superscript is used to distinguish

¹⁷Since the buyer does not observe the contract T^s or the report κ , the buyer will have to best-respond to a menu of contracts indexed by possible type reports. This is not a menu in the usual sense of the term as the agent is not choosing from this menu.

agent profit in this full delegation game from its profit in the partial delegation game. Let

$q^f(\kappa) = \text{argmax}_q a^f(q, \kappa)$ subject to $a^f(q, \kappa) \geq 0$ and let $A(\kappa) \equiv a^f(q^f(\kappa), \kappa)$. With differentiable contracts, a strictly positive value of $q^f(\kappa)$ will satisfy

$$T_q^b(q) = T_q^s(q, \kappa) \text{ and } T_{qq}^b(q) \leq T_{qq}^s(q, \kappa). \quad (24)$$

Differentiation of the agent's payoff, $A(\kappa)$, yields incentive compatibility conditions

$$dA(\kappa)/d\kappa = -T_{\kappa}^s(q^f(\kappa), \kappa) \text{ and } T_{q\kappa}^s(q^f(\kappa), \kappa)q^{f'}(\kappa) \leq 0. \quad (25)$$

From (25), we can see that the seller's contract affects the agent's incentive compatibility conditions in two ways. The first is through the relationship between the agent's type and its net return. For instance, low-cost types must earn lower profits than high-cost types if the seller chooses to extract significantly more rents from low-cost types (i.e., $T_{\kappa}^s(q^f(\kappa), \kappa) < 0$). The second effect is through the marginal valuation of q . If $T_{q\kappa}^s > 0$, then a high cost seller charges the agent a higher marginal price and $q^{f'}(\kappa) \leq 0$ is required for implementation. However, if $T_{q\kappa}^s < 0$ the quantity schedule must be non-decreasing.

Since the seller's contract is not observable by the buyer, the first and second stages are formally simultaneous. Thus, we need to look for a Bayes-Nash equilibrium in $(T^s(q, \kappa), \kappa(\theta))$ and $T^b(q)$ given (25). We begin with the seller's best-response problem. As in (19), the seller's profit is

$$s(q^f(\kappa), \kappa, \theta) = T^s(q^f(\kappa), \kappa) - c(q^f(\kappa), \theta). \quad (26)$$

Given $T^b(q)$, the seller's problem is to choose $T^s(q, \kappa)$ and $\kappa(\theta)$ to induce a schedule $q^s(\theta) = q^f(\kappa(\theta))$ that maximizes (26) subject to $T_{q\kappa}^s(q^s(\theta), \kappa(\theta))q^{f'}(\kappa(\theta)) \leq 0$ and $A(\kappa(\theta)) \geq 0$. The seller will not need to leave the agent with any information rent because the seller sets the agent's type, κ . $A(\kappa) \equiv 0$ means the seller's problem is equivalent to maximizing $T^b(q^s(\theta)) - c(q^s(\theta), \theta)$ subject to $q^s(\theta) = q^f(\kappa(\theta))$ and $T_{q\kappa}^s(q^s(\theta), \kappa(\theta))q^{f'}(\kappa(\theta)) \leq 0$. We show in the appendix (Lemma A1) that the second constraint is always satisfied due to the agent's zero-profit condition. Consequently, at any strictly positive quantity, the seller's best-response must induce the agent to choose $q^s(\theta)$ for all θ such that

$$T_q^b(q^s(\theta)) = c_q(q^s(\theta), \theta) \quad (27)$$

and such that agent profit is zero. This can be accomplished by setting $T_q^s(q, \kappa)$ to solve (24) and using the κ -component of T^s to extract all the agent's profit. Eq. (27) differs from (20) in that the buyer's contract cannot depend directly on κ and so neither does (27). Since the seller's best-response quantity will in general depend non-trivially on θ , implementation via (24) requires a strictly monotone reporting strategy, $\kappa(\theta)$. These observations lead to the same source of payoff-equivalent multiple equilibria as in the last section. Denote the seller's indirect profit from any best-response as

$$\Sigma^f(\theta) = T^s(q^s(\theta), \kappa(\theta)) - c(q^s(\theta), \theta). \quad (28)$$

Based on our analysis of the seller's problem, we wish to focus on equilibria for which $\kappa(\theta)$ is strictly increasing. This restriction can only serve to limit the number of equilibria that we derive. Nonetheless, we will show that the set of equilibria satisfying this restriction is quite large. Once we focus on equilibria for which $\kappa(\theta)$ is strictly increasing, we can, without loss of generality take $\kappa(\theta) \equiv \theta$.

Turning to the buyer's problem, take as given $T^s(q, \kappa)$, $\kappa(\theta) \equiv \theta$, and $q^s(\theta) = q^*(\theta)$ such that $T_{q\kappa}^s(q^*(\theta), \theta)q'^*(\theta) \leq 0$. The buyer must choose $T^b(q)$ to induce $q^b(\theta) = q^f(\kappa(\theta))$ which maximizes $\mathcal{E}_\theta [b(q^b(\theta)) - T^b(q^b(\theta))]$ subject to (25) and $A(\theta) \geq 0$. Using (23) and (25), the buyer's objective is equivalent to $\mathcal{E}_\theta [b(q^b(\theta)) - T^s(q^b(\theta), \theta) - A(\theta)]$ where

$$A(\theta) = A(\bar{\theta}) + \int_{\bar{\theta}}^{\theta} T_{\kappa}^s(q^b(t), t) dt. \quad (29)$$

Eq. (29) highlights the key difference between partial delegation and full delegation. With full delegation the rent extraction incentives of the agent, as captured by $A(\theta)$, are linked with the contract offered by the seller. Thus, the seller cannot provide its agent with efficient output incentives that do not provide the agent with a rent extraction motive as well. With partial delegation, this link was absent.

In equilibrium, $q^b(\theta)$ must equal $q^*(\theta)$. Thus, we need to determine when $q^*(\theta)$ solves

$$\begin{aligned} \max_{q(\cdot)} \quad & \mathcal{E}_\theta [b(q(\theta)) - T^s(q(\theta), \theta) - A(\theta)] \\ \text{s.t.} \quad & a) A'(\theta) = -T_{\kappa}^s(q(\theta), \theta) \\ & b) A(\theta) \geq 0 \\ & c) T_{q\kappa}^s(q(\theta), \theta)q'(\theta) \leq 0. \end{aligned} \quad (30)$$

Necessary and sufficient conditions for a solution to (30) are derived in the Appendix. Informally, the buyer's best-response reflects the buyer's desire to limit the information rents earned by the agent offset by a commitment effect created by the seller's contract to the agent. In the absence of a seller contract, the buyer would seek to implement the bargaining solution from Proposition 1 when $\gamma=1$. Suitably constructed seller contracts make this solution infeasible due to negative agent profit. Thus, the commitment effect reduces the buyer's ability to lower the quantity traded in equilibrium.

Proposition 5 reveals that the commitment effect associated with unobservable contracts supports economically distinct multiple equilibria. The proof provided in the Appendix is constructive and uses

very simple contracts to support a wide range of equilibrium quantity schedules. In particular, to support the quantity schedule $q^*(\theta)$ contracts of the following form will suffice:

$$T^s(q, \kappa) = \Sigma^*(\kappa) + c(q, \kappa) \quad (31)$$

and

$$T^b(q) = \Sigma^*(\bar{\theta}) + \int_{t=q^{*-1}(q)}^{\bar{\theta}} c_{\theta}(q^*(t), t) dt + c(q, q^{*-1}(q)) \quad (32)$$

where $\Sigma^*(\kappa) = \Sigma^*(\bar{\theta}) + \int_{t=\kappa}^{\bar{\theta}} c_{\theta}(q^*(t), t) dt$. This construction ensures that $T_{\kappa}^s(q, \kappa) = 0$, which from (25) is

necessary for the agent to be driven to zero profits for all type reports.

Proposition 5. Let $q^*(\theta)$ be a strictly decreasing, differentiable function satisfying

$$w_q(q^*, \theta) = \frac{F(\theta) - \beta^*(\theta)}{f(\theta)} \quad (33)$$

where $\beta^*(\theta)$ is a non-decreasing function such that $0 \leq \beta^*(\theta) \leq 1$ and let

$$\Sigma^*(\bar{\theta}) \in \left[0, \int_{\theta=\bar{\theta}}^{\bar{\theta}} (w(q^*(\theta), \theta) f(\theta) + w_{\theta}(q^*(\theta), \theta) F(\theta)) d\theta \right] \quad (34)$$

Then (31), $\kappa(\theta) = \theta$, and (32) define a differentiable, full-delegation equilibrium in non-linear contracts in which $q^*(\theta)$ is traded.

Proposition 5 identifies a wide range of equilibria that are consistent with the full delegation equilibria. For a given quantity schedule $q^*(\theta)$ satisfying (33), equation (34) identifies a continuum of equilibria in which the payoff to sellers differ only by a constant. This is analogous to the multiplicity that arises in the partial delegation game when seller contracts are not observable. A second source of multiplicity is the large range of equilibrium quantity schedules. This type of multiplicity did not arise in the partial delegation game, where only the efficient quantity schedule was supportable as an equilibrium. These additional equilibrium quantity schedules arise because, under full delegation, the buyer cannot condition its quantity schedule on the agent's type. As a result, there will be equilibria in which the buyer offers a contract with $T_q^b(q) \neq b'(q)$ in order to extract information rents from the agent. The choice of $\Sigma^*(\kappa)$ in the seller's contract determines how large this wedge between $T_q^b(q)$ and $b'(q)$ can be in a given

equilibrium.

How large is the set of set of quantity schedules that can be supported as equilibria in the full delegation game? It should be noted that Proposition 5 provides only a partial characterization of equilibria because we have restricted attention to equilibria with contracts of the form defined by (31) and (32). However, our next result shows that the subset of differentiable equilibria is quite large even with this restriction. In particular, the set of equilibria includes all incentive efficient allocations with strictly decreasing quantity schedules associated with the direct bargaining problem in section 2. It also includes an open set of incentive inefficient allocations.

Proposition 6. *The set of full-delegation equilibrium allocations includes all incentive-efficient, direct-bargaining allocations with strictly decreasing quantity schedules.*

Proof. The set of strictly decreasing, incentive efficient quantity schedules, defined by (A.16) and (A.18), are described by functions of conditional welfare weights, $\hat{\alpha}(\theta) = \mathcal{E}_\theta[\alpha(t) | t \leq \theta]$ and a Lagrange multiplier (λ^* or λ^{**}). First, fix the welfare weights $\alpha(\theta)$ and γ such that $\gamma \geq 1/2$ and such that (A.16) and (A.17) define a strictly decreasing quantity schedule. This quantity schedule is a full-delegation equilibrium quantity for $\beta(c) = \hat{\alpha}(c)F(c)/(\gamma + \lambda^*)$. Second, fix the welfare weights $\alpha(\theta)$ and γ such that $\gamma < 1/2$ and such that (A.20) and (A.21) define a strictly decreasing quantity schedule. This schedule is a full-delegation equilibrium schedule for $\beta(c) = \hat{\alpha}(c)F(c)/(\hat{\alpha}(\bar{c}) + \lambda^{**})$. Simple inspection of both cases reveals that the $\beta(\cdot)$ functions are non-negative, non-decreasing, and always less than or equal to one.

Q.E.D.

A simple example in which the full-delegation equilibrium is not incentive efficient arises when $\beta(\theta) \equiv 1$. By (33), the quantity traded when $\theta = \underline{\theta}$ is greater than the first-best level. Yet the derivation of incentive efficient allocations in the Appendix shows that the quantity traded when $\theta = \underline{\theta}$ must be first best.

Therefore, the advantage of partial delegation over full delegation is that partial delegation permits an informed principal to refine the set of equilibrium outcomes in a way that ensures higher profit. Since private communication between the seller and its agent cannot, by its very nature, be observed by the buyer, it is also possible that a seller might wish to avail itself of private and public reports. In this case, a result similar to Proposition 6 would arise.¹⁸ However, the description of

¹⁸In an earlier version of this paper, we considered a partial equilibrium game in which the buyer and the seller simultaneously announce their contracts and then the seller reports θ to its agent and κ to

equilibria must now include buyer beliefs about the probability that each seller type makes a private report to its agent. Starting from a pure public reporting equilibrium, if a seller type can improve its utility through private communication, it can do so just as well by offering a different non-linear contract and not resorting to private communication. If such an opportunity existed, the initial strategy profile could not have been a pure public reporting equilibrium. As a result, there will exist an equilibrium of the game with the potential for public and private reports in which the buyer places zero probability on any seller type making private reports.

Finally, Laffont and Martimort (1998) point out that collusion between the agent and the buyer might undermine the seller's effort to increase its rents. This possibility is absent in our partial delegation and full delegation games. Any rents the agent might earn from a side contract with the buyer cannot be retained in equilibrium because the seller has full information. To be effective, incentive compatibility of the side-contract would require the buyer to pay the agent information rents the seller cannot anticipate in equilibrium.¹⁹ In a model in which the agent also has some private information, some scope for collusion may arise. We leave this question for future research.

5. Concluding Remarks.

As an alternative to recent papers that focus on the potential for delegation to mitigate the impact of collusion among privately-informed agents, this paper considers the potential for delegation by an informed party in influencing bargaining outcomes. We believe our paper offers several new results. Both our partial delegation and full delegation games extend the literature on delegated bargaining by incorporating the effects of persistent private principal information. With full delegation the commitment effect induced by the introduction of a delegate can, but need not, allow an informed seller to induce favorable rent shifting relative to the case of direct bargaining. With partial delegation, our analysis shows that an informed seller can guarantee higher profit except when it is of the worst type. Thus, with a privately informed principal in a bargaining problem, it is not simply the act of delegation that confers strategic advantage to an informed party. Rather it is delegation wherein the informed party retains

the buyer. Because the agent's profit depends on θ (as well as κ), the buyer must pay the agent an information rent. Even though this rent will be appropriated by the seller, it will introduce the same range of equilibrium quantity distortions as in Propositions 5 and 6.

¹⁹The way in which Laffont and Martimort (1998) introduce communication limits does not have any effect in our model. Thus, our argument is equivalent to their Theorem 1 asserting no equilibrium loss from side-contracting.

control of its private information that creates strong incentives for the buyer and the seller to trade the first-best quantity. To the best of our knowledge, this is the first paper to address the issue of how the control of private information influences the returns to delegation received by an informed principal. Moreover, our results show that the benefits of partial delegation to the informed party are robust to the observability of the delegation contracts and to delegation by the uninformed party as long as the uninformed party does not or cannot delegate first.

We believe our specific results are also related to several important strands of the literature. First, we believe our results add to our understanding of the economic implications of vertical integration, specifically Bork's Thesis (1978). Bork's Thesis is the term used by Bernheim and Whinston (1998) and Prat and Rustichini (2002) to describe Judge Bork's argument that vertical relationships, while viewed as anti-competitive, can facilitate efficient production. While Gal-Or (1991) and Martimort (1996) look at the role of vertical integration with a privately informed agent, our work focuses on an environment with a privately informed principal. Thus, our work can be interpreted as extending the support for Bork's Thesis as found in Bernheim and Whinston (1998) and Prat and Rustichini (2002) in a direction we do not believe has not yet been addressed. Again, in the context of this application, we find our results on partial delegation intriguing in light of the conventional wisdom that private information induces inefficiencies due to information rents. Since our model is not a direct extension of the work of Fershtman and Judd (1987), Katz (1991), and Fershtman and Kalai (1997), an interesting extension would be to apply this analysis to oligopoly models.

Second, our motivation for focusing on the structure and role of delegation is related to Spulber's (1999, 2002) arguments on the importance of paying attention to the microstructure of markets in which bilateral agreements arise. For example, he demonstrates how the separation intermediation creates between investment and pricing decisions in business-to-business transactions can ameliorate hold-up problems and leads to more efficient investment. Partial delegation creates a similar separation; only now it is between trade/production and rent-seeking.

Third, from a game-theoretic perspective, the full-delegation and partial-delegation games we solve have the technical structure of common agency games with private principal information as opposed to private agent information.²⁰ Thus, our work can also be seen as complementing the analysis of

²⁰Common agency games with a privately informed agent and symmetrically uninformed principals have been studied settings by Laffont and Tirole (1991), Martimort (1992), and Stole (1992), Bond and Gresik (1996), Mezzetti (1997), Epstein and Peters (1999), Peters (2002), Martimort and Stole

informed principal problems such as Myerson (1983) and Maskin and Tirole (1990) to include competition, via the agent, with another principal.²¹ Unlike in these classic papers, the choice of a contract by the informed seller influences competition between the buyer and the agent. Because the buyer has the power to make take-it-or-leave-it offers, agent beliefs about the seller's type do not play a role in deriving equilibria in this paper. Similarly, buyer beliefs about the seller's type do not play a role in our analysis because the buyer is constrained by the participation constraint of the agent. Buyer beliefs could play a role if the seller chose to delegate for some types and not for others. The seller has no incentive to bargain directly in this paper because doing so would only improve the capability of the buyer to minimize the rents the seller earns.

Fourth, we wish to point out a possible corporate finance application. The finance literature has documented short-run excess stock price returns of 2.4% to 4.3% related to spin-off announcements.²² Numerous explanations for this phenomenon exist. The one that is closest in spirit to this paper is from Aron (1991). She argues that with a multi-division firm, share price does not track the performance of any one division very closely. Spinning off a division allows owners to provide the manager with stronger incentives based on stock price. Seward and Walsh (1996), however, do not find any evidence that the excess returns are related to stronger managerial incentives. Our results suggest an alternative explanation. In the short run, the information of the parent with respect to its interactions with the spun-off division (e.g. Ford and Visteon) remains accurate while the parent can now extract its surplus through an arm's-length relationship. Our analysis implies that by creating this arm's-length relationship, the parent eliminates the incentives faced by suppliers that in equilibrium created quantity distortions. This is inherently a short-term advantage. Over time the division's economic characteristics will likely change

(2003), and Peters (2003). Baron (1985) studies a regulation problem with asymmetrically uninformed principals in a hierarchical environment, e.g. the EPA and price-control boards. Neither principal has private information in his model.

²¹More recently, Segal and Whinston (2003) use menus of contracts (in which the agent chooses a contract from the menu) to capture competition in a principal-multiple-agent model. In their model, the principal does not have any exogenously inherited private information. Bond and Gresik (1997) study a common agency problem with an informed and an uninformed principal but do not allow the uninformed principal to elicit information from the agent or the informed principal.

²²See Hite and Owers (1983), Miles and Rosenfeld (1983), Schipper and Smith (1983), and Rosenfeld (1984).

and its reliance on its parent for business will diminish.

Our paper suggests a number of extensions. First, in more general environments such as those in which the buyer and the seller have private information, we expect the first-best trade result and its associated benefits to be reduced but not eliminated. Second, the category management example suggests the need to include a moral hazard component to the agent's utility. In our full delegation game, renegotiation between the seller and the agent after information is shared may give the seller an additional incentive to strategically manage what it tells its agent. Once the agent learns the seller's private information, it may have an incentive to use that information to bargain for positive surplus.²³ Again, one obvious way to respond to this possibility is partial delegation. However, we do not at this time rule out alternative responses to agent renegotiation in a full delegation setting as there may be important economic settings in which partial delegation is not feasible. We hope to address these and other extensions in future work.

²³We thank Matt Mitchell for this suggestion.

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Appendix

Lemma A1. Fix the buyer's contract, $T^b(q)$. Since for any seller best-response, $T^s(q^f(\kappa), \kappa) = T^b(q^f(\kappa))$ for all κ , it is also true that $T_{q\kappa}^s(q^f(\kappa), \kappa)q^{f'}(\kappa) \leq 0$.

Proof. Totally differentiating $T^s(q^f(\kappa), \kappa) = T^b(q^f(\kappa))$ twice implies

$$2T_{q\kappa}^s(q^f(\kappa), \kappa)q^{f'}(\kappa) = (T_{qq}^b - T_{qq}^s)q^{f'}(\kappa)^2 + (T_q^b - T_q^s)q^{f''}(\kappa). \quad (\text{A.1})$$

By (24), the second term in the sum on the right-hand side of (A.1) is zero and the first term is non-positive. ***Q.E.D.***

The buyer's best-response in the full-delegation game. In contrast to the standard principal agent problem in which (30b) binds only for the least cost type, the solution to the seller's problem dictates that in equilibrium $A(\theta)$ will equal zero for all θ and thus can bind on open sets of types. Thus, rather than characterizing the buyer's best-response to any seller strategy, we instead seek to derive necessary and sufficient conditions for which the buyer's best-response implies $A(\theta) = 0$. To do this we formulate the supplier's problem as an optimal control problem with a pure state constraint. Following Seierstad and Sydsaeter, we form the Hamiltonian

$$\mathcal{H}(q, A, \mu, \theta) = (b(q(\theta)) - T^s(q(\theta), \theta) - A(\theta))f(\theta) - \mu(\theta)T_{\kappa}^s(q(\theta), \theta) \quad (\text{A.2})$$

and the associated Lagrangian

$$\mathcal{L}(q, A, \mu, \lambda, \theta) = \mathcal{H}(q, A, \mu, \theta) + \lambda(\theta)A(\theta).$$

Theorem 5.1 in Seierstad and Sydsaeter shows that $(q^*(\theta), A(\theta) = 0)$ will maximize (30) if there exists a piecewise continuous function $\lambda(\theta) \geq 0$ and a continuous and piecewise continuously differentiable function $\mu(\theta)$ satisfying the following conditions:

$$\begin{aligned} (a) \quad & q^*(\theta) \in \operatorname{argmax}_q \mathcal{H}(q, A, \mu, \theta), \\ (b) \quad & \dot{\mu}(\theta) = f(\theta) - \lambda(\theta), \text{ and} \\ (c) \quad & \mu(\theta) \leq 0 \text{ and } \mu(\bar{\theta}) \geq 0. \end{aligned} \quad (\text{A.3})$$

When (A.2) is concave in q , condition (A.3.a) is equivalent to

$$(b_q(q^*(\theta)) - T_q^s(q^*(\theta), \theta))f(\theta) = \mu(\theta). \quad (\text{A.4})$$

It reveals that the equilibrium quantities will depart from the buyer's unconstrained first best if $\mu(\theta) \neq 0$.

This will occur if it is optimal for the buyer to distort the quantity schedule in order to reduce the information rents it must pay the agent.

Define the non-decreasing function,

$$\beta(\theta) = \beta(\underline{\theta}) + \int_{t=\underline{\theta}}^{\theta} \lambda(t) dt.$$

By incentive compatibility constraint (30a), if the buyer wishes to change agent θ 's profit by one dollar, it

must also adjust the profit for all lower type agent's by one dollar. Thus, $\beta(\theta)$ is the aggregate shadow price associated with the non-negative profit constraint, (30b). Condition (A.3.b) implies that $\mu(\theta)=F(\theta) - \beta(\theta) + k$ where k is a constant. Without loss of generality, we can set k equal to 0. Because $\beta(\cdot)$ is non-decreasing, (A.3.c) implies that $0 \leq \beta(\theta) \leq 1$ for all θ . Note that higher values of $\beta(\cdot)$ reduce the buyer's incentive to limit the agent's marginal information rents by trading a lower quantity (via $\mu(\cdot)$). Therefore, if $T^s(q, \kappa)$ implies that (A.2) is concave in q , then $(q^*(\theta), A(\theta) \equiv 0)$ maximizes (30) only if

$$\text{there exists a non-decreasing function } \beta(\theta) \text{ such that, for all } \theta, 0 \leq \beta(\theta) \leq 1, \text{ and} \quad (\text{A.5})$$

$$b_q(q^*(\theta)) - T_q^s(q^*(\theta), \theta) = (F(\theta) - \beta(\theta))/f(\theta). \quad (\text{A.6})$$

Furthermore, because the Hamiltonian and the state constraint, $A(\theta) \geq 0$, are linear in the state variable, $A(\cdot)$, Theorem 3 in chapter 5 of Seierstad and Sydsæter (1987) implies that these conditions are also sufficient. Finally, the buyer can always refuse to offer the agent a contract and earn zero profit. The buyer will do so only if its expected profit is negative. Thus, implementing $(q^*(\theta), A(\theta) \equiv 0)$ is a best response for the buyer to any seller strategy $(T^s, \kappa(\theta) \equiv \theta)$ that for which (A.2) is concave in q if, and only if, (A.5) and (A.6) are satisfied and

$$\mathcal{E}_\theta[b(q^*(\theta)) - T^s(q^*(\theta), \theta)] \geq 0. \quad (\text{A.7})$$

Proof of Proposition 5. Consider a candidate equilibrium quantity schedule, $q^*(\theta)$, that is strictly decreasing. We begin by showing that using (31) to induce $q^*(\theta)$ with $\kappa(\theta) \equiv \theta$ is a best-response to (32) by the seller. As previously discussed, the seller's objective, $T^b(q) - c(q, \theta)$, is independent of κ . This observation gives us great latitude in choosing $\kappa(\theta)$. Eq. (32) also implies that the derivative of the seller's objective with respect to q (equivalent to (27)) is

$$c_q(q, q^{*-1}(q)) - c_q(q, \theta). \quad (\text{A.8})$$

Because $c_{q\theta}(q, \theta) > 0$, (A.8) is strictly positive for $q < q^*(\theta)$ and strictly negative for $q > q^*(\theta)$. Thus, (27) is maximized at $q^*(\theta)$. Moreover, at $q=q^*(\theta)$, (31) implies zero agent profit and $T_q^b(q) = T_q^s(q, \theta)$. The assumption $c_{q\theta} > 0$ ensures that agent's profit is also maximized at $q^*(\theta)$. To implement $q^*(\theta)$, $\kappa(\theta)$ must be strictly monotone. Without loss of generality, we can set $\kappa(\theta) \equiv \theta$. Consequently, $T^s(\cdot, \cdot)$ defined by (31) and $\kappa(\theta) \equiv \theta$ is a best-response to (32) as long as $\Sigma^*(\theta)$ is non-negative (to guarantee non-negative seller profit for all θ).

Next we need to determine when using (32) to induce $q^*(\theta)$ is a best-response for the buyer to the seller strategy of (31) and $\kappa(\theta) \equiv \theta$. First note that (31) makes the buyer's Hamiltonian, (A.2), strictly concave in q . Then note that substituting (31) into (A.6) implies that it is optimal for the buyer to offer a contract that implements $q^*(\theta)$ and results in zero agent profit if, and only if, there exists a non-decreasing

function $\beta(\theta)$ that maps into $[0,1]$ such that

$$b_q(q^*(\theta)) - c_q(q^*(\theta), \theta) = (F(\theta) - \beta(\theta))/f(\theta). \quad (\text{A.9})$$

Since the left-hand side of (A.9) is $w_q(q^*(\theta), \theta)$, (A.9) is equivalent to (33). Expected equilibrium buyer profit will be non-negative if

$$\Sigma^*(\bar{\theta}) \leq \int_{\underline{\theta}}^{\bar{\theta}} (w(q^*(\theta), \theta)f(\theta) + w_\theta(q^*(\theta), \theta)F(\theta))d\theta$$

which is the upper bound in (34).

Q.E.D.

Characterization of incentive efficient allocations for the direct bargaining problem with strictly decreasing quantity schedules.²⁴

To set-up the appropriate optimization problem, we first substitute (3) into (7) to get

$$\hat{W} = (\hat{\alpha}(\bar{\theta}) - \gamma)S(\bar{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} [(\hat{\alpha}(\theta) - \gamma)q(\theta)F(\theta)/f(\theta) + \gamma w(q(\theta), \theta)]/f(\theta)d\theta \quad (\text{A.10})$$

where $\hat{\alpha}(\theta) = \mathcal{E}_\theta[\alpha(t) | t \leq \theta]$ is a conditional welfare weight reflecting the cumulative welfare effects of shifting profit to or from a seller of type θ due to incentive compatibility constraints.

Second, (6) is equivalent to

$$\int_{\underline{\theta}}^{\bar{\theta}} [w(q(\theta), \theta) - (S(\theta) - S(\bar{\theta}))]dF(\theta) \geq S(\bar{\theta}). \quad (\text{A.11})$$

Using (3), integrating the left-hand side of (A.11) by parts produces

$$\int_{\underline{\theta}}^{\bar{\theta}} (d[w(q(\theta), \theta)F(\theta)] - w_q(q(\theta), \theta)q'(\theta)F(\theta)d\theta) \quad (\text{A.12})$$

so (A.11) is equivalent to

$$S(\bar{\theta}) \leq \int_{\underline{\theta}}^{\bar{\theta}} [w(q(\theta), \theta)f(\theta) + w_\theta(q(\theta), \theta)F(\theta)]d\theta. \quad (\text{A.13})$$

²⁴This section complements the analysis in Spulber (1988) which does not provide an explicit characterization of strictly decreasing, incentive efficient allocations for multi-unit bargaining problems. The focus in his paper was to describe allocations that involved pooling (constant quantities over an open set of types) and to describe conditions on welfare weights for which a first-best quantity schedule would be incentive efficient.

Since $\mathcal{S}(\bar{\theta}) \geq 0$, (6) can be satisfied as long as

$$\int_{\underline{\theta}}^{\bar{\theta}} [w(q(\theta), \theta) f(\theta) + w_{\theta}(q(\theta), \theta) F(\theta)] d\theta \geq 0. \quad (\text{A.14})$$

As a result, incentive efficient bargaining allocations can be derived by choosing $q(\cdot)$ and $\mathcal{S}(\bar{\theta})$ to maximize (A.10) subject to (A.13), $\mathcal{S}(\bar{\theta}) \geq 0$, (A.14), and $q'(\cdot) \leq 0$.

As is common in the literature, the third step in the proof is to solve the optimization problem without the last constraint and then verify that it is satisfied. We need to consider two cases.

Case 1: $\gamma \geq 1/2$. With $\gamma > 1/2$, it is optimal to set $\mathcal{S}(\bar{\theta}) = 0$ as $\hat{\alpha}(\bar{\theta}) < \gamma$. If $\gamma = 1/2$, then it is optimal to choose $\mathcal{S}(\bar{\theta})$ from the interval defined by (34). In either case, the Lagrangian is

$$\mathcal{L} = (\hat{\alpha}(\theta) - \gamma + \lambda)(F(\theta)/f(\theta))q(\theta) + (\gamma + \lambda)w(q(\theta), \theta) \quad (\text{A.15})$$

where $\lambda \geq 0$ is the multiplier associated with (A.14). Note that (A.15) is strictly concave in $q(\cdot)$. Define the quantity schedule $q'(\theta, \lambda)$ from the first-order condition with respect to $q(\cdot)$ from (A.15). Thus, $q'(\theta, \lambda)$ is the solution to

$$w_q(q(\theta), \theta) = \left(1 - \frac{\hat{\alpha}(\theta)}{\gamma + \lambda} \right) \frac{F(\theta)}{f(\theta)}. \quad (\text{A.16})$$

Define λ^* to be the smallest value of $\lambda \geq 0$ such that (A.14) is satisfied by $q'(\cdot, \lambda)$. Because the integrand in (A.14) is continuous in $q(\cdot)$, λ^* is well-defined. If $\partial q'(\theta, \lambda^*)/\partial \theta \leq 0$, then $q'(\cdot, \lambda^*)$ is the incentive efficient quantity schedule. Totally differentiating (A.16) implies $\partial q'(\theta, \lambda^*)/\partial \theta \leq 0$ if, and only if,

$$\left(\left(1 - \frac{\hat{\alpha}(\theta)}{\gamma + \lambda^*} \right) \frac{F(\theta)}{f(\theta)} \right)' + 1 \geq 0. \quad (\text{A.17})$$

Case 2: $\gamma < 1/2$. With $\theta < 1/2$, $\hat{\alpha}(\bar{\theta}) > \gamma$. Thus, (A.13) will bind,

$$\hat{W} = \int_{\underline{\theta}}^{\bar{\theta}} [\hat{\alpha}(\bar{\theta})w(q(\theta), \theta) + (\hat{\alpha}(\theta) - \hat{\alpha}(\bar{\theta}))q(\theta)F(\theta)/f(\theta)] f(\theta) d\theta, \quad (\text{A.18})$$

and

$$\mathcal{L} = (\hat{\alpha}(\theta) - \hat{\alpha}(\bar{\theta}) - \lambda)q(\theta)F(\theta)/f(\theta) + (\hat{\alpha}(\bar{\theta}) + \lambda)w(q(\theta), \theta). \quad (\text{A.19})$$

Again, \mathcal{L} is strictly concave in $q(\cdot)$. Define $q^2(\cdot, \lambda)$ as the solution to the first-order condition with respect to $q(\cdot)$ from (A.19). Thus, $q^2(\cdot, \lambda)$ is the solution to

$$w_q(q(c), c) = \left(1 - \frac{\hat{\alpha}(\theta)}{\hat{\alpha}(\bar{\theta}) + \lambda} \right) \frac{F(\theta)}{f(\theta)}. \quad (\text{A.20})$$

Define λ^{**} to be the smallest value of $\lambda \geq 0$ such that (A.14) is satisfied by $q^2(\cdot, \lambda)$. As with λ^* above, λ^{**} is well-defined. If $\partial q^2(\theta, \lambda^{**})/\partial \theta \leq 0$, then $q^2(\cdot, \lambda^{**})$ is the incentive efficient quantity schedule for this case.

Totally differentiating (A.20) shows that $\partial q^2(\theta, \lambda^{**})/\partial \theta \leq 0$ if, and only if,

$$\left(\left(1 - \frac{\alpha(\theta)}{\alpha(\bar{\theta}) + \lambda^{**}} \right) \frac{F(\theta)}{f(\theta)} \right)' + 1 \geq 0. \quad (\text{A.21})$$

Q.E.D.