

Econ 30010
Intermediate Microeconomic Theory
Answers to Chapter 7 Problems

3. The marginal value of labor is $1000/10$ or 100 gallons from an extra dollar spent on labor. The marginal value of fermentation capacity is $200/.25 = 800$ gallons from an extra dollar spent on fermentation capacity. Since the marginal value of fermentation capacity is greater than the marginal value of labor, the winery can lower costs without changing output by increasing fermentation capacity and decreasing the amount of labor.

7. At a cost-minimizing output, the following two equations must be satisfied -

$$\frac{MP_K}{r} = \frac{MP_L}{w} \quad (1)$$

and

$$Q = (\sqrt{L} + \sqrt{K})^2. \quad (2)$$

Substituting the formulas for MP_K and MP_L and the values of Q , w , and r into (1) and (2) creates two equations and two unknowns. Solving this system of equations produces the following cost-minimizing quantities of capital and labor to produce 121,000 airframes: $L = 1000$ and $K = 100,000$.

15. a. The marginal value of capital equals $1/50$ while the marginal value of labor equals $.5L^{-1/2}$. These two marginal values will be equal only if $L = 625$. For L less than 625, the marginal value of labor is always greater than the marginal value of capital and the firm should produce using only labor. For L greater than 625, the firm should produce using only capital. Since you need at most 100 units of labor to produce 10 units of output, the cost-minimizing input combination will use only labor and hence no capital.

b. In order for the marginal value condition to hold with equality, it must be that

$1/r = .5L^{-1/2}$ or $L = r^2/4$. This amount of labor and no capital would allow one to produce $r/2$ units of output. If $Q = 10$, r would have to fall to 20. For r less than 20, the firm would now want to use some capital as $K^* = Q - L^{1/2} = Q - r/2$ implies $K^* > 0$ when $r < 20$.

c. Based on the answer to (a), output would have to increase to 25.

16. The formulas do not correspond to input demand curves for a cost-minimizing firm because a proportional change in r and w is not neutral. If input prices change by the same proportion, the firm should demand the same amounts of capital and labor. Given the input demand formulas, an $x\%$ change in r and w will cause the quantity-demanded of capital and labor to change by $x^3\%$.

17. To begin, assume that w will imply that the cost-minimizing amounts of capital and labor are strictly positive. This means the optimal amounts of K and L must satisfy the marginal value equation

$$\frac{K+1}{w} = \frac{L+1}{1} \quad (3)$$

and the equation for the 5-unit isoquant

$$KL + K + L = 5. \quad (4)$$

Solving (3) and (4) simultaneously for K and L results in

$$K^* = \sqrt{6w} - 1$$

and

$$L^* = \sqrt{\frac{6}{w}} - 1.$$

Notice that K^* will be positive only when $w > 1/6$ and L^* will be positive only when $w < 6$.

a. $w \leq 1/6$

b. $w \geq 6$

c. $1/6 < w < 6$

24. Since the firm's capital is fixed, the isoquant equation tells us that the firm has no flexibility in how much labor it employs. Thus, L must satisfy

$$Q = 10\bar{K}L^{1/3}$$

or

$$L = \left(\frac{Q}{10\bar{K}} \right)^3.$$

26. a) Since there are three inputs, K , L , and M , we will have a system of three equations (two marginal value equations and the isoquant equation) and three unknowns.

$$\frac{MPL}{w} = \frac{MPK}{k} \Rightarrow K = L, \quad (5)$$

$$\frac{MPL}{w} = \frac{MPM}{m} \Rightarrow L = M, \quad (6)$$

and

$$Q = K^{1/3} L^{1/3} M^{1/3}. \quad (7)$$

The solution to (5), (6), and (7) is $K^*=L^*=M^*=Q$.

b) Since K is fixed, the firm must choose an optimal combination of labor and materials by solving the following system of equations simultaneously for L and M ,

$$\frac{MPL}{w} = \frac{MPM}{m} \Rightarrow M = L \quad (8)$$

and

$$Q = \bar{K}^{1/3} L^{1/3} M^{1/3} \quad (9)$$

The short-run cost-minimizing input combination is

$$M^* = L^* = \left(\frac{Q}{\bar{K}^{1/3}} \right)^{3/2} . \quad (10)$$

c) In the long-run, $L^*=M^*=4$. In the short-run, L and M are defined by (10) which implies $L^*=M^*=4$.

28. The firm is not employing the optimal input bundle for its current output. Since $MP_K/r = 150/10 = 15$ and $MP_L/w = 200/25 = 8$, the firm should increase the amount of capital and decrease the amount of labor it uses.