Econ 30010

Intermediate Microeconomic Theory

Answers to Chapter 8 Problems

3.

Q	**TC**	**TVC**	**TFC**	**AC**	**MC**	**AVC**
1	18	8	10	18	8	8
2	30	20	10	15	12	10
3	46	36	10	15.33	16	12
4	66	56	10	16.5	20	14
5	90	80	10	18	24	16
6	118	108	10	19.67	28	18

7. Economies of scale and diseconomies of scale regions correspond to regions in which average total cost is decreasing with output and increasing with output. In this case, $AC(Q) = 40 – 10Q + Q^2$. Average total cost is minimized where average total cost and marginal costs are equal or when

$$40 – 10Q + Q^2 = 40 – 20Q + 3Q^2.$$

Solving for Q, one gets $Q = 5$. Thus, for $Q < 5$, the production function will exhibit economies of scale and for $Q > 5$, it will exhibit diseconomies of scale.

12. a. The marginal value condition requires that $MP_L/MP_K = w/r = 2$. Since $MP_L/MP_K = (K/L)^{1/2}$, the marginal value equation implies $K = 4L$. Substituting this equation into the isoquant equation implies $Q = 9L$ or $L^* = Q/9$ and $K^* = 4Q/9$.

b. The firm's long-run cost function is $TC(Q) = wL^* + rK^* = 2Q/3$.

c. $AC(Q) = TC(Q)/Q = 2/3$.

d. When capital is fixed at 9, the isoquant equation alone determines the amount of labor used. Thus,

$$Q = (\sqrt{L} + \sqrt{9})^2$$

or

$$L^* = (\sqrt{Q} - 3)^2.$$

f. The short-run total cost function is $STC(Q) = (1)(9) + (2)(\sqrt{Q} - 3)^2$. Since $SAC(Q) = STC(Q)/Q$,

$$SAC(Q) = \frac{9 + 2(\sqrt{Q} - 3)^2}{Q}.$$

15. The cost-minimizing input combination must satisfy the isoquant equation

$$Q = KL + K$$

and the marginal value equation

$$\frac{K}{w} = \frac{L+1}{r}.$$

Solving these two equations simultaneously produces the optimal input quantities

$$K^* = \sqrt{\frac{wQ}{r}}$$

and

$$L^* = \sqrt{\frac{rQ}{w}} - 1.$$

Therefore, the firm's long-run total cost function is

$$TC(Q, w, r) = \sqrt{rwQ} + \sqrt{rwQ - w^2}. \quad (1)$$

To verify that total cost doubles when input prices double, we need to show that $TC(Q,2w,2r) = 2TC(Q,w,r)$. Given (1), note that

$$\begin{aligned} TC(Q,2w,2r) &= \sqrt{4rwQ} + \sqrt{4rwQ - 4w^2} \\ &= 2\sqrt{rwQ} + 2\sqrt{rwQ - w^2} \\ &= 2TC(Q, w, r). \end{aligned}$$