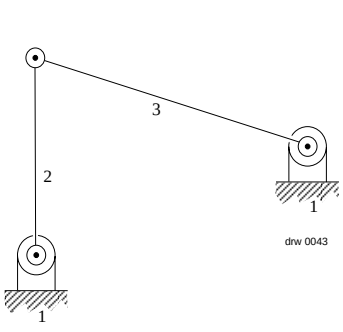
AME 339 - Kinematics and Dynamics of Machinery

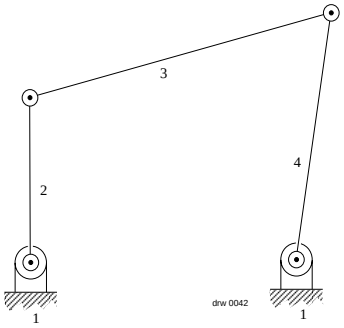
Grüebler's Criterion

Grüebler's Criterion calculates the theoretical number of degrees-of-freedom within a mechanism. This is also known as the mechanism's F number.

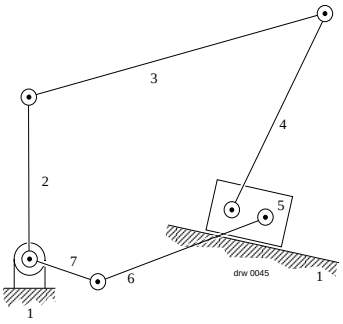
Definition: *degrees-of-freedom* - the number of independent joint variables which must be specified in order to define the position of all links within a mechanism. Examples,



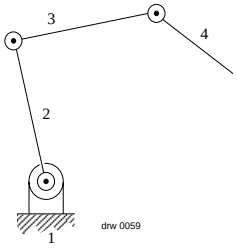
a truss (F=0)



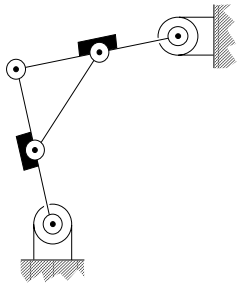
a four bar (F=1)



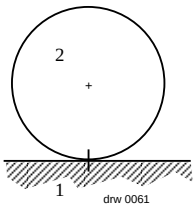
a seven bar (F=2)



planar robot (F=3)



indeterminate structure (F=-1)



rolling disc (F=1)

Figure 1

Definition: *planar motion* - a motion where all points on all links move on parallel planes (that plane is known as the plane of motion) and all bodies rotate about an axis which is perpendicular to the plane of motion.

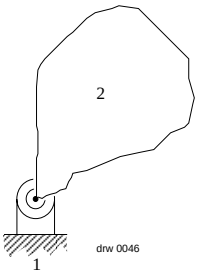
A body restricted to planar motion has at most three degrees-of-freedom.

Definition: *link* - a rigid body

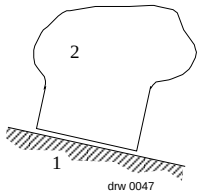
Definition: *joint* - a contact (or permanent connection) between two links. The type of joint (or connection) defines the relative motion of the two connected links.

There are two categories of contacts in planar motion, they are P_1 contacts and P_2 contacts, known as such because the joints (contacts) belonging to each of these two categories allow for one and two degrees-of-freedom respectively between the connected bodies. (A P_3 contact would be a non-contact!)

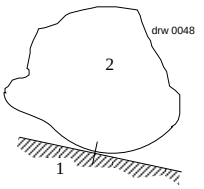
The three known types of P_1 joints (contacts) are:



a pin joint



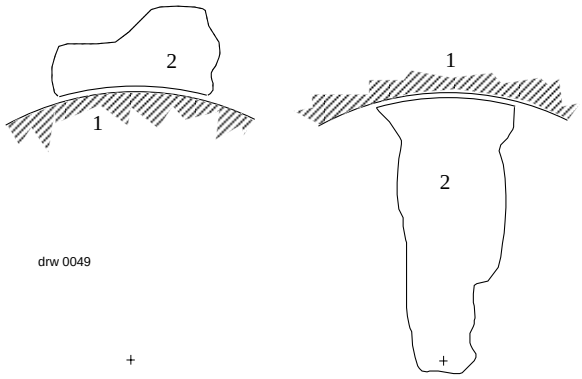
a sliding joint



a rolling joint

Figure 2

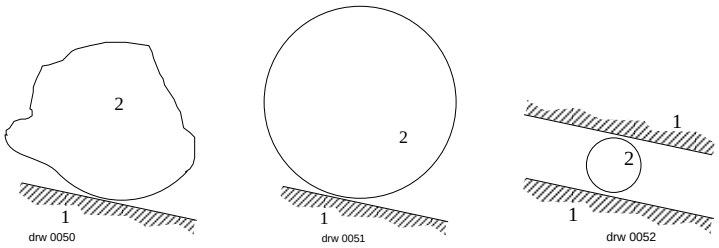
A rolling contact will always be indicated by a hashmark, as above (you will see why soon). There is a variation in the sliding joint where the bodies appear to rotate relative to each other, as in a pin joint.



Variations of the Sliding Joint
Figure 3

The above P_1 contacts allow only one relative degree-of-freedom between the contact bodies. Thus they eliminate 2 degrees-of-freedom of relative movement.

The only known P_2 contact is referred to as a slipping joint. It has several possible forms some of which are shown below. Note these are all the same slipping joint.



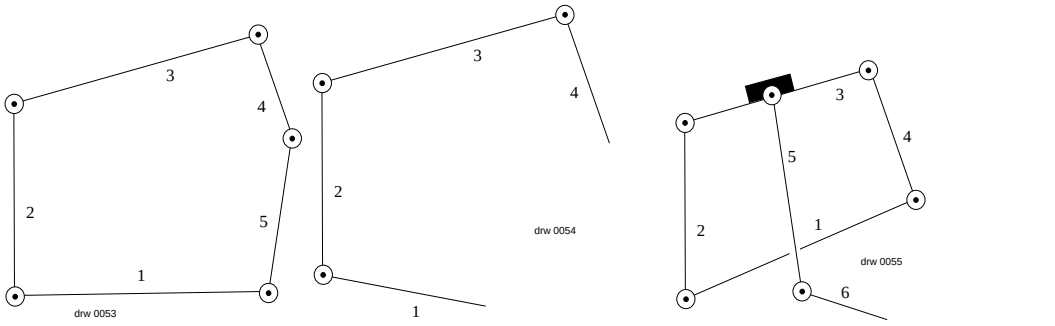
slipping joints

Figure 3

Note that the hashmark is not used here. The hashmark is our way of communicating to each other whether a contact is rolling or slipping. Otherwise, there would be no means of distinguishing them in such drawing.

The P_2 contact allows two relative degrees of freedom between the contacting bodies. Thus, they eliminate 1 degree-of-freedom of relative movement.

Definition: *kinematic chain* - a system of links connected by joints



a closed chain an open chain mixed open/closed chain

Figure 4

Definition: *mechanism* - a kinematic chain which has one link fixed to ground. E.g. the three kinematic chains in Figure 4 can be used as the following mechanisms.

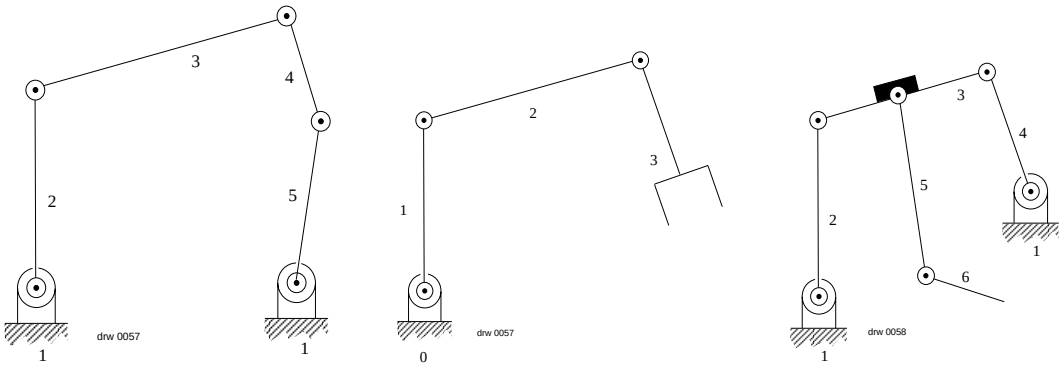


Figure 5

Grüebler's Criterion: From what we have seen in this discussion, we can now calculate the number of degrees-of-freedom belonging to a mechanism.

$$F = 3(N-1) - 2P_1 - P_2$$

where P_1 and P_2 are the number of P_1 and P_2 contacts in the mechanism and N is the total number of links in the mechanism.

It is important to realize that the F number is a *theoretical* result, and Grübler's Criterion can be easily fooled. The *actual* number of degrees-of-freedom in a mechanism is known as the Mobility, or M number and it can only come by inspection.

Definition *Mobility* - the actual number of degrees-of-freedom in a mechanism.

Working Definition: *Mobility* - the minimum number of single d.o.f. joints which must be removed from a mechanism in order to reduce it to a determinate structure.

Finally, what is a machine?

Definition: *machine* - a mechanism used to transmit power from a source (or from sources) to a load (or loads).