

**M20550 Calculus III Tutorial
Worksheet 6**

1. Write an equation of the tangent line to the curve of intersection between the two surfaces defined by $z = x^2 + y^2$ and $x^2 + 2y^2 + z^2 = 7$ at the point $(-1, 1, 2)$.
Hint: Think about the geometry of the gradient vectors. You don't have to parametrize the curve to do this problem.
2. Find the tangent plane and the normal line to the surface $x^2y + xz^2 = 2y^2z$ at the point $P = (1, 1, 1)$.
3. Find a point on the surface $z = x^2 - y^3$ where the tangent plane is parallel to the plane $x + 3y + z = 0$.
4. Find all the critical points of $f(x, y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2$.
5. Find the local maximum and the local minimum value(s) and saddle point(s) of the function $z = x^3 + y^3 - 3xy + 1$.
6. Identify the absolute maximum and absolute minimum values attained by $g(x, y) = x^2y - 2x^2$ within the triangle T bounded by the points $P(0, 0)$, $Q(2, 0)$, and $R(0, 4)$.
7. Identify the absolute maximum and absolute minimum values attained by $z = 4x^2 - y^2 + 1$ within the region R bounded by the curve $4x^2 + y^2 = 16$.
8. Find the point(s) on the surface $y^2 = 9 + xz$ that are closest to the origin.