Name: Date: 09/29/2016

M20550 Calculus III Tutorial
Worksheet 5

1. Find $\dfrac{dz}{dt}$ when $t = 2$, where $z = x^2 + y^2 - 2xy$, $x = \ln(t-1)$ and $y = e^{-t}$.

2. (a) Let $f(x, y, z) = x^2 - yz$. If $\mathbf{v} = \langle 1, 1, 0\rangle$, find the directional derivative of f in the direction of $\mathbf{v}$ at the point $(1, 2, 3)$.

   (b) Interpret your result in part (a) by filling in the blanks and circling the correct word of the statement below:

   At the point ____________, the value of the function f is *increasing* / *decreasing* at the rate of ____________ as we move in the direction given by the vector ____________.

3. Let $f(x, y) = \ln(xy)$. Find the maximum rate of change of f at $(1, 2)$ and the direction in which it occurs.

4. If $h = x^2 + y^2 + z^2$ and $y \cos z + z \cos x = 0$, find $\dfrac{\partial h}{\partial x}$ assuming that x and y are the independent variables.

5. A cylinder containing an incompressible fluid is being squeezed from both ends. If the length of the cylinder is *decreasing* at a rate of 3m/s, calculate the rate at which the radius is changing when the radius is 2m and the length is 1m. (Note: An incompressible fluid is a fluid whose volume does not change.)

6. Let $r = r(x, y)$, $x = x(s, t)$, and $y = y(t)$. Given that

$$\begin{array}{lll} x(1,0) = 2, & x_s(1,0) = -1, & x_t(1,0) = 7, \\ y(0) = 3, & y(1) = 0 & y'(0) = 4, \\ r(2,3) = -1, & r_x(2,3) = 3, & r_y(2,3) = 5, \\ r_x(1,0) = 6, & r_y(1,0) = -2, & \end{array}$$

   calculate $\dfrac{\partial r}{\partial t}$ at $s = 1, t = 0$.

7. Suppose that over a certain region of space the electrical potential V is given by

$$V(x, y, z) = 5x^2 - 3xy + xyz.$$

   (a) Find the rate of change of the potential at $P(1, 1, 0)$ in the direction of the vector $\mathbf{v} = \mathbf{i} + \mathbf{j} - \mathbf{k}$.

   (b) In which direction does V decrease most rapidly at P?

   (c) What is the maximum rate of change at P?

8. Find **<u>all</u>** points at which the direction of fastest change of the function $f(x, y) = x^2 + y^2 - 2x - 4y$ is $\mathbf{i} + \mathbf{j}$.

9. (a) Find an equation for the tangent line (in vector or parametric form) at the point $(2, 2, 1)$ to the curve of intersection of the two surfaces $g(x, y, z) = 2x^2+2y^2+z^2 = 17$ and $h(x, y, z) = x^2 + y^2 - 3z^2 = 5$.

   (b) Suppose $f(x, y, z)$ is a function with $\nabla f = \langle 1, 0, 0 \rangle$ at the point $(2, 2, 1)$. Starting at $(2, 2, 1)$, which direction should one travel along the curve of intersection in order to increase f? (*Note: You can give a tangent vector to the curve at* $(2, 2, 1)$ *that points in the desired direction.*)