

**M20550 Calculus III Tutorial
Worksheet 9**

1. Compute $\iint_R \frac{1}{2} dA$ where R is the region bounded by $2x^2 + 2xy + y^2 = 8$ using the change of variables given by $x = u + v$ and $y = -2v$.
2. Let R be the parallelogram enclosed by the lines $x + 3y = 0$, $x + 3y = 2$, $x + y = 1$, and $x + y = 4$. Evaluate the following integral by making appropriate change of variables

$$\iint_R \frac{x + 3y}{(x + y)^2} dA.$$

3. Evaluate the line integral $\int_C (z - 2xy) ds$ along the curve C given by $\mathbf{r}(t) = \langle \sin t, \cos t, t \rangle$, $0 \leq t \leq \frac{\pi}{2}$.
4. Find $\int_C 2xy^3 ds$ where C is the upper half of the circle $x^2 + y^2 = 4$.
5. Calculate the line integral $\int_C (y^2 + x) dx + 4xy dy$ where C is the arc of $x = y^2$ from $(1, 1)$ to $(4, 2)$.
6. Evaluate the line integral $\int_C z^2 dx + x dy + y dz$ where C is the line segment from $(1, 0, 0)$ to $(4, 1, 2)$.
7. Compute $\int_C x^2 ds$ where C is the intersection of the surface $x^2 + y^2 + z^2 = 4$ and the plane $z = \sqrt{3}$.