Econ 30010

Intermediate Microeconomic Theory

Answers to Chapter 13 Problems

12. a. The Cournot equilibrium arises when each firm maximizes profit given the output of its competitor. This means each firm chooses its output so that its marginal revenue equals its marginal cost. For firm 1, $MR_1(X,Y) = 280 - 4X - 2Y$. For firm 2, $MR_2(X,Y) = 280 - 2X - 4Y$. Then $MR_1(X,Y) = MC_1(X)$ implies $4X + 2Y = 240$ and $MR_2(X,Y) = MC_2(Y)$ implies $2X + 4Y = 240$. Solving these two equations simultaneously gives us $X^* = Y^* = 40$ and $P^* = 120$. Each firm earns profit of 3200.
b. Solving $MR_2(X,Y) = MC_2(Y)$ gives us firm 2's reaction function to any output choice of firm 1: $Y = 60 - X/2$. Firm 1 can anticipate firm 2's reaction function which means firm 1's profit is just a function of X. Namely, $\pi_1(X,BR_2(X)) = (280 - 2X - 2(60-X/2))X - 40X = (120 - X)X$. To find the profit-maximizing value of X, calculate $d\pi_1(X,BR_2(X))/dX$ and set it equal to zero. Doing so results in $X^* = 60$, $Y^* = 30$, $P^* = 100$, $\pi_1^* = 3600$, and $\pi_2^* = 1800$.

26. a. If American sets a price of \$200, United has a demand curve of $Q_U = 1200 - 2P_U$. This implies profits for United of $\pi_U = (1200-2P_U)P_U - 10(1200-2P_U) = (P_U - 10)(1200 - 2P_U)$. Since $d\pi_U/dP_U = 1200 - 4P_U + 20$, the profit-maximizing price for United is $P_U = 1220/4 = 305$.
b and c. In general, United's profit is $\pi_U = (P_U - 10)(1000 - 2P_U + P_A)$ and $\partial\pi_U/\partial P_U = 1020 - 4P_U + P_A$. Setting this partial derivative equal to zero gives us United's reaction function: $P_U = (1020 + P_A)/4$. This means that when $P_A = 400$, United should charge $P_U = 355$.
d. Similar calculations will show that American's reaction function is $P_A = (1020 + P_U)/4$. Solving the reaction function equations for American and United simultaneously implies that in equilibrium each firm will charge $P^* = 1020/3 = 340$.