AME 339 - Kinematics and Dynamics of Machinery

Grashoff's Criterion

The four bar linkage, shown below, is a basic mechanism which is quite common. Further, the vast majority of planar one degree-of-freedom (dof) mechanisms have "equivalent" four bar mechanisms. Hence the study of the four bar mechanism is important. The four bar has two rotating links ("levers") which have fixed pivots,

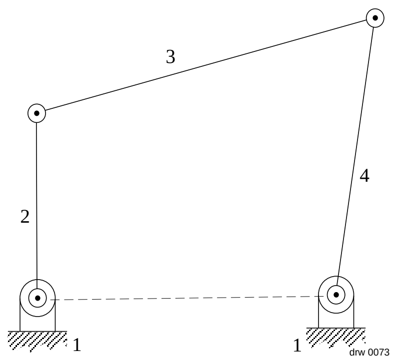

Figure 1: A Four Bar Linkage

(bodies 2 and 4 above). One of the levers would be an input rotation, while the other would be the output rotation. The two levers have their fixed pivots with the "ground link"(body 1) and are connected by the "coupler link" (body 3).

Definition: *crank*- a ground pivoted link which is continuously rotatable.

Definition: *rocker*- a ground pivoted link that is only capable of oscillating between two limit positions and cannot rotate continuously.

Cranks are convenient as inputs, since most drivers (engines, motors, servos, actuators, etc.) provide a continuous rotation about a fixed axis. Clearly, the dimension (length) of each link will effect the relative motion of all the links. Let a, b, c and d represent the length of each of the four links in a four bar kinematic chain as shown below.

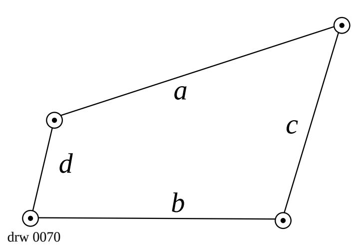

Figure 2: A Four Bar Kinematic Chain

Grashoff's Criterion will allow us to determine whether the levers of a particular four bar mechanism are cranks or rockers.

Grashoff's Criterion:

When the sum of the lengths of the longest and shortest links is *less than* the sum of the lengths of the intermediate pair of links, the four bar kinematic chain is a *Class I chain*. In all Class I four bar kinematic chains, the shortest link is capable of making a complete revolution relative to each of the three longer links. The three longer links can only oscillate relative to each other. Class I four bar kinematic chains can result in three different types of mechanisms, depending only upon the location of the fully rotatable (shortest) link, as illustrated in Figures 3 $(b - e)$.

When the sum of the lengths of the longest and shortest links is *greater than* the sum of the lengths of the intermediate pair of links, the four bar kinematic chain is a *Class II chain*. In all Class II four bar kinematic chains, no link is capable of making a complete revolution relative to any of the other three remaining links. All four links can only oscillate relative to each other. Class II four bar kinematic chains *always* result in mechanisms with rocker inputs and outputs, since no link is capable of a complete revolution relative to any other link. Figures 4 $(b - e)$ illustrate the rocker-rocker mechanisms which come from a Class II four bar kinematic chain.

Finally, there is a "borderline" case, where the sum of the lengths of the longest and shortest links equals the sum of the lengths of the intermediate links. The mechanisms which come from this anomaly are not sure what they want to be. The chain can be flattened, or folded onto itself, so that all the links are overlapping. Such mechanisms are typically avoided, however they do appear in situations as parallelogram four bar mechanisms, or isosceles four bar mechanisms, for example, as shown in Figures 5 and 6 respectively. As with anything, they have their advantages and disadvantages. Be careful with these borderline cases.

Kinematic Inversions

The mechanisms in Figures 3(b-e) are *kinematic inversions* of each other. This means that they are all derived from the *same kinematic chain* and they differ from one another only in which of the links in the kinematic chain is fixed to ground, or made the ground link (a.k.a. frame). Likewise, the mechanisms in Figures 4(b-d) are kinematic inversions of each other.

Kinematic inversions exist for *any* kinematic chain, not just for the four bar chain. Clearly, the number of kinematic inversions for any particular kinematic chain is equal to the number of links in the chain. The *relative* motion of the links in a set of kinematic inversions is the same, however, the *absolute* motion of the links varies, since they are referenced to a different fixed body in each case.

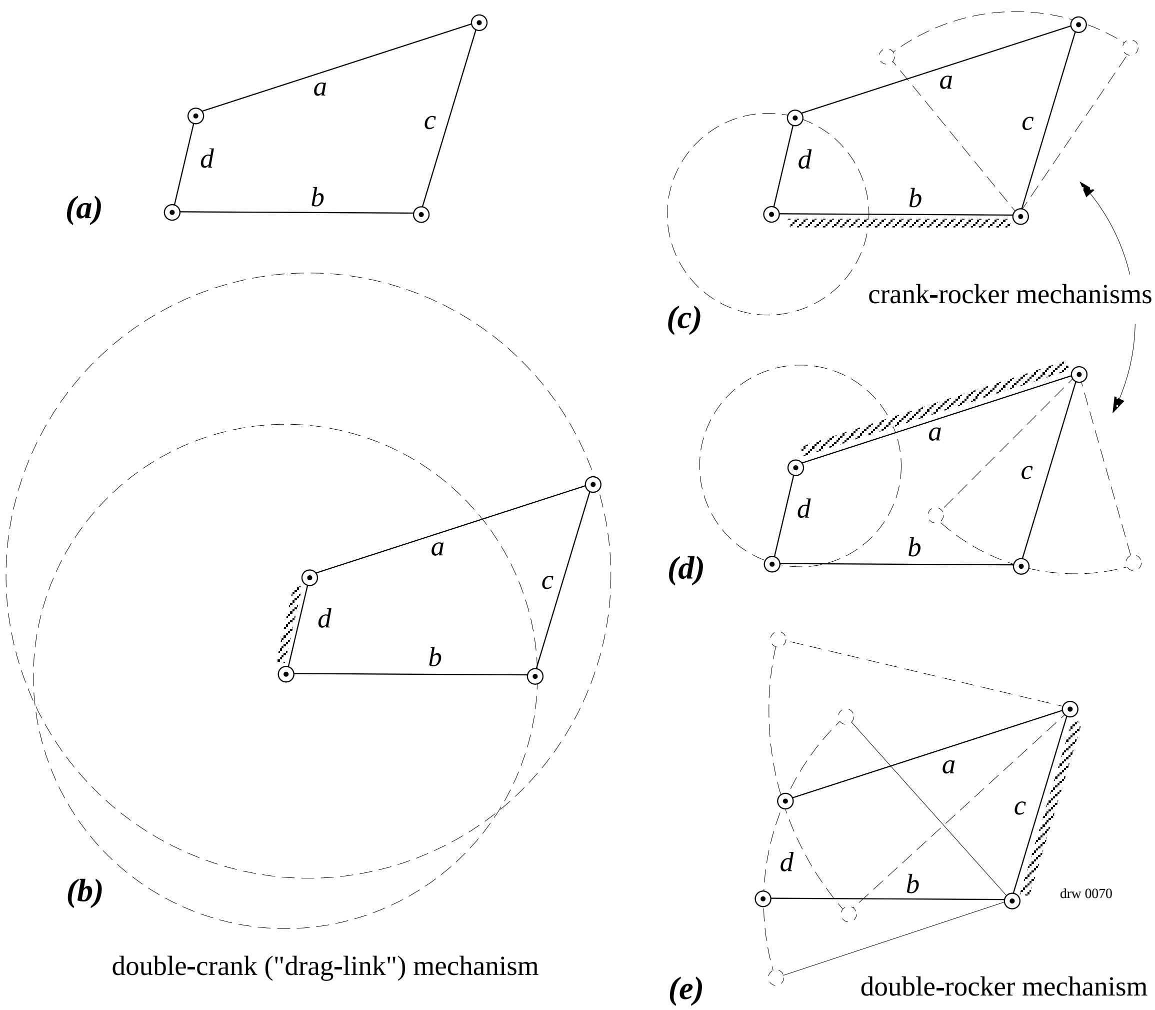

Figure 3: Four Bar Mechanisms from a Class I Kinematic Chain

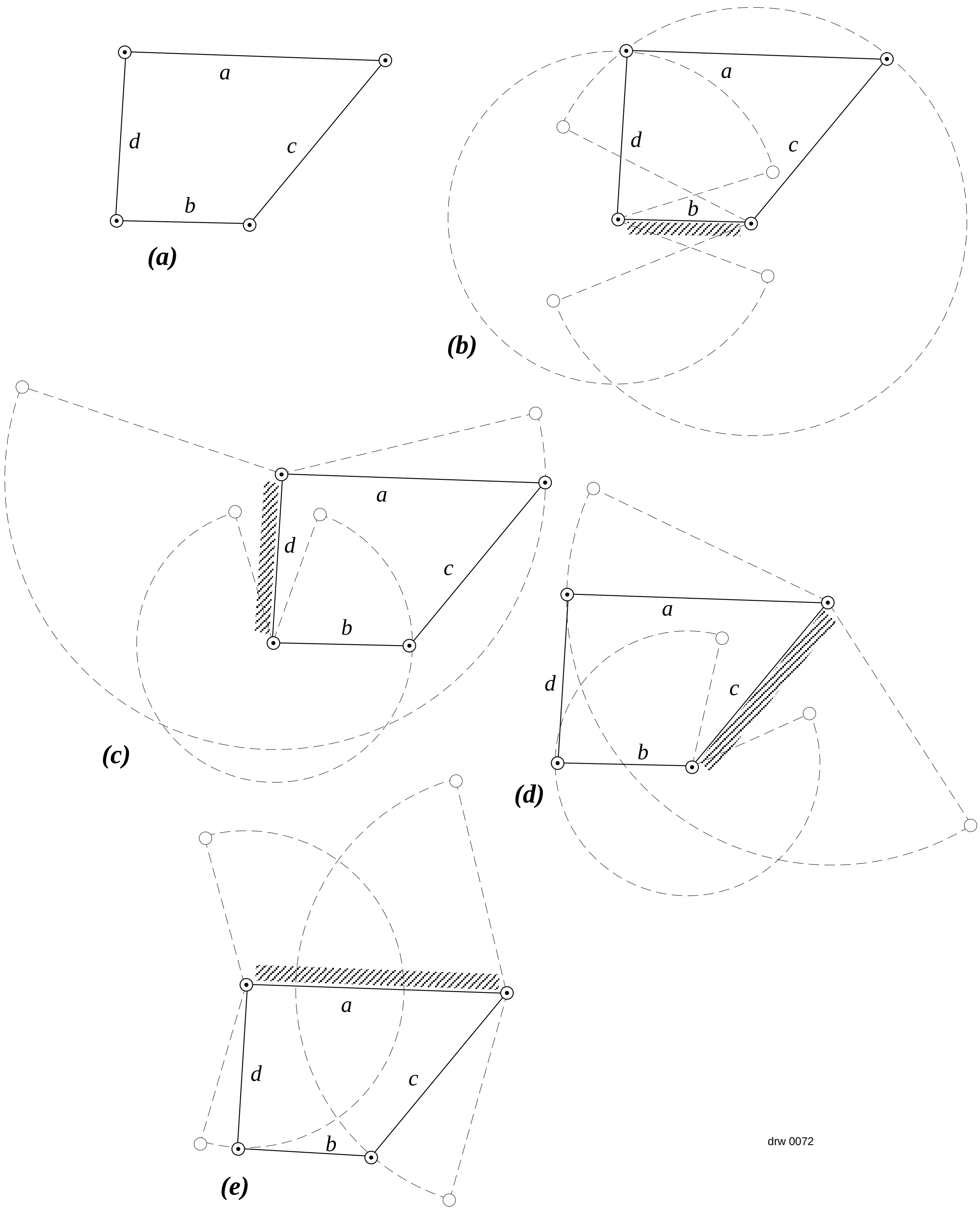

Figure 4: Four Bar Mechanisms from a Class II Kinematic Chain

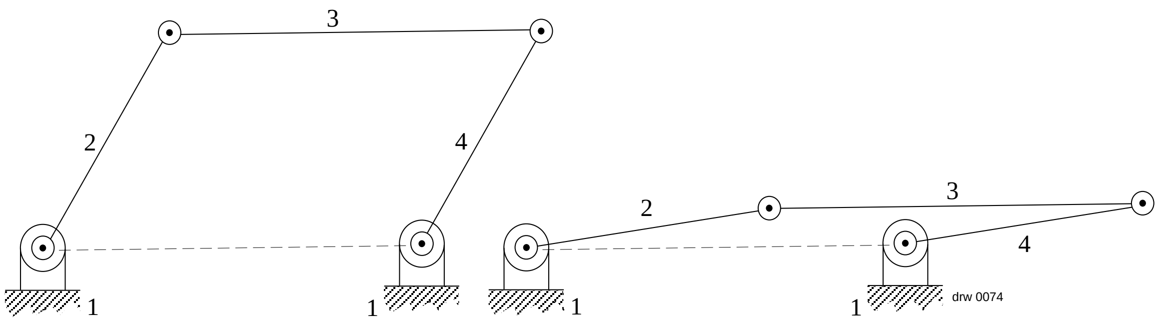

Figure 5: Example of Parallelogram Four Bar Mechanism (Borderline Case)

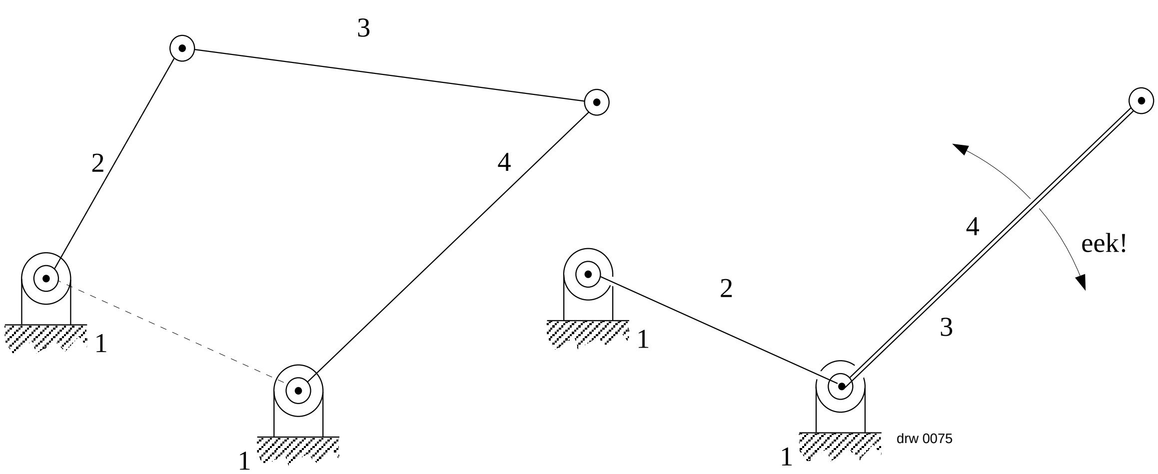

Figure 6: Example of an Isosceles Four Bar Mechanism (Borderline Case)