1 Structural Design

- The structural design of the rotor and tower naturally follows from the aerodynamic design from which the aerodynamic loads are derived.
- As it often happens in the design of aerodynamic systems, their needs to be a compromise between the aerodynamic optimum and the structural optimum.
  - The **structural design** seeks to optimize strength, weight and cost.
  - Catastrophic failures of wind turbine structures are rare, but not impossible.

Figure 1: Examples of rare structural failures of horizontal axis wind turbines.

- Conditions leading to structural failures include
  1. extreme winds,
  2. an inadequate control system,
  3. cyclic-load fatigue that leads to cracks in the structure.
- Fatigue is a very important issue since wind turbines are designed to operate for a minimum of 20 year **over which the rotor will rotate on the order of** 10^9 **revolutions!**
- Some of the loads repeat with every revolution of the rotor which results in a cyclic straining of the structure that could lead to strain hardening and brittle fracture.

- There are four primary sources of loads that are relevant to horizontal axis wind turbines. These are

  1. aerodynamic loads,
  2. gravitational loads,
  3. dynamic loads, and
  4. control loads.

- **Aerodynamic loads** include the lift, drag and pitch moment on the rotor such as can be determined by the BEM method.

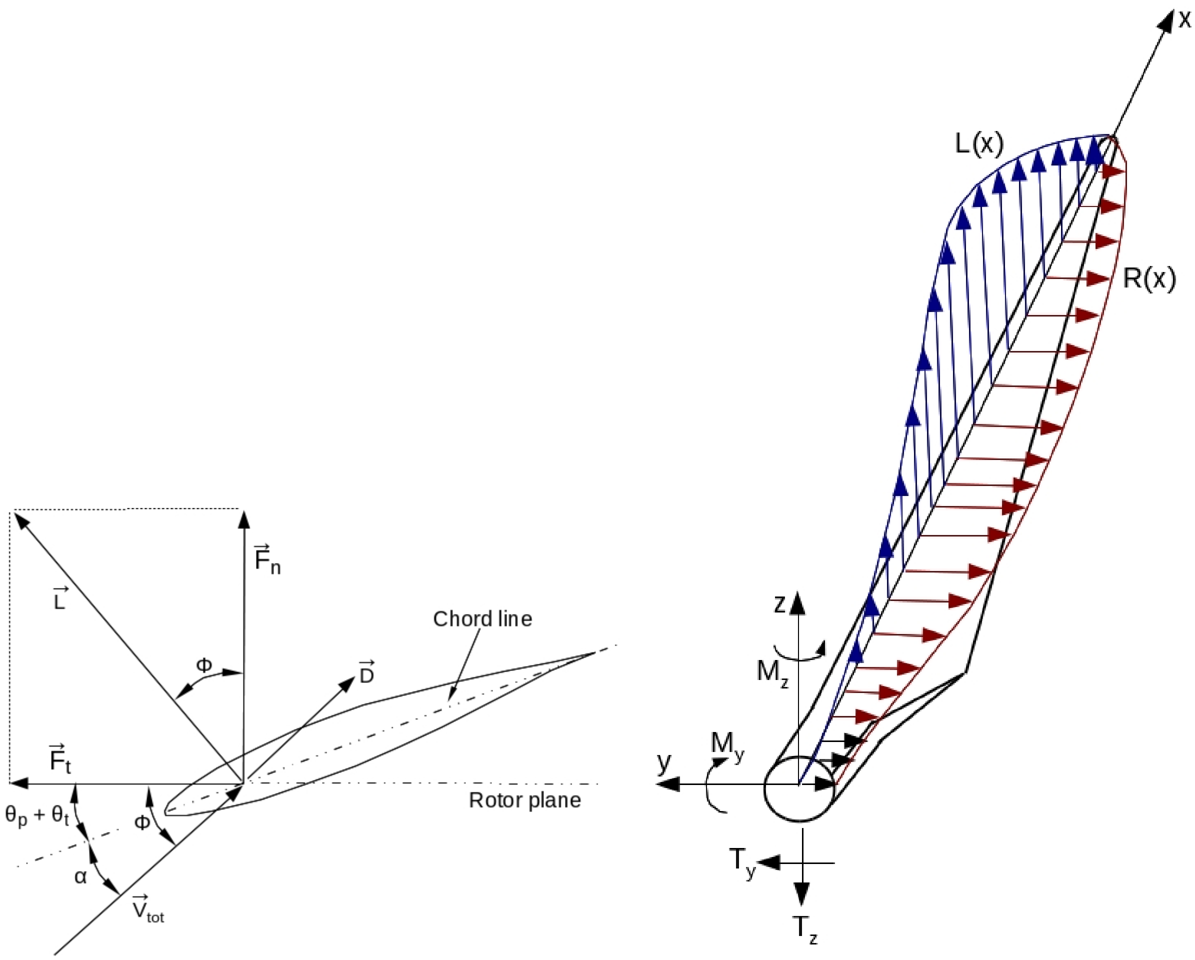

Figure 2: Force vectors based on BEM analysis (left) and illustration of 3-D lift and drag force distribution resulting in maximum shear forces and bending moments at the rotor root.

- Structurally, the rotor is a cantilever beam with a fixed attachment at the rotor hub.
- The material stresses associated with these loads determines the structural design.
- The forces that act on the rotor can be transmitted through the rotor shaft to the gear box and tower.
  - Structural failure of the gear box continues to be an important issue.

- **Gravitational loads** are primarily associated with the weight of the rotor blades.
- This is a cyclic loading whose magnitude on a radial element is

$$dF_g = \vec{g} dm \cos(\psi) \tag{1}$$

- The cyclic gravitational loading on the rotor is converted into a **cyclic torque variation** on the rotor shaft that is then transmitted to the gear box.

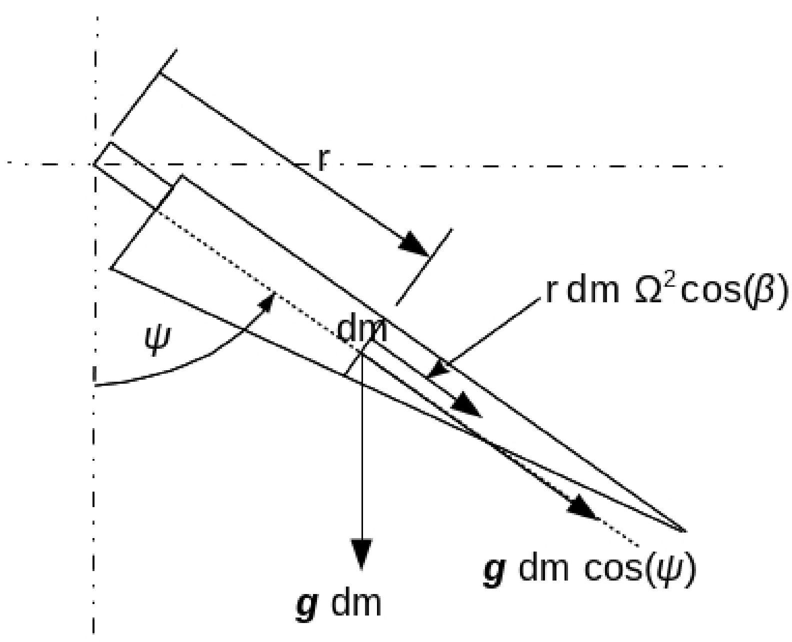

Figure 3: Illustration of gravitational and centrifugal loads acting on a spinning wind turbine rotor.

- The gravitational loading generally acts through the rotor plane axis, except if the rotor bends out of plane, which is referred to as **"flapping"**.

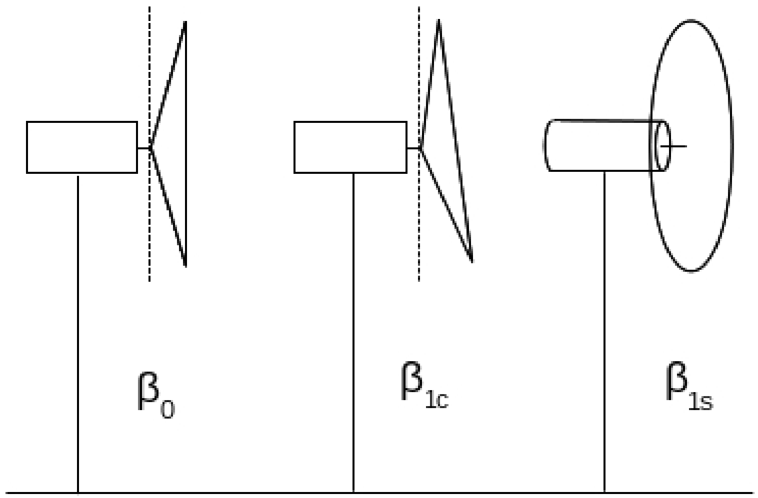

Figure 4: Illustration of types of coned or "flapping" rotor conditions of the horizontal axis wind turbine.

- Out of plane or flapping angle is defined as β.
  - β_0 shows a rotor plane that is aligned with the wind direction. *The loading on the blades is steady with respect to the rotor rotation angle, psi.*
- β_{1c}, has the axis of the rotor aligned with the wind direction, but the coned rotor plane is canted upward
  - The rotor location that is tilted upwind (bottom portion) will have a larger effective angle of attack compared to the rotor that is tilted downwind.
  - This will produce a cyclic loading with a magnitude that varies as $cos(\psi)$, where again $\psi = 0$ corresponds to the bottom of the rotation cycle.

- β_{1s}, has the axis of the coned rotor yawed with respect to the wind direction.
  - This produces a cyclic loading whereby the rotor that tilts upwind (right portion) will have an effectively larger angle of attack compared to the rotor that tilts downwind.
  - This will produce a cyclic loading with a magnitude that varies in this case, as $\sin(\psi)$.
- It is reasonable to sum the effects of the three coned rotor conditions to obtain an effective flapping angle, β given as

$$\beta = \beta_0 + \beta_{1c}\cos(\psi) + \beta_{1s}\sin(\psi). \tag{2}$$

- In this case β_0 represents the collective or coned response, and $\beta 1c$ and β_{1s} are the coefficients representing the respective cosine and sine cyclic responses.

- **Dynamic loading** is the result of changes in the motion of rotor.
- One example is the centrifugal force generated by the rotation of the rotor.

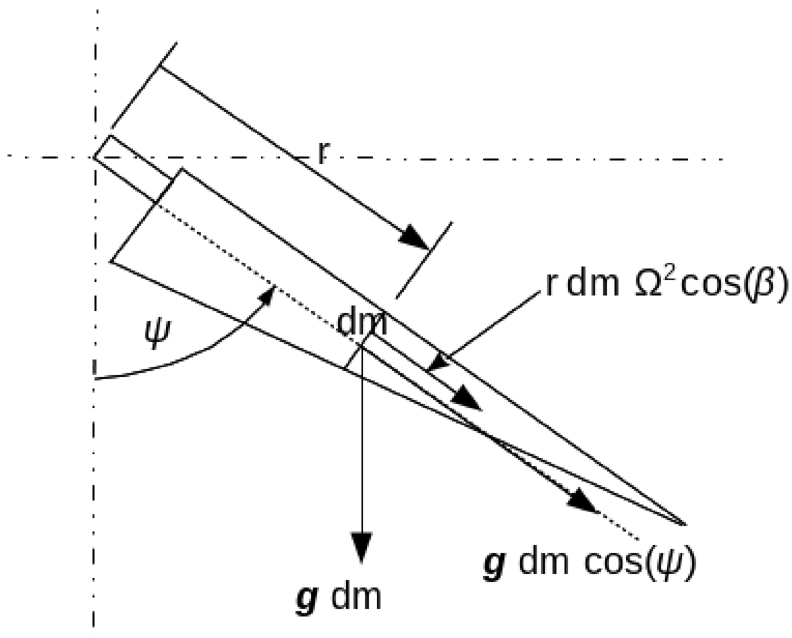

Figure 5: Illustration of gravitational and centrifugal loads acting on a spinning wind turbine rotor.

- The centrifugal force acting on a radial element of the rotor at some radius is

$$dF_c = r dm \Omega^2 \cos(\beta) \quad (3)$$

  - Again β is the effective flapping angle
- The centrifugal force can be considered as a point load that acts on the center of mass of the rotor blade, and is directed perpendicular to the axis of rotation.

- The moment produced by the centrifugal force acting on a differential element at radius r is

$$dM_c = r\sin(\beta)\left[rdm\Omega^2\cos(\beta)\right]. \tag{4}$$

- **Gyroscopic loads** are produced by yaw or flapping motions of the spinning rotor.

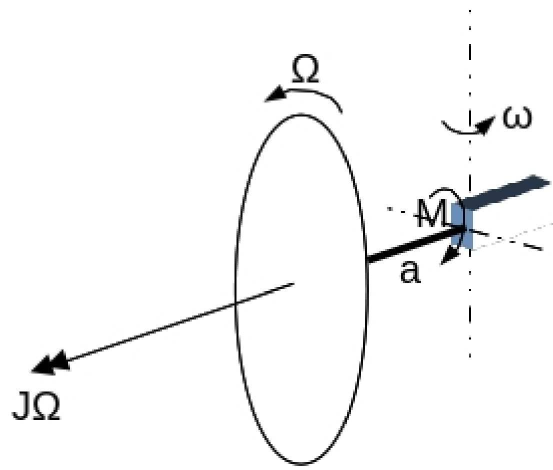

Figure 6: Illustration of the gyroscopic restoring moment produced by the yawed motion of the rotor.

- Assuming that the rotor has a polar moment of inertia of J, and spins at a rate Ω, it will have an angular momentum of $J\Omega$.
- Based on the theory of gyroscopes, if a body with angular momentum of $J\Omega$ is rotated about an axis that is perpendicular to the rotor Ω plane, it will generate a moment equal to the cross product, $\omega \times J\Omega$, where ω is the yawing rate.
- The generated bending moment acts on the bearing block
- These bending moments put stress on the rotor shaft and bearing block that could lead to structural failure.

- **Control loads** result from continuous changes in blade pitch and torque used to maintain the optimum tip-speed-ratio
- These control operations can produce intermittent loads on the rotor, shaft and gear box

2 Rotor Response to Loads

- The horizontal axis wind turbine rotor is designed to be stiff and light weight.

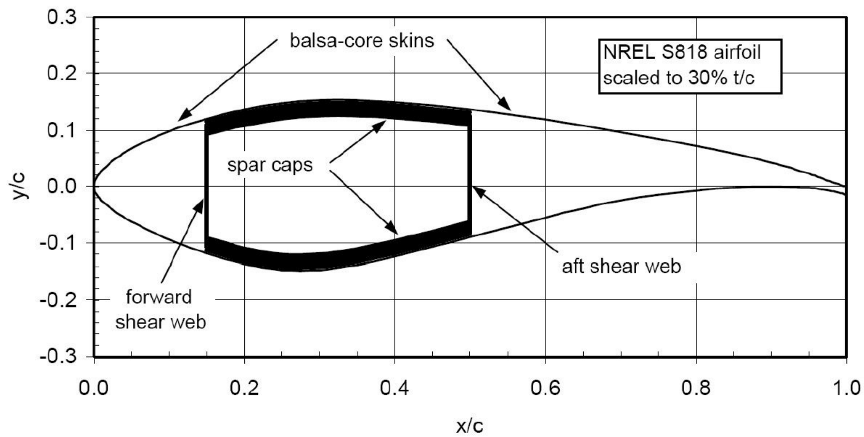

Figure 7: Section view of a HAWT rotor illustrating the internal structure.

- The rotor blade can be modeled as a cantilever beam.
- Like the BEM approach, the rotor blade is divided into small spanwise segments
  - The external loading of a rotor segment, pdx is known from the BEM analysis.
  - Loading results in shear forces, T and $T + dT$, and bending moments, M and $M + dM$ on each element.

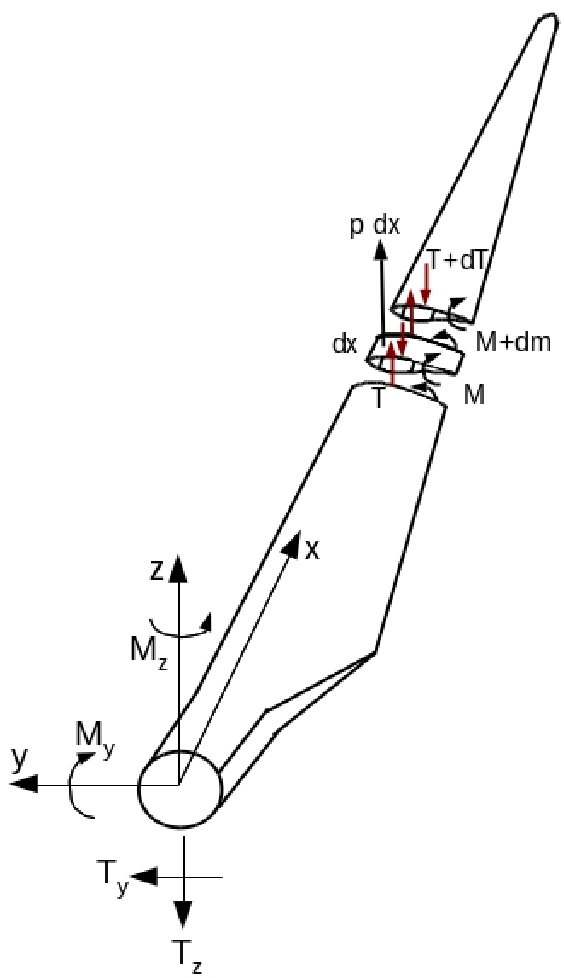

Figure 8: Illustration of shear force and bending moment on a small spanwise element of the loaded rotor.

- A balance of forces and moments gives the following equations.

$$\frac{dT_z}{dx} = -p_z(x) + m(x)\frac{d^2u_z(x)}{dt^2} \quad (5)$$

$$\frac{dT_y}{dx} = -p_y(x) + m(x)\frac{d^2u_y(x)}{dt^2} \quad (6)$$

$$(7)$$

  – Time derivative terms represent the inertia in the blade motion (BENDING).

- The bending moments are then found from

$$\frac{dM_y}{dx} = T_z \quad (8)$$

$$\frac{dM_z}{dx} = -T_y \quad (9)$$

$$(10)$$

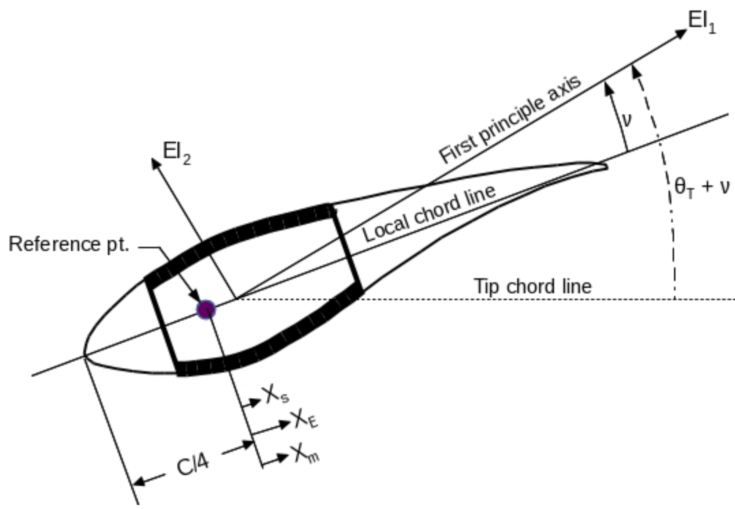

Figure 9: Spanwise element of rotor blade used in beam analysis to determine principle bending axis.

- **Principle bending axis** is the point of bending elasticity where a normal force (out of the plane) does not produce bending of the beam.
  - If the airfoil section is symmetric (no camber) the first principle axis lies along the chord line, that is $\nu = 0$.
  - For normally twisted blades, $theta_T \leq 0$, although $(\theta_T + \nu)$ is considered to be positive.
- The transformation of the bending moments due to the loads to those along the principle axes is

$$M_1 = M_y \cos(\theta_T + \nu) - M_z \sin(\theta_T + \nu) \tag{11}$$

and

$$M_2 = M_y \sin(\theta_T + \nu) - M_z \cos(\theta_T + \nu). \tag{12}$$

- From beam theory, the curvatures about the principle axes are

$$\kappa_1 = \frac{M}{EI_1} \tag{13}$$

and

$$\kappa_2 = \frac{M}{EI_2}. \tag{14}$$

- These curvatures are transformed back to the y and z axes by

$$\kappa_z = -\kappa_1 \sin(\theta_T + \nu) + \kappa_2 \cos(\theta_T + \nu) \tag{15}$$

and

$$\kappa_y = \kappa_1 \cos(\theta_T + \nu) + \kappa_2 \sin(\theta_T + \nu). \tag{16}$$

- The angular deformations are then calculated as

$$\frac{d\theta_y}{dx} = \kappa_y \tag{17}$$

and

$$\frac{d\theta_z}{dx} = \kappa_z. \tag{18}$$

- The deflections, u_z and u_y are found by integrating

$$\frac{du_z}{dx} = -\theta_y \tag{19}$$

and

$$\frac{du_y}{dx} = -\theta_z. \tag{20}$$

- **Numerical approach**: Consider a rotor blade divided into N spanwise elements, where the N^{th} element is at the rotor tip
- **Shear force**:

$$T_y^{i-1} = T_y^i + \frac{1}{2}\left(p_y^{i-1} + p_y^i\right)\left(x^i - x^{i-1}\right) \; ; \; i = N, N-1, \cdots 2 \tag{21}$$

and

$$T_z^{i-1} = T_z^i + \frac{1}{2}\left(p_z^{i-1} + p_z^i\right)\left(x^i - x^{i-1}\right) \; ; \; i = N, N-1, \cdots 2. \tag{22}$$

- **Bending Moments**:

$$M_y^{i-1} = M_y^i - T_z^i\left(x^i - x^{i-1}\right) - \left(\frac{1}{6}p_z^{i-1} + \frac{1}{3}p_z^i\right)\left(x^i - x^{i-1}\right)^2 \; ; \; i = N, N-1, \cdot \tag{23}$$

and

$$M_z^{i-1} = M_z^i - T_y^i\left(x^i - x^{i-1}\right) - \left(\frac{1}{6}p_y^{i-1} + \frac{1}{3}p_y^i\right)\left(x^i - x^{i-1}\right)^2 \; ; \; i = N, N-1, \cdot \tag{24}$$

- **Rotor Deflections**:

$$u_y^{i+1} = u_y^i + \theta_z^i\left(x^{i+1} - x^i\right) + \left(\frac{1}{6}\kappa_z^{i+1} + \frac{1}{3}\kappa_z^i\right)\left(x^{i+1} - x^i\right)^2 \; ; \; i = 1, 2, \cdots N-1 \tag{25}$$

and

$$u_z^{i+1} = u_z^i + \theta_z^i\left(x^{i+1} - x^i\right) + \left(\frac{1}{6}\kappa_y^{i+1} + \frac{1}{3}\kappa_y^i\right)\left(x^{i+1} - x^i\right)^2 \; ; \; i = 1, 2, \cdots N-1 \tag{26}$$

where

$$\theta_y^{i+1} = \theta_y^i + \frac{1}{2}\left(\kappa_y^{i+1} + \kappa_y^i\right)\left(x^{i+1} - x^i\right) \; ; \; i = 1, 2, \cdots N-1 \quad (27)$$

and

$$\theta_z^{i+1} = \theta_z^i + \frac{1}{2}\left(\kappa_z^{i+1} + \kappa_z^i\right)\left(x^{i+1} - x^i\right) \; ; \; i = 1, 2, \cdots N-1 \quad (28)$$

- Boundary conditions on the shear force are

$$T_y^N = 0 \tag{29}$$
$$T_z^N = 0 \tag{30}$$
$$T_y^1 = \sum_i^N (R^i) \tag{31}$$
$$T_z^1 = \sum_i^N (L^i). \tag{32}$$
$$\tag{33}$$

- The boundary conditions on the moments are

$$M_y^N = 0 \tag{34}$$
$$M_z^N = 0 \tag{35}$$
$$M_y^1 = \sum_i^N (L^i)(x^i) \tag{36}$$
$$M_z^1 = \sum_i^N (R^i)(x^i). \tag{37}$$
$$\tag{38}$$

- Assuming a rigid rotor support, the boundary conditions on the displacements are

$$u_y^1 = 0 \tag{39}$$
$$u_z^1 = 0. \tag{40}$$
$$\tag{41}$$