Econ 30010
Intermediate Microeconomic Theory
Answers to Monopoly Problems

1)For this problem, $MR(Q) = 18000 - 2000Q$ and $MC(Q) = 3Q^2$. Profit is maximized at the output level that solves $18000 - 2000Q = 3Q^2$. Using the quadratic formula, we get $Q^M = 8.88$ and $P^M = \$9120$. This quantity and price imply profit of $\pi = (9120)(8.88) - 20000 - (8.88)^3 = \$60,285$.

2) The publisher will want to charge the higher price. The publisher will maximize book profit by equating marginal revenue and marginal cost. The author will maximize her earnings by maximizing book revenues which occurs where marginal revenue is zero. This means the author would like to sell more books than the publisher. To sell more books, the author would need a lower price than the publisher is willing to charge.

3) a) The publisher should choose Q^{US} and Q^E to satisfy $MR^{US}(Q^{US}) = MR^E(Q^E)$ and $MR^{US}(Q^{US}) = MC(Q^{US} + Q^E)$. The demand curves imply that $P^{US} = 25 - Q^{US}/2000$ and $P^E = 20 - Q^E/500$. Thus, $MR^{US}(Q^{US}) = 25 - Q^{US}/1000$ and $MR^E(Q^E) = 20 - Q^E/250$. The cost function implies that $MC = (Q^{US} + Q^E)/1000$.

Substituting these formulas into the two equations implies $Q^{US} = 11{,}667$, $Q^E = 1{,}667$, $P^{US} = 19.17$, and $P^E = 16.67$.

b) Since the books have already been printed, the objective for the publisher is to maximize revenue. The books should be allocated between the two countries so that the last book sold in each country increases revenues equally. That is, the quantities should be chosen so that $MR^{US}(Q^{US}) = MR^E(Q^E)$ and $Q^{US} + Q^E = 12000$. These equations imply $Q^{US} = 10{,}600$, $Q^E = 1400$, $P^{US} = 19.70$, and $P^E = 17.20$.

4.a. The optimal complete information quantities and taxes will imply price equal to marginal cost and zero post-tax profit for each the monopolist with each possible cost level.

For the monopolist with $\theta = 5$, the socially optimal quantity must imply $250 - .5Q = 5$ or $Q = 490$. At this quantity, the monopolist's pre-tax profit equals $(5)(490) - 200 - (5)(490) = -200$. Thus, the tax should in fact be a subsidy equal to \$200.

For the monopolist with $\theta = 10$, the socially optimal quantity must imply $250 - .5Q = 10$ or $Q = 480$. At this quantity, the monopolist's pre-tax profit equals $(10)(480) - 200 - (10)(480) = -200$. Thus, the tax will again be a subsidy equal to \$200.

b. Since the government cannot observe θ, the quantities and taxes it sets must satisfy two incentive constraints: (ICL) $\Pi(Q_5,T_5,5) = \Pi(Q_{10},T_{10},5)$ and (PH) $\Pi(Q_{10},T_{10},10) = 0$. The ICL constraint implies $(P(Q_5) - 5)Q_5 - 200 - T_5 = (P(Q_{10}) - 5)Q_{10} - 200 - T_{10}$ and the PH constraint implies $(P(Q_{10}) - 10)Q_{10} - 200 - T_{10} = 0$.

Because the government cannot observe θ, it must choose the quantities and taxes to maximize expected welfare which is

$$EW = .6[\int_{x=0}^{Q_5} P(x)dx - P(Q_5)Q_5 + T_5] + .4[\int_{x=0}^{Q_{10}} P(x)dx - P(Q_{10})Q_{10} + T_{10}] .$$

Since $T_{10} = (P(Q_{10}) - 10)Q_{10} - 200$ and $T_5 = (P(Q_5) - 5)Q_5 - 200 - (10 - 5)Q_{10,}$ we can write expected welfare as

$$EW = .6[\int_{x=0}^{Q_5} P(x)dx - 5(Q_5 + Q_{10}) - 200] + .4[\int_{x=0}^{Q_{10}} P(x)dx - 10Q_{10} - 200] .$$

Taking the partial derivative of EW with respect to Q_5 implies $P(Q_5) = 5$ (price equals marginal cost as in (a)) or $Q_5 = 490$.

Taking the partial derivative of EW with respect to Q_{10} implies $P(Q_{10}) = 10 + 5(.6)/(.4) = 17.50$. Now price will be set above marginal cost to reduce the quantity the high marginal cost monopolist sells. Thus, $Q_{10} = 465$.

Using the tax equations, the low marginal cost monopolist will receive a subsidy of $200 + 5Q_{10}$ or \$2525. The high marginal cost monopolist will pay a tax $(17.50 - 10)Q_{10} - 200$ or \$3287.50 resulting in zero post-tax profit.

Calculate the optimal quantity the government should allow the monopolist to produce and the tax it should charge when θ=5 and when θ=10.

c. To discourage a low-cost firm from claiming to have high costs, the optimal regulations must adjust the optimal complete information regulations in two ways. It must make the regulation intended for the low-cost firm look more attractive to the low-cost firm (i.e. more profitable). This is accomplished by paying the low-cost firm an information rent equal to $(H - L)Q_H$. It also has to make the regulations intended for the high-cost firm look less attractive to the low-cost firm. This is accomplished by reducing the quantity the high-cost firm is allowed to produce and by reducing the subsidy offered to the high-cost firm - so much so that the subsidy becomes a tax! Thus Q_5 is unchanged. The low-cost firm still produces the optimal complete information quantity but T_5 goes from -\$200 to -\$2525. For the high-cost firm, Q_{10} falls from 480 to 465 and T_{10} increases from -\$200 to \$3287.50.