Homework Assignment # 8, due March 28

1. Let $H\colon X\times I\to Y$ be a homotopy between the maps $f\colon X\to Y$ and $g\colon X\to Y$. Let $f_\#, g_\#\colon C_*(X)\to C_*(Y)$ be the chain maps induced by f, g on the singular chain complexes. Show that the *prism operator* $P\colon C_n(X)\to C_{n+1}(Y)$ defined by

$$P(\sigma) := \sum_{i=0}^{n}(-1)^i H\circ(\sigma\times \mathbb{1})\circ[v_0,\dots,v_i,w_i,\dots,w_n]$$

is a chain homotopy from $f_\#$ to $g_\#$, i.e.,

$$\partial P + P\partial = g_\# - f_\#$$

2. Suppose that

$$A \xrightarrow{f} B \longrightarrow C \longrightarrow D \xrightarrow{g} E$$

is an exact sequence of abelian groups. Show that it gives rise to a short exact sequence

$$0\longrightarrow \operatorname{coker} f \longrightarrow C \longrightarrow \ker g \longrightarrow 0$$

3. Let

$$0\longrightarrow A \xrightarrow{f} B \xrightarrow{g} C \longrightarrow 0$$

be a short exact sequence of abelian groups. We say that it is *split exact* (or that *the sequence splits*) if there is a homomorphism $s\colon C\to B$ which is a right-inverse to g in the sense that $gs = \mathbb{1}$. Show that if the sequence splits, then B is isomorphic to $A\oplus C$.

4. Show that if X is a finite CW complex, then its suspension ΣX has the structure of a finite CW complex (this is true without the finiteness assumption, but this avoids pointset topology issues). If C_* is a chain complex, define its *suspension* ΣC_* in such a way that the cellular chain complex $C_*^{CW}(\Sigma X)$ is isomorphic to the suspension of the celluar chain complex $C_*^{CW}(X)$.