Homework Assignment # 1

1. Show that for any pointed topological space (X, x_0), the n-homotopy group $\pi_n(X, x_0)$ is a group. More precisely, this involves showing the following:

associativity: Let f, g, and h be maps from $(I^n, \partial I^n)$ to (X, x_0) and let $f + g\colon (I^n, \partial I^n) \to (X, x_0)$ be defined by

$$(f+g)(t_1, \ldots, t_n) = \begin{cases} f(2t_1, t_2, \ldots, t_n) & 1 \le t_1 \le \frac{1}{2} \\ g(2t_1 - 1, t_2, \ldots, t_n) & \frac{1}{2} \le t_1 \le 1 \end{cases}$$

Then we need to show that $f+(g+h)$ is homotopic to $(f+g)+h$.

unit property: If $c\colon (I^n, \partial I^n) \to (X, x_0)$ is the constant map, we need to show $f + c \sim f$ and $c + f \sim f$.

inverse property: If $\bar{f}\colon (I^n, \partial I^n) \to (X, x_0)$ is given by $\bar{f}(t_1, \ldots, t_n) = f(1 - t_1, t_2, \ldots, t_n)$, we need to show $\bar{f} + f \sim c$ and $f + \bar{f} \sim c$.