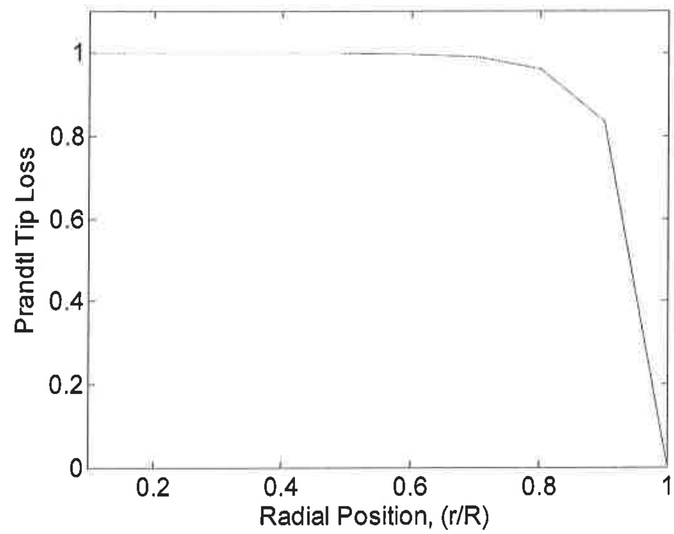
50/50

Melissa Drury
Wind Turbines Project-Part 1
Due: 3/1/16

All figures and values below were found with the Matlab script attached. The code follows the flow chart listed in Figure 4.14 to iteratively solve the BEM equations along a wind turbine rotor.

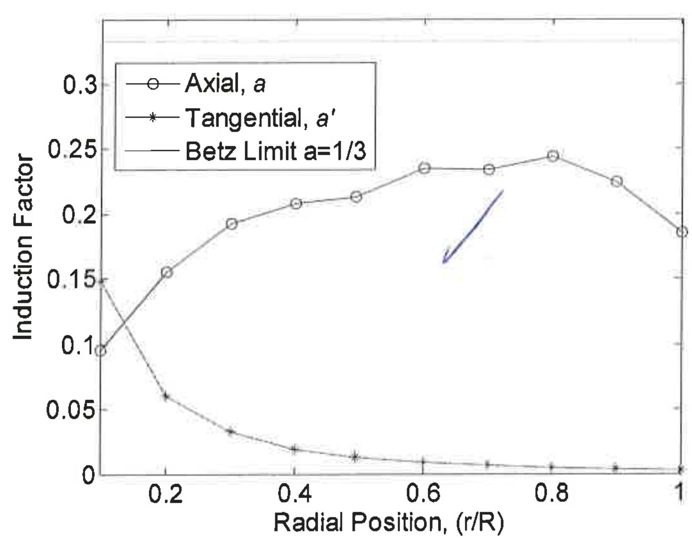
1.

Prandtl Tip Loss



a and a′, less than Betz optimum, a=1/3, as expected

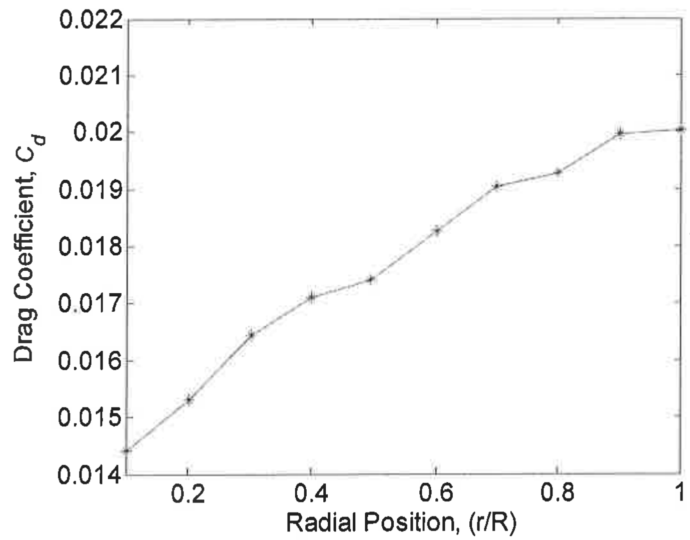
Radial Position, r/R	a	a′
0.1	0.095	0.148
0.2	0.155	0.061
0.3	0.193	0.033
0.4	0.208	0.019
0.5	0.213	0.013
0.6	0.235	0.009
0.7	0.234	0.007
0.8	0.244	0.005
0.9	0.225	0.004
1	0.185	0.003



10/10

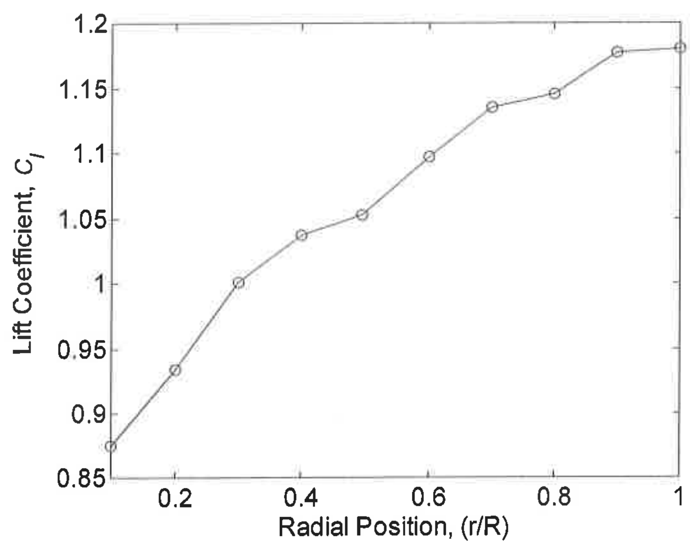
2. Lift and Drag Coefficients

Radial Position, r/R	Drag Coef. C_d
0.1	0.014
0.2	0.015
0.3	0.016
0.4	0.017
0.5	0.017
0.6	0.018
0.7	0.019
0.8	0.019
0.9	0.019
1	0.020



✓ 5/5

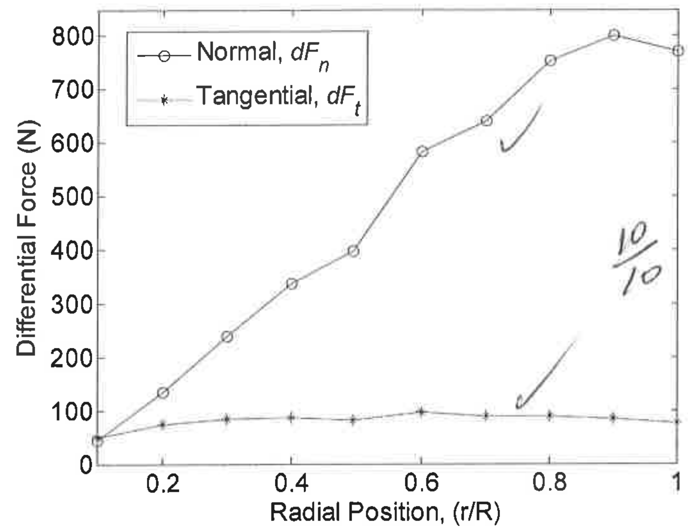
Radial Position, r/R	Lift Coef. C_l
0.1	0.876
0.2	0.934
0.3	1.000
0.4	1.037
0.5	1.053
0.6	1.097
0.7	1.135
0.8	1.146
0.9	1.177
1	1.180



✓ 5/5

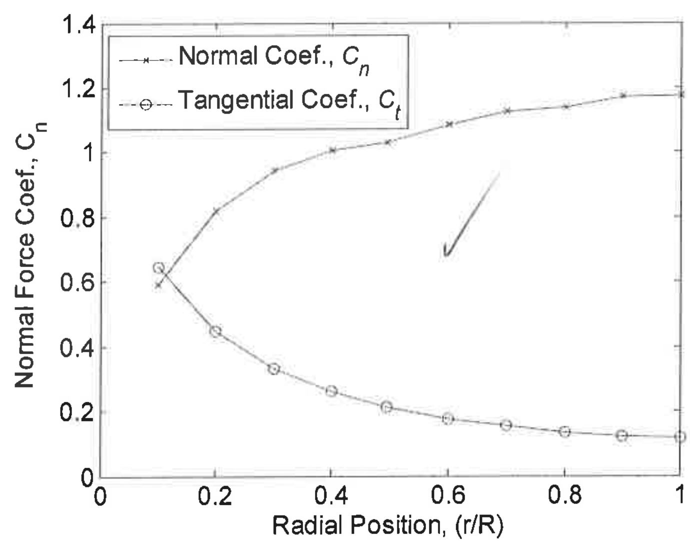
3. Differential normal, dF_n, and differential tangential force, dF_t

Radial Position, r/R	dF_n (N)	dF_t (N)
0.1	43.77	47.68
0.2	134.0	73.28
0.3	238.2	84.26
0.4	336.4	87.37
0.5	398.0	82.62
0.6	583.5	95.41
0.7	639.9	88.58
0.8	752.7	88.31
0.9	801.5	84.54
1	770.7	76.29



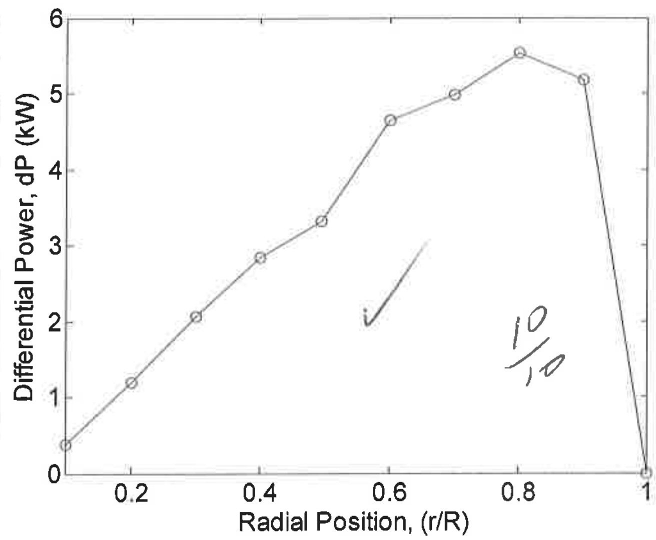
Normal, C_n, and tangential force coefficients, C_t

Radial Position, r/R	C_n	C_t
0.1	0.592	0.645
0.2	0.820	0.448
0.3	0.944	0.334
0.4	1.004	0.261
0.5	1.031	0.214
0.6	1.083	0.177
0.7	1.123	0.156
0.8	1.14	0.134
0.9	1.171	0.124
1	1.175	0.116



4. Differential Power, dP, no power generated at very tip due to losses, as expected

Radial Position, r/R	Differential Power, dP (kW)
0.1	0.388
0.2	1.19
0.3	2.06
0.4	2.84
0.5	3.32
0.6	4.64
0.7	5.0
0.8	5.52
0.9	5.17
1	0



5. Total Power Generated by wind turbine

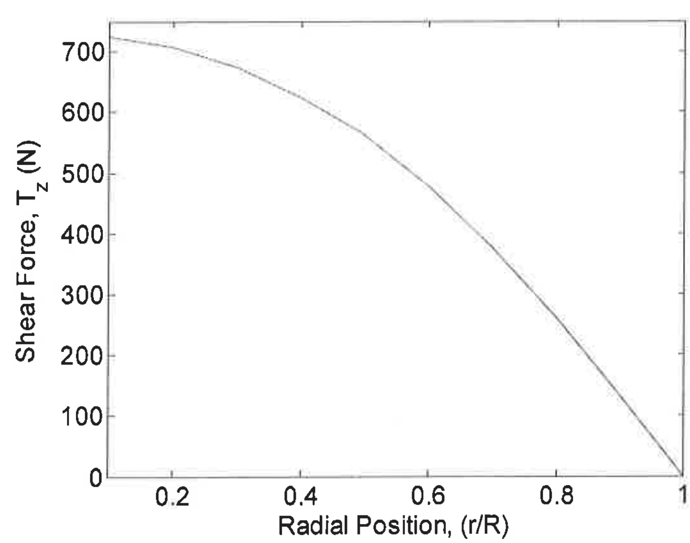
P=90.4 kW

10/10

Part 2

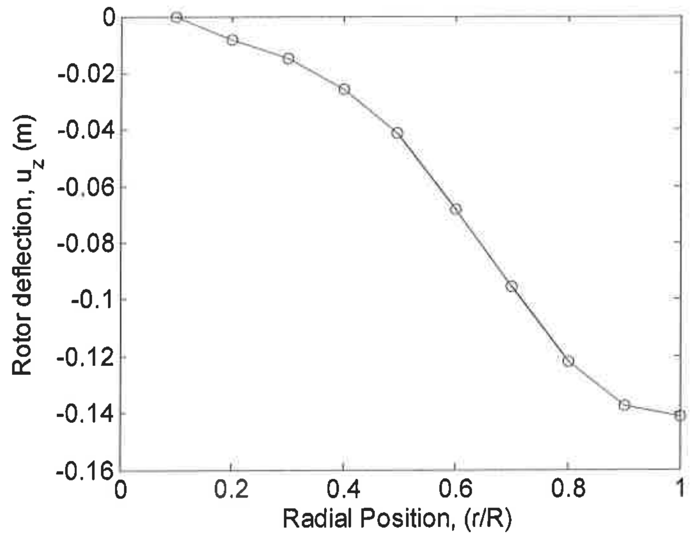
1. Shear Force

Radial Position, r/R	Shear Force, T_z (N)
0.1	724.26
0.2	706.29
0.3	672.85
0.4	623.33
0.5	564.78
0.6	477.48
0.7	375.42
0.8	259.62
0.9	130.37
1	0



2. Rotor Deflection, u_z, assuming the principle bending axis is the airfoil section mean chord line, V=0

Radial Position, r/R	Rotor deflection, u_z (m)
0.1	0
0.2	-0.008
0.3	-0.015
0.4	-0.026
0.5	-0.041
0.6	-0.068
0.7	-0.096
0.8	-0.122
0.9	-0.138
1	-0.141




```
%Melissa Drury
%Wind Turbines
%Project, Part 1
%Due 3/2/16

close;
clear;
clc;

%given info
ro=1.225;%density of air, kg/m^3
B=3;%number of blades
R=4.953; %blade radius, m
lambda=7; %tip-speed-ratio
V=11.62; %rated wind speed, m/s
thetacp=-2;%blade pitch, fixed, degrees
E=9E9;%modulus of elasticity for thin glass/epoxy, Pa
tau=0.15;%thickness to cord

%also given
r=[0.495, 0.991, 1.486, 1.981, 2.447, 2.972, 3.467, 3.962,
4.458, 4.953];%segment of interest, m
c=[0.411, 0.455, 0.384, 0.311, 0.259, 0.223, 0.186, 0.167,
0.137, 0.107];%cord length, m
thetaT=[45, 25.6, 15.7, 10.4, 7.4, 4.5, 2.7, 1.4, 0.4, 0];
%angle of twist, degrees
I1=[280.89, 421.91, 214.04, 92.09, 44.30, 24.34, 11.78, 7.66,
3.46, 1.29]; %moment of inertia in z
```

```
I2=[280.89, 421.91, 214.04, 92.09, 44.30, 24.34, 11.78, 7.66,
3.46, 1.29]; %set moment of inertia in y= Iz, because it was not
given on handout

N=length(r);

%set all variables to zero
phi=zeros(1,N);
alpha=zeros(1,N);
Cl=zeros(1,N);
Cd=zeros(1,N);
Cn=zeros(1,N);
Ct=zeros(1,N);
sigmar=zeros(1,N);
aprimenew=zeros(1,N);
anew=zeros(1,N);
a=zeros(1,N);
aprime=zeros(1,N);
f=zeros(1,N);
F=zeros(1,N);

%Follow Flow Chart, Fig. 14, pg 115

Omega=lambda*V/R;

%calculate dr
for i=2:10
dr(i)=r(i)-r(i-1);
end
dr(1)=r(1);


check1=ones(1,N);
check2=ones(1,N);

for i=1:10
    I1(i)=I1(i)*1E-8;
    I2(i)=I2(i)*1E-8;
    while abs(check1(i)) > 10E-4 || abs(check2(i)) > 10E-4

        phi(i)=atand((V*(1-a(i)))/(Omega*r(i)*(1+aprime(i))));
%angle that the resultant velocity makes with respect to the
plane of rotation
        alpha(i)=phi(i)-(thetaT(i)+thetacp);%local angle of
attack

        %%NACA 4415 airfoil
```

```
        Cl(i)=0.368+0.0942.*alpha(i); %lift coef
        Cd(i)=0.00994+0.000259.*alpha(i)+0.0001055.*alpha(i)^2;
%drag coef
        Cn(i)=Cl(i)*cosd(phi(i))+Cd(i)*sind(phi(i)); %normal
force coef.
        Ct(i)=Cl(i)*sind(phi(i))-Cd(i)*cosd(phi(i));

        sigmar(i)=B.*c(i)/(2*pi.*r(i));

        %accounting for tip loss
        f(i)=(B/2)*((R-r(i))/(r(i)*sind((phi(i)))));
        F(i)=(2/pi)*acos(exp(-f(i)));



aprimenew(i)=1/(((4.*sind(phi(i))*cosd(phi(i)))/(sigmar(i).*Ct(i
)))-1);

anew(i)=1/(((4.*(sind(phi(i))^2))/(sigmar(i).*Cn(i)))+1);

        %check for convergence
        check1(i)=anew(i)-a(i);
        check2(i)=aprimenew(i)-aprime(i);

       %update a, and anew
       a=anew;
       aprime=aprimenew;

   end

    Vr(i)=sqrt((V*(1-a(i)))^2+(Omega*r(i)*(1+aprime(i)))^2);
%resultant velocity, m/s
    dFn(i)=B/2*ro*Vr(i).^2*Cn(i)*c(i)*dr(i);%differential normal
force, N
    dFt(i)=B/2*ro*Vr(i).^2*Ct(i)*c(i)*dr(i);%differential
tangential force, N


    dQ(i)=2*F(i)*aprime(i)*(1-
a(i))*ro*V*Omega*(r(i)^2)*(2*pi*r(i))*dr(i);%differential
torque, Nm
    dP(i)=Omega*dQ(i); % differential power, Watts
    dT(i)=2*F(i)*ro*V^2*a(i)*(1-a(i))*2*pi*r(i)*dr(i);%
differential  thrust, N
    pz(i)=Cl(i)*ro/2*(Vr(i)^2)*c(i)*dr(i);% differential  lift
force generated by each segment(N), eq. 4.46
```

```
    py(i)=Cd(i)*ro/2*(Vr(i)^2)*c(i)*dr(i);% differential  drag
force generated by each segment(N)

end


Tz(N)=0; % shear force in z, N
Ty(N)=0; % shear force in y, N
My(N)=0; % Moment in y
Mz(N)=0; % Moment in z

for i=N:-1:2 % start from the tip and step in
    Tz(i-1)=Tz(i)+0.5*(pz(i-1)+pz(i))*(r(i)-r(i-1));
    Ty(i-1)=Ty(i)+0.5*(py(i-1)+py(i))*(r(i)-r(i-1));
    My(i-1)=My(i)-Tz(i)*(r(i)-r(i-1))-((1/6)*pz(i-
1)+(1/3)*pz(i))*((r(i)-r(i-1))^2);
    Mz(i-1)=Mz(i)-Ty(i)*(r(i)-r(i-1))-((1/6)*py(i-
1)+(1/3)*py(i))*((r(i)-r(i-1))^2);
end

for i=1:N % start from the hub and go out to tip
M1(i)=My(i)*cosd(thetaT(i))-Mz(i)*sind(thetaT(i));
M2(i)=My(i)*sind(thetaT(i))-Mz(i)*cosd(thetaT(i));
k1(i)=M1(i)/(E*I1(i));
k2(i)=M2(i)/(E*I2(i));
kz(i)=-k1(i)*sind(thetaT(i))+k2(i)*cosd(thetaT(i));
ky(i)=k1(i)*cosd(thetaT(i))+k2(i)*sind(thetaT(i));
end

uz(1)=0; % deflection in z
thetaz(1)=0;
for i=1:N-1
    thetaz(i+1)=thetaz(i)+0.5*(kz(i+1)+kz(i))*(r(i+1)-r(i))
    uz(i+1)=uz(i)+thetaz(i)*(r(i+1)-
r(i))+((1/6)*ky(i+1)+(1/3)*ky(i))*(r(i+1)-r(i))^2;
end


    Q=sum(dQ) %total torque
    P=sum(dP) %total power, 1 blade
    T=sum(dT) %total thrust

    WTP=3*P %total power generated by wind turbine

    abetz(1:N)=1/3;

    figure(1)
```

```
    plot(r/R, a, 'o-k')
    hold on;
    plot(r/R, aprime, '*-')
    hold on;
    plot(r/R, abetz)
    axis([0.1 1 0 0.35]);
    set(gca, 'Fontsize', 14)
    xlabel('Radial Position, (r/R)')
    ylabel('Induction Factor')
    legend('Axial, \it{a}', 'Tangential, \it{a''}', 'Betz Limit
a=1/3')


    figure(2)
    plot(r/R, Cl, 'o-k')
    set(gca, 'Fontsize', 14)
    axis([0.1 1 0.85 1.2]);
    xlabel('Radial Position, (r/R)')
    ylabel('Lift Coefficient, \it{C_l}')

    figure(3)
    plot(r/R, Cd, '*-')
    set(gca, 'Fontsize', 14)
    xlabel('Radial Position, (r/R)')
    ylabel('Drag Coefficient, \it{C_d}')



    figure(4)
    plot(r/R, dFn, 'o-k')
    axis([0.1 1 0 850]);
    hold on;
    plot(r/R, dFt, '*-')
    set(gca, 'Fontsize', 14)
    xlabel('Radial Position, (r/R)')
    ylabel('Differential Force (N)')
    legend('Normal, \it{dF_n}', 'Tangential, \it{dF_t}')

    figure(5)
    plot(r/R, dP/1000, 'o-k')
    axis([0.1 1 0 6]);
    set(gca, 'Fontsize', 14)
    xlabel('Radial Position, (r/R)')
    ylabel('Differential Power, dP (kW)')


    figure(6)
```

```
plot(r/R, uz, 'o-k')
axis([0.1 1 -0.16 0]);
set(gca, 'Fontsize', 14)
xlabel('Radial Position, (r/R)')
ylabel('Rotor deflection, u_z (m)')

figure(7)
plot(r/R, F)
axis([0.1 1 0 1.1]);
set(gca, 'Fontsize', 14)
xlabel('Radial Position, (r/R)')
ylabel('Prandtl Tip Loss')

figure(8)
plot(r/R, Tz)
axis([0.1 1 0 750]);
set(gca, 'Fontsize', 14)
xlabel('Radial Position, (r/R)')
```