Homework Assignment # 3, due Feb. 6

1. Show that the singular chain complex of a topological space X is in fact a chain complex, i.e., that $\partial_q \circ \partial_{q+1} = 0$, where $\partial_q \colon C_q(X) \to C_{q-1}(X)$ is the boundary map.

2. Calculate the singular homology groups of the one point space.

3. Let X be a topological space with path components X_α, $\alpha \in A$. Show that $H_q(X)$ is isomorphic to $\bigoplus_{\alpha\in A} H_q(X_\alpha)$.

4. a) Show that the Hurewicz map $h\colon \pi_1(X, x_0) \to H_1(X)$ given by $[\gamma] \mapsto [[\gamma]]$ is a homomorphism.
b) Let $\Psi\colon C_1(X)/B_1(X) \to \pi_1(X, x_0)^{ab}$ be the map defined by $[[\gamma]] \mapsto [\lambda_{\gamma(0)}\gamma\lambda_{\gamma(1)}]$ for any singular 1-simplex γ (as in class we have chosen for every point $x \in X$ a path λ_x from the basepoint x_0 to x). Show that the restriction of Ψ to $H_1(X) \subset C_1(X)/B_1(X)$ provides an inverse to the map $\bar{h}\colon \pi_1^{ab}(X, x_0) \to H_1(X)$.