Econ 30010

Intermediate Microeconomics

Second-degree Price Discrimination Example

This is a summary of the example from class.

Consumer Information

1. All consumers have utility functions of the form $u(q, p, \theta) = \theta\sqrt{q} - p$. Different customers may have different values of the preference parameter, θ.

2. The preference parameter θ can take on two different values, 1 or 2. The probability that a customer has preferences defined by θ=1 is denoted by λ. The probability that a customer has preferences defined by θ=2 is then 1-λ.

Firm Information

3. With two types of potential customers, second-degree price discrimination means the monopolist needs to choose two combinations of prices and quantities: (q_L,p_L) for customers with θ=1 and (q_H,p_H) for customers with θ=2.

4. The firm's objective to maximize its expected profit,

$$E\pi = \lambda(p_L - cq_L) + (1-\lambda)(p_H - cq_H).$$

5. The firm's marginal cost of production equals

Incentive Constraints

Because the monopolist cannot observe any customer's type, the best it can do is let each customer choose either (q_L,p_L) or (q_H,p_H). To ensure that θ=1 types choose (q_L,p_L) and that θ=2 types choose (q_H,p_H), the quantities and prices the monopolist chooses must satisfy several incentive constraints.

1. Incentive compatibility. Each customer must get at least as much utility from the quantity-price combination the monopolist would like a customer with his or her type to choose than he or she gets from any other quantity-price combination the monopolist offers.

(IC-H) $Hv(q_H) - p_H \geq Hv(q_L) - p_L$

(IC-L) $Lv(q_L) - p_L \geq Lv(q_H) - p_H$

2. Individual rationality or participation. Each customer must get at least as much utility from the quantity-price combination the monopolist would like a customer of his or her type to choose than the customer would get from making no purchase.

(IR-H) $Hv(q_H) - p_H \geq 0$

(IR-L) $Lv(q_L) - p_L \geq 0$.

From the analysis we did in class, the binding constraints will be (IC-H) and (IR-L). (Make sure you can explain why?)

Thus, using (IR-L) the monopolist will charge θ=1 customers the price, $p_L = Lv(q_L) = \sqrt{q_L}$ and using (IC-H) the monopolist will charge the θ=2 customers the price,

$$p_H = Hv(q_H) - Hv(q_L) + Lv(q_L) = Hv(q_H) - (H-L)v(q_L) = 2\sqrt{q_H} - \sqrt{q_L} \text{ .}$$

Expected profit-maximizing quantities

Using the prices we just calculated, expected profit for the monopolist can now be written as

$$E\pi = \lambda(Lv(q_L) - cq_L) + (1-\lambda)(Hv(q_H) - (H-L)v(q_L) - cq_H) \text{ .}$$

To calculate the optimal quantities, we need to set the partial derivative of expected profits with respect to each quantity equal to zero and solve the resulting equations. Doing so yields

$$\begin{aligned} \partial E\pi / \partial q_L &= \lambda(Lv'(q_L) - c) - (1-\lambda)(H-L)v'(q_L) \\ &= (2\lambda - 1)v'(q_L) - \lambda c = 0 \end{aligned} \qquad (1)$$

and

$$\partial E\pi / \partial q_H = (1-\lambda)(Hv'(q_H) - c) = (1-\lambda)(2v'(q_H) - c) = 0 \text{ .} \qquad (2)$$

Equation (1) implies $\dfrac{1}{2\sqrt{q_L}} = \dfrac{\lambda c}{2\lambda - 1}$ and equation (2) implies $\dfrac{1}{\sqrt{q_H}} = c$. Thus, $q_H = 1/c^2$ (which is the quantity at which the θ=2 customer's marginal utility equals the monopolist's marginal cost) and $q_L = \dfrac{(2\lambda-1)^2}{4\lambda^2 c^2}$.

If λ=1/2 and c=1, then the monopolist would sell 1 unit to high-type customers at a price of 2 and it would not sell anything to low-type customers (q_L =0). Low-types would not be willing pay $2 for one unit.

If λ=3/4 and c=1, then the monopolist would sell 1 unit to high-type customers and 1/9 units to low-type customers. The price for one unit would be 2 – 1/3 or 5/3. The price for buying 1/9 units would be 1/3.