Econ 30010

Intermediate Microeconomic Theory

Answers for Practice Problems on Firm Costs

Production Functions to be used in the following problems.

A. $q = 50\ L^{1/4}\ K^{1/5}$

B. $q = KL+L^2$

C. $q = 75LK$

D. $q = 30L^{1/3}\ K^{2/3}$

E. $q = 4L + 3K$

1. Cost Minimization

Find the cost minimizing input combination for each set of production function, factor prices, and output level.

a. Production function A, q = 1000, w = 2, and r = 5.

  K* = 413, L* = 1292

b. B, q = 500, w = 6, and r = 1.

  K* = 40, L* = 10

c. C, q = 650, w = 3, and r = 4.

  K* = 2.55, L* = 3.4

d. D, q = 10000, w = 2, and r = 2.

  K* = 420, L* = 210

e. E, q = 40, w = 7, and r = 10.

  K* = 0, L* = 10

2. Long-run Cost Functions - For each production function/factor price combination in section 1, derive the long-run cost function.

a. $TC(q) = .001\ q^{2.22}$

b. $TC(q) = 10\ (q/5)^{1/2}$

c. $TC(q) = .8\ q^{1/2}$

d. $TC(q) = .126q$

e. $TC(q) = 7q/4$

3**. Short-run Cost Functions -** For each production function/factor price combination in section 1, derive the short run total cost function given the associated restriction below. Then derive the formulas for fixed costs, variable costs, and marginal costs.

a. (a) plus K=5.

$TC(q) = 25 + 2q^4/22{,}667{,}121$

$FC(q) = 25$

$VC(q) = 2q^4/22{,}667{,}121$

$MC(q) = 8q^3/22{,}667{,}121$

b. (b) plus L=10.

If $q \leq 100$, $TC(q) = 60$. If $q > 100$, $TC(q) = 50 + q/10$.

For $q \leq 100$, $FC(q)= 60$ and $VC(q)=MC(q) = 0$.

For $q > 100$, $FC(q) = 60$, $VC(q) =q/10-10$, and $MC(q) = 1/10$.

c. (c) plus $L \geq 20$.

If $q \leq 22{,}500$, $TC(q) = 60 + 4q/1500$. If $q > 22{,}500$, $TC(q) = .8q^{1/2}$.

For $q \leq 22{,}500$, $FC(q) = 60$, $VC(q) = 4q/1500$, and $MC(q) = 4/1500$.

For $q > 22{,}500$, $FC(q) = 60$, $VC(q) = .8q^{1/2} - 60$, and $MC(q) = .4q^{-1/2}$.

d. (d) plus $L \geq 10$ and $K \geq 5$.

(Step 1) From the calculations for (4d), the long-run cost-minimizing values for labor and capital are $L = q/47.62$ and $K = q/23.81$. Using these values, $L \geq 10$ and $K \geq 5$ only when $q \geq 476.2$. So for $q \geq 476.2$, $TC(q) = .126q$.

$FC(q) = 30$, $VC(q) = .126q - 30$, and $MC(q) = .126$.

(Step 2) $L = 10$ and $K = 5$ imply $q= 189$. So for $q \leq 189$, $TC(q) = 30$.

$FC(q) = 30$ and $VC(q) = MC(q) = 0$.

(Step 3) For $189 < q < 476.2$, only one of the two constraints, $L \geq 10$ and $K \geq 5$, will be binding. If $L = 10$, then K must equal $q^{3/2}/520$ and $TC(q) = 20 + q^{3/2}/260$. If $K = 5$, then L must equal $q^3/675{,}000$ and $TC(q) = 10 + 2q^3/675{,}000$. Of these two cost equations, the first one is always smaller given the relevant range of quantities. Thus, $TC(q) = 20 + q^{3/2}/260$.

$FC(q) = 30$, $VC(q) = q^{3/2}/260 - 10$, and $MC(q) = 3q^{1/2}/520$.

e. (e) plus $K \geq 24$.

If $q \leq 72$, $TC(q) = 240$. If $q > 72$, $TC(q) = 240 + 7(q-72)/4$.

For $q \leq 72$, $FC(q) = 240$ and $VC(q) = MC(q) = 0$.

For $q > 72$, $FC(q) = 240$, $VC(q) = 7(q-72)/4$, and $MC(q) = 7/4$.

4. We need to first recover the input demand curves. For labor, $L* = \partial C/\partial w = \sqrt{rq}/\sqrt{w}$ and for capital,

$K* = \partial C/\partial r = \sqrt{wq}/\sqrt{r} - 3$. Solving the labor demand equation for r/w implies

$$r/w = {L^*}^2/q \tag{1}$$

and solving the capital demand equation for r/w implies

$$r/w = q/(K^* + 3)^2. \tag{2}$$

Setting the right-hand sides of (1) and (2) equal to each other and solving for q results in

$$q = L(K + 3).$$

Thus the production function that generated the given cost function is $f(K,L) = L(K+3)$.