$\rho = 1.225 \ kg/m^3$

AME 40530 Wind Turbine Performance, Control and Design

Spring, 2016

Final Exam

Name: Master

Problem 1. Please answer if the following are Important (I) or Not-important (N) to sizing a wind turbine to produce a specific amount of power at a site.

1. The terrain surface roughness, z_0. I
2. The height of the rotor above the ground. I
3. The time-averaged (mean) wind speed. I
4. The cut-in wind speed. I
5. The rated wind speed. I
6. The cut-out wind speed I
7. The probability of wind speeds between rated and cut-out. I
8. ~~8.~~ The probability of wind speeds between rated and cut-out.
9. The coefficients used in a Weibull wind frequency model. I
10. The diameter of the rotor. I
11. The airfoil types used for the rotor. I
12. The effect of the rotor surface finish. I
13. The coefficient of power of the rotor. I
14. The efficiency of the electric power conversion I

Problem 2. A wind turbine is proposed for the South Bend, IN site. The airport data obtained at an elevation of $z_{ref} = 10$ m., gave a Weibull wind frequency distribution fit with coefficients of $k = 2.0$ and $c = 5.34$ m/s. The proposed wind turbine will have a hub height of 15 m.

1. Based on the Weibull coefficient at the reference 10 m. elevation, determine the new Weibull coefficients for the 15 m. hub height. Assume $n = 0.23$.

2. The rated wind speed for the wind turbine is $V_{rated} = 4$ m/s. What is the probability for the wind to exceed the rated wind speed at the site?

3. If the cut-out wind speed is $V_{cut-out} = 10$ m/s., How many hours in a 24 hour day will the wind turbine produce rated power?

1. Eq 2.17: $K = K_{ref} \frac{[1 - 0.088 \ln(z_{ref}/10)]}{[1 - 0.088 \ln(z/10)]} = \boxed{2.074 = k}$

Eq 2.18 $c = c_{ref} \left[\frac{z}{z_{ref}} \right]^n$; $n = 0.23 \Rightarrow \boxed{c = 5.864}$

$z_{ref} = 10$ m

$k_{ref} = 2.0$

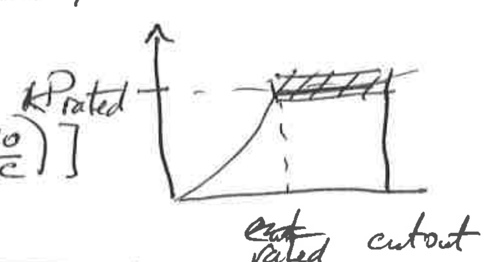
$c_{ref} = 5.34$ m/s

2. Weibull probability: $p(V \geq V_p) = \exp\left[-\left(V_p/c\right)^k\right]$

using $k = 2.074$ and $c = 5.864$

$p(V > 4\text{ m/s}) = \boxed{0.636}$ (63.6%)

3. $V_{cutout} = 10$ m/s. Wind turbine generates rated power between V_{rated} & V_{cutout}.



$p(4 \leq V \leq 10) = \exp\left[-(4/c)^k\right] - \exp\left[-\left(\frac{10}{c}\right)\right]$

$= 0.636 - 0.0485$

Hours $= 24(0.636 - 0.0485) = \boxed{14.10\text{ hrs}}$

Problem 3. For the wind turbine in Problem 2, determine the **minimum rotor diameter** to produce a rated power of 10 kW.

$$P = C_P \frac{1}{2} \rho A V_\infty^3 = C_P \frac{1}{2} \rho \frac{\pi D^2}{4} V_\infty^3$$

$$\therefore D^2 = \frac{8P}{\rho \pi V_\infty^3 C_P}$$

For the minimum rotor size, we want the maximum (Betz) $C_P = 0.593$.

$V_\infty = V_{rated}$ in Problem 2 $= 4\,m/s$.

$$\therefore D^2 = \frac{8 \cdot 10{,}000\,W}{(1.225\,\frac{kg}{m^3})(\pi)(4\,m/s)^3(0.593)} = 549.98\,m^2 \Rightarrow \boxed{D = 23.45\,m}$$

$$\boxed{R = 11.75\,m}$$

Problem 4. If a wind turbine were placed directly down-wind of the wind turbine with the minimum rotor diameter found in Problem 3,

1. What is the velocity of the wind approaching the downstream wind turbine?
2. If the rotor diameter is the same as the upstream wind turbine, what is the maximum power that could be generated by the downstream wind turbine?

(1). From problem (3), the rotor is ideal (Betz), therefore $a = \frac{1}{3}$.

Based on "actuator disk theory", $V_w = V_\infty [1 - 2a] = \frac{1}{3} V_\infty$ (with $a = 1/3$)

$\therefore$ for $V_\infty = V_{rated} = 4\,m/s$, $\boxed{V_w} = \frac{4\,m/s}{3} = \boxed{1.36\,m/s}$

(2) $P = C_P \frac{1}{2} \rho A V_\infty^3$

everything else being the same

$$\frac{P_2}{P_1} = \frac{(V_{\infty 2})^3}{(V_{\infty 1})^3} \Longrightarrow P_2 = P_1 \frac{(1.36\,m/s)^3}{(4.0\,m/s)^3} = \boxed{393.04\,W}$$

($P_1 = 10000\,W$)