Cooling + Humidification Operations

Definitions

1- Humidity Y

It is the mass of vapor carried by a unit mass of vapor free gas. Thus the humidity depends on it's partial pressure of the vapor in the gas-vapor mixture when the total pressure is constant, 1 atm.

$$Y = \frac{M_A \bar{P}_A}{M_B (1 - \bar{P}_A)}$$

where M is molecular wt

$A \Rightarrow$ liquid (water vapor)

$B \Rightarrow$ Gas (dry gas)

2- Saturated gas

It is the gas in which the vapor is in equilibrium with the Liquid at gas temperature.

By using Dalton's Law, $\bar{P}_A$ partial pressure of vapor in Saturated gas is equal to the vapor pressure P of Liquid at gas temp.

$$Y_s \text{ (saturated humidity)} = \frac{M_A P_A}{M_B (1 - P_A)}$$

3- Relative humidity: Y_R

It is the ratio of the partial pressure of vapor to the vapor pressure of the Liquid at gas temp.

$$Y_R = 100 \cdot \frac{\bar{P}_A}{P_A}$$

4- Percentage humidity Y_A

It is the ratio of the humidity to the Saturated gas humidity at gas temperature.

$$Y_A = 100\frac{Y}{Y_s} = Y_R\,\frac{1-P_A}{1-P_A}$$

5- Humid heat C_s

It is the amount of heat needs to increase the temp. of 1 gm of gas-vapor mixture 1 °C

$$C_s = C_{p_B} + C_{p_A}\cdot Y$$

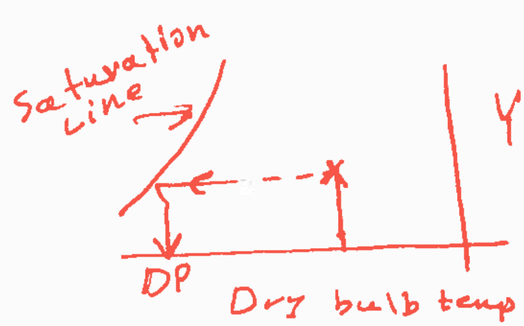
where C_p ⇒ Specific heats of gas & vapor

6- Humid volume V_H

It is the volume of unit mass of dry gas, plus the vapor content, at 1 atm and gas temp.

7- Dew Point

It is the temperature to which the vapor gas mixture should cool down at constant humidity to be come saturated.



Some useful information to the Cooling Tower

1- Energy transfer to water by the pump = 0.1 KW and it has been added to the cooling Load by the Software.

2- Water Capacity of the system = 3.0 L [excluding the make-up tank]

3- The Column dimentions = $0.15 \times 0.15 \times 0.6\ m^3$ and it contains 4 Sections

4- Number of decks in each section (2)

total decks number = $2 \times 4 = 8$

Total number of plates = $18 \times 8 = 144$

5- Total surface area of the packing = $2.16\ m^2$

6- Packing hight = 0.48 m

7- Packing Volume = $0.15 \times 0.15 \times 0.48 = 0.0108\ m^3$

8- Packing density $= \left(\dfrac{\text{total surface area}}{\text{volume}}\right)_{\text{packing}} = 200\ m^{-1}$

9- The Tower is Computer Linked and designed to simulate the operational characteristic of modren air-water evaporating Cooling System.

10- The Cooling Tower unit is an open system through which two streams of fluid flow (air-Water) Counter Current and in which there are mass and heat transfer from one stream to each other.

11- This unit has much same Configuration as a full size forced draugh Cooling tower.

12- This unit allows multiple configurations of air and water flowrates and heating Load.

Useful info --- Continue

1- Cooling range $= (T_2 - T_1)_{water}$

2- makeup tank

Provides a fresh water to the system (distilled water only) due to the evaporation losses.

3- Drift

Droplets of water are entrained by the air stream leaving the tower (causing som error in the data)

4- Approch to Wet-Bulb Temperature

is the difference between the temperature of water leaving the tower and inlet air wet-bulb temp. $\doteq T_1 - t_{(w-b)1}$

In the cooling tower

5- Heat transfer by:

1- Radiation ⇒ Very small (neglected)

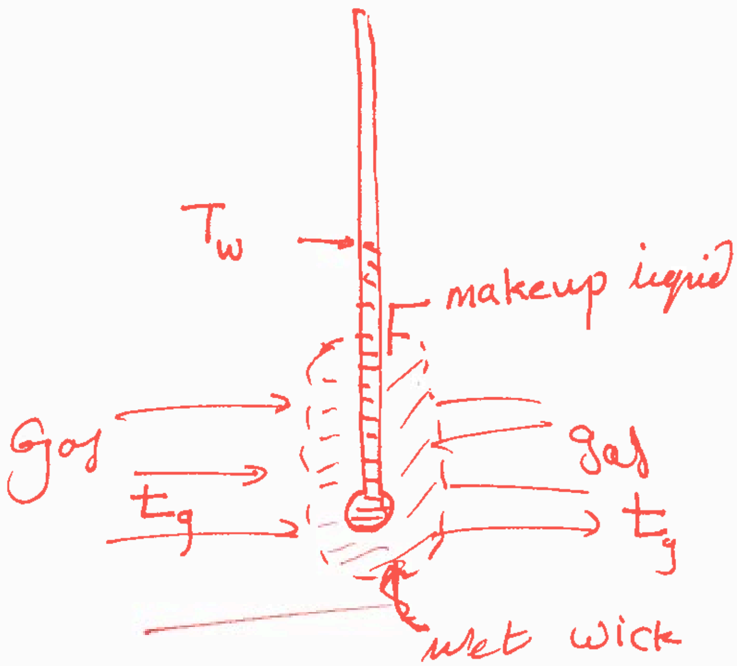
2- Conduction } These are controlled by Δt and a (contact area)
3- Convection

4- Evaporation

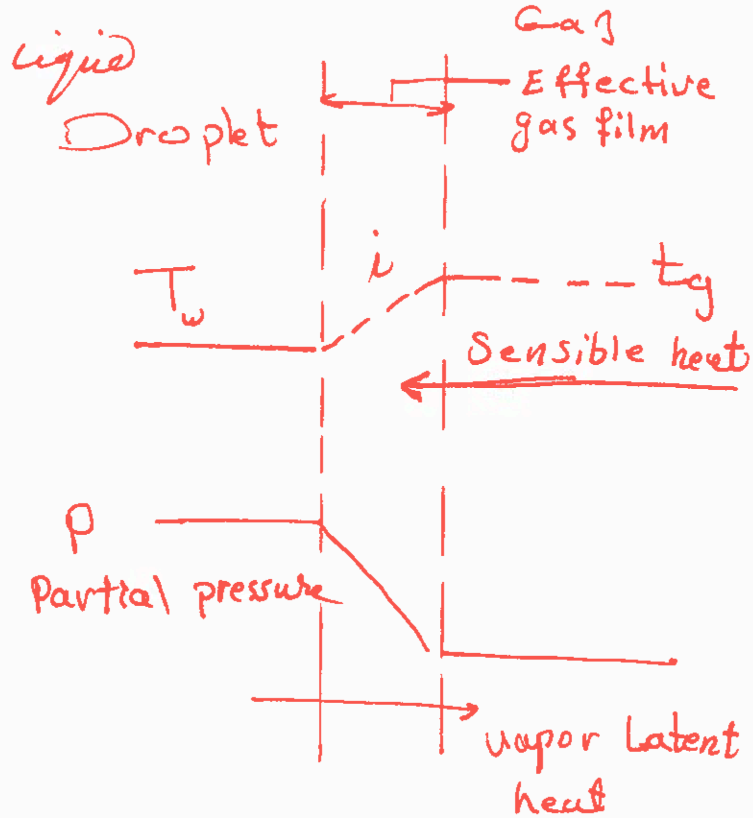
In the cooling tower this part is the most effective.

Wet-Bulb Temperature Theory

W-B Temp is the Steady-state of temp. reach by a small amount of liquid evaporates into large amount of Unsaturated vapor-gas mixture.



Wet - bulb Thermometer



When the Liquid temperature reach below the dry-bulb temp. tg of the gas, heat will transfer from gas to the Liquid at increasing rate depending on $\Delta T \Rightarrow (t_g - T_w)$.

In fact this rate of heat transfer is equal to the rate of heat removed from the Liquid by evaporation and the temperature of the Liquid will remain constant at some Low value.

AT this temp [wet-bulb temp], the rate of

1- heat transfer is q_s (sensible heat)

$$q_s = N_A M_A \left(\lambda_w + C_p (t_g - T_w) \right) \quad --- \ (1)$$

where N_A molal rate of evaporation

λ_w latent heat of liquid at wet-bulb temp.

q_s can be also expressed in terms of

(a) area of contact, heat transfer coefficient (h_g) and temp.

$$q_s = h_g \cdot a \, (t - t_i) \quad --- \ (2)$$

where

$t_i \Rightarrow$ gas-liquid interface temp.

2- In the second hand the rate of mass-transfer may expressed in term of mass transfer (k_y')

and the driving force in mole fraction of vapor (y)

$$N_A = \frac{K'_y}{\overline{(1-y)}} \cdot (y_i - y) \cdot a \quad --- \quad (3)$$

where $\overline{(1-y)}$ one way diffusion factor and

y_i is mole fraction of vapor in saturated gas at T_w where T_w in fact equal to t_i or Δt at water bulk $= 0$

K'_y = mass transfer coeff. = kgmole/s m²

(a) Replace the mole fraction (y) in equation (3) by the humidity (Y)

∴ y_i will be coresponding to Y_s (saturation humidity at wet-bulb temp)

(b) Combine equations (2 & 3)

$$h_g\ (t - T_w) = \frac{K'_y}{\overline{(1-y)}_L}\left[\left(\frac{Y_s}{1/M_B + Y_s/M_A}\right) - \left(\frac{Y}{1/M_B - \cancel{Y}/M_A}\right)\right]\left[\lambda_w + C_{P_A}\ (t - T_w)\right] \quad ---- \quad (4)$$

Equation (4) may simplified with out a big error:

① $\overline{(1-y)}_L \Longrightarrow 1$

② $C_{p_A}(t - T_w)$ very small compared with latent heat λ_w

③ Both terms $\frac{Y_s}{M_A}$ & $\frac{Y}{M_A}$ are very small in comparison with $1/M_B$

∴ Eq. (4) will be reduce to

$$h_g\,(t - T_w) = M_B\,k_y'\,\lambda_w\,(Y_s - Y) \quad \text{---} ⑤$$

re arrange Eq. ⑤

$$\boxed{\frac{Y - Y_s}{t - T_w} = -\frac{h_g}{M_B\,k_y'\,\lambda_w}} \quad \text{----} ⑥$$

Note: For given T_w (wet-bulb temp) both λ_w & Y_s are fixed.

Consequently the relation between gas temp t and -9 humidity Y depends on the ratio of $\frac{h_g}{M_B K'_y}$

Reynolds and Schmidt numbers

If we go back to the relation of heat transfer by Convection and Conduction between two streams of fluid and it's dependence on Rynolds number (Re) $\frac{\rho D v}{\mu}$ or $\left(\frac{D \mathring{m}}{\mu}\right)$ and Prandtl number P_r $\frac{C_p \mu}{k}$

And mass transfer Coefficient relation to (Re) and Schmidt number S_c $\frac{\mu}{\rho D_e}$

For Same boundry Layer and turbulant flow

$$\frac{h_g}{C_p \cdot \mathring{m}_a} = b\, R_e^{n}\, P_r^{-C} \quad --- \quad (7)$$

$$\frac{\bar{M}_B K'_y}{\mathring{M}_a} = b\, R_e^{n}\, S_c^{-C} \quad --- \quad (8)$$

$\mathring{m}_a$ = mall velocity of air (in your case)
M = molecular (aurage) wt. of gas (air)

b, n, & C constants

Substitute Eqs (7 + 8) into (6)

$$\frac{Y - Y_s}{t - T_w} = -\frac{h_g}{M_B K'_y \lambda_w} = -\frac{C_p}{\lambda_w}\left[\frac{S_c}{P_r}\right]^m \quad \text{---} (9)$$

where $m \simeq 2/3$ for air - water system

Lewis + Psychrometric Line relation

For water - air system

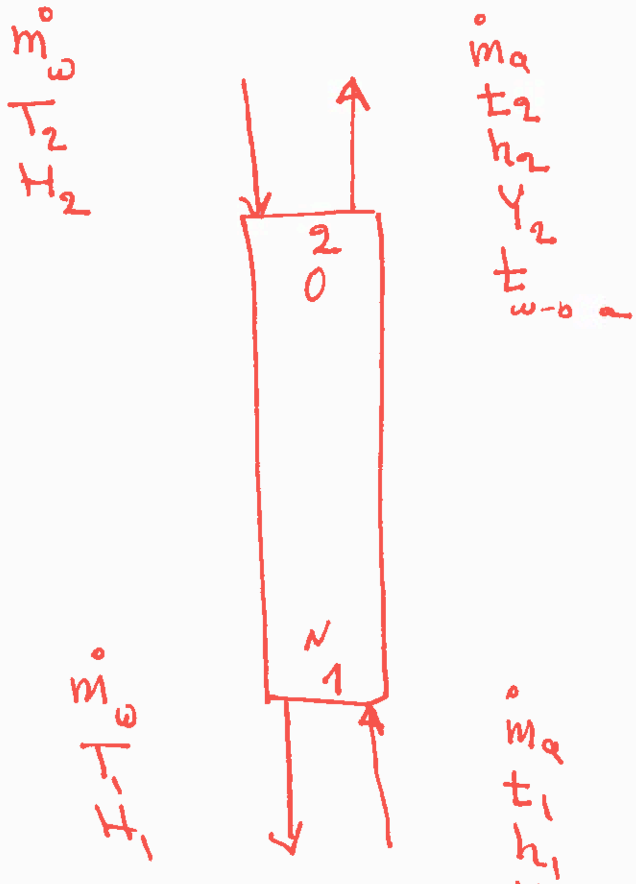
$$\boxed{\frac{h_g}{M_B K'_y} = C_s = C_{p_B} + C_{p_A} \cdot Y \quad \text{---} (10)}$$

This relation holds, the Psychrometric line and the adiabatic - Saturation Line become essantialy the same.

Cooling Tower [Air – Water]

For a Countercurrent packed Tower and adiabatic condition:

Fig 1 ⟹



Where

$\dot{m}_a$ = [air] mass velocity kg /(s m²)

t = air temp

h = Enthalpy J/kg dry air

Y = absolute humidity kg vapor/kg dry air

$\dot{m}_w$ = [water] mass velocity kg/(s m²)

T = Water temp.

H = Enthalpy of water J/kg dry air

Take mass balance over a differential section

* mass velocity of the air is so large, it will remain constant compared with the humidity gained

$$\dot{m}_w - \dot{m}_{w1} = \dot{m}_a (Y - Y_1) \quad --- \; (1)$$

$$d\dot{m}_w = \dot{m}_a \, dY \quad ---- \; (2)$$

Take Enthalpy balance

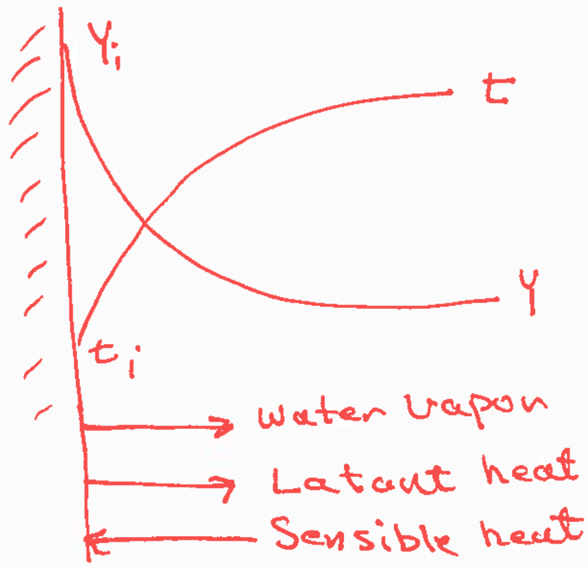
$$\dot{m}_w \cdot H + \dot{m}_a h_1 = \dot{m}_{w1} H_1 + \dot{m}_a h \quad --- \; (3)$$

* The Latent heat (λ_w) of water is so Large that only a small amount of evaporation produce Large Cooling effect, and the rate ~~thus~~ of mass transfer is usually small

∴ $\dot{m}_w$ can be assumed Constant

$$\dot{m}_w (H - H_1) = \dot{m}_a (h_1 - h) \quad ---- \; (4)$$

Fig 2 ⇒



* Sensible heat can be ignored [very small compared with Latent heat] without consequence error.

Then eq. 4 can be rewritten in term of heat balance

$$\overset{\circ}{m}_w \, c_p \cdot dT = \underbrace{\overset{\circ}{m}_a \, c_p \, dt + \overset{\circ}{m}_a \, \lambda_w \cdot dY}_{\overset{\circ}{m}_a \, \Delta h} \quad \text{--- (5)}$$

For entire Column equation 5 becoms

$$\boxed{\dot{m}_w \, c_p \,(T_2 - T_1) = \dot{m}_a \,(h_2 - h_1) \quad \text{---} \ (6)}$$

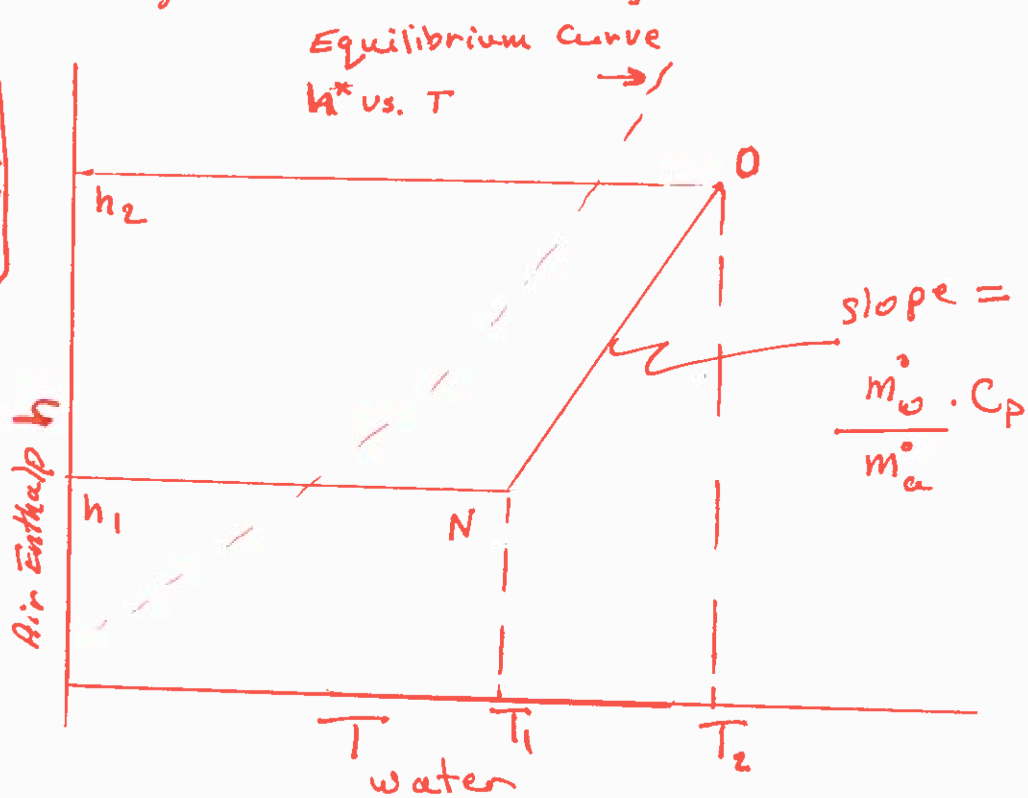
Equation 6 may be represented graphically by ploting air enthalpies versus water Temp. see Fig 3

where the line ON represents equation 6 and it is the operating line (straight) of the cooling tower

h^* : enthalpy of a saturated vapor-gas mixture in equilibrium with bulk liquid Temp.

see the powerpoint

Fig 3



Mass balance in term of mass transfer coefficient

The right side of equation (2) can be then :-

$$\dot{m}_a \cdot dY = k_g \cdot a \left(Y_i - Y \right) dz \quad --- (7)$$

where k_y = mass transfer coefficient
$= kg\, S^{-1} m^{-2}$

a = interface area per unit volume of packing m^{-1}

Similarly Heat balance in term of heat transfer coefficient.

$$\dot{m}_a \, C_{p_{air}} \cdot dt = h_g \cdot a \left[t_i - t \right] \quad --- (8)$$

where h_g = heat transfer Coeff. W/m^2 in the gas Side

and

$$\dot{m}_w \, C_{p_{water}} \, dT = h_L \, a \left(T - t_i \right) dz \quad --- (9)$$

h_L = heat transfer coeff in liquid side if we use equ. (7 + 8)

$\dot{m}_a \cdot dh$ becoms

$$\dot{m}_a \cdot dh = h_g \cdot a \, (t_i - t) dz + \lambda_w K_y a (Y_i - Y) dz \quad \text{---- (10)}$$

Now if you take

$$r = \frac{h_g \cdot}{C_{p_{air}} K_y} = r$$

then eq. 10 becom

$$\dot{m}_a \cdot dh = K_y \cdot a \left(\frac{h}{K_y \cdot a} (t_i - t) + \lambda_w (Y_i - Y) \right) dz$$

$$= K_y \cdot a \left[r C_{p_{air}} (t_i - t) + \lambda_w (Y_i - Y) \right] dz$$

$$\dot{m}_a \, dh = K_y \cdot a \left[\left(r C_{p_{air}} t_i + \lambda_w Y_i \right) - \left(r C_{p_{air}} t + \lambda_w Y \right) \right] dz \quad \text{--- (11)}$$

But for air-water system and continuous irrigated tower

$$r = 1 = \frac{h_g}{k_y \cdot C_{p_{air}}}$$

$\therefore$ the eq 11 reduce to

$$\dot{m}_a\, dh = k_y \cdot a\left[\left(\underbrace{c_p t_i + \lambda_w y_i}_{h_i}\right) - \left(\underbrace{c_p t + \lambda_w y}_{h}\right)\right] dz$$

$$\therefore\ \dot{m}_a\, dh = k_y \cdot a\ (h_i - h)\, dz \quad --- (12)$$

and $\dot{m}_a \cdot dh = \dot{m}_w\, c_{p_L}\, dT = h_y a (T - t_i) dz$

--- (12a)

$$\frac{dh}{(h_i - h)} = \frac{k_y \cdot a}{\dot{m}_a} dz \quad --- (13)$$

take integral over entire Tower

$$\int_{h_1}^{h_2} \frac{dh}{h_i - h} = \frac{k_y \cdot a}{\dot{m}_a} \int_0^{Z} dZ = \frac{k_y a Z}{\dot{m}_a}$$

--- (14)

and equ. 14 becomes

$$\frac{h_2 - h_1}{(h_i - h)_{av.}} = N_{tG}$$ Number of gas Enthalpy transfer units

$$\boxed{N_{tG} = \frac{k_y . aZ}{m^o_a}} \quad --- \quad 15$$

N_{tG} Could be evaluated graphically

$$\boxed{Z = H_{tG} \times N_{tG}} \quad --- \quad 16$$

where H_{tG} is the height of gas enthalpy

$\boxed{\text{transfer units} = \dot{m}_a / k_y \cdot a = H_{tG}}$ (17)

where H_{tG} is a way to measure packing performance

The over all driving force in the Cooling tower

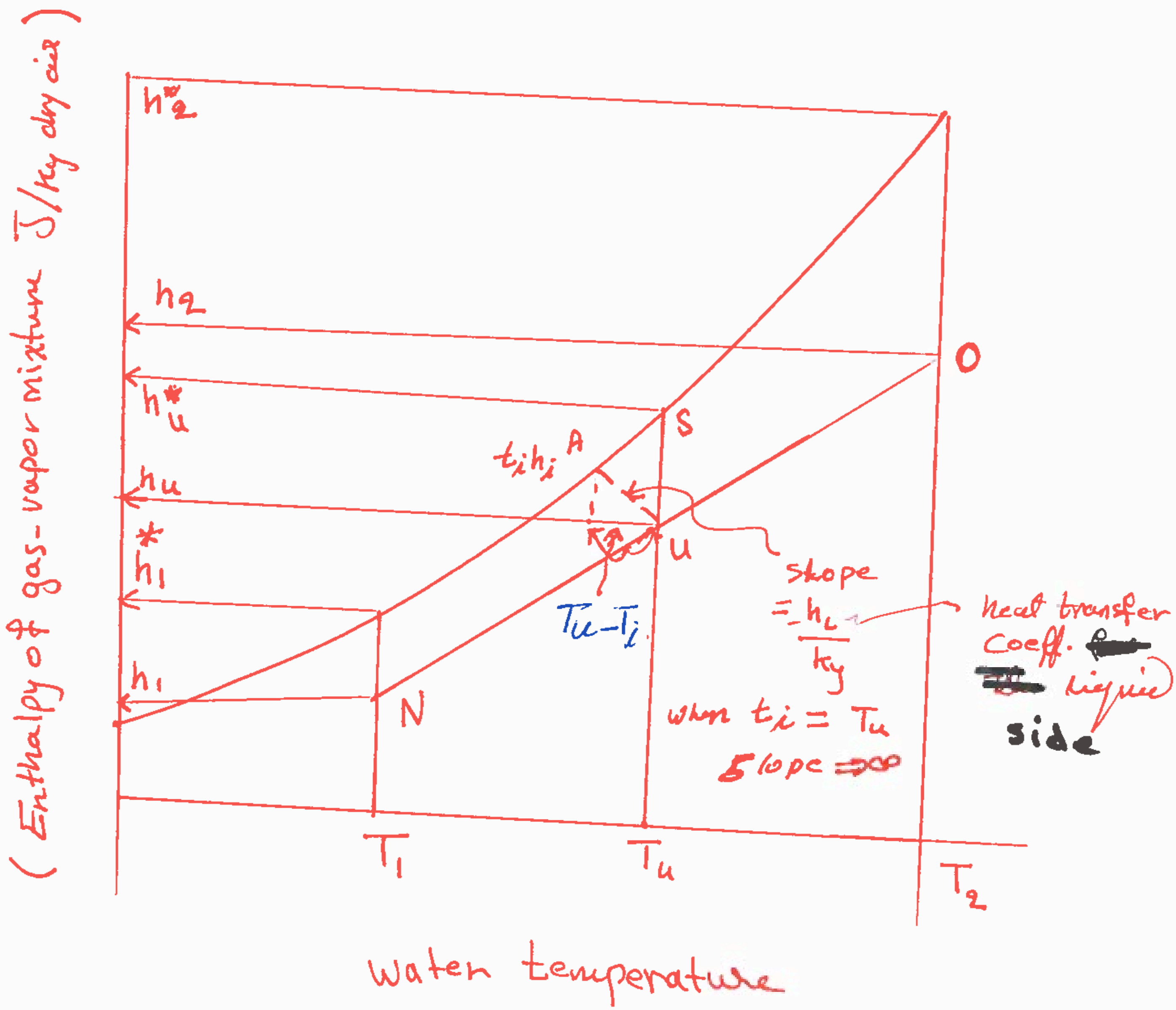
the over all driving force representing the Enthalpy difference for the bulk phases such as the vertical distance SU " where U is any point on the operating Line for example

$$T_U = T_3 = \frac{T_1 + T_2}{2} \qquad \text{See Fig.4}$$

and this will Lead to over all numbers and heights of transfer units

$$\boxed{N_{toG} = \int_{h_1}^{h_2} \frac{dh}{h^* - h} = \frac{K_y a Z}{m^\circ_a} = \frac{Z}{H_{toG}}} \quad \text{-- 18}$$

In industry for water Cooling tower they use equation 18 and liquid term



$$T_u = \frac{T_1 + T_2}{2}$$

Fig. 4 Driving Force Diagram

$$\frac{K_y \cdot a Z}{m^\circ_w} \int_{T_1}^{T_2} \frac{dT}{h^* - h} \quad --- (19)$$

Which results from combining eq (18) and (12a) or (6)

$$\boxed{\frac{K_y \cdot a Z}{m^\circ_w} = \frac{C_{p_w}(T_2 - T_1)}{\Delta h_{av.}}} \quad * \quad (20)$$

Take

$$\Delta h = (h^* - h)_{at} \quad T_m = \frac{T_1 + T_2}{2}$$

See Fig 8

Then the Characteristic equation of the Cooling tower can be obtained by ploting

$$\frac{K_y a Z}{m^\circ_w} \quad \text{vs} \quad \frac{\dot{m}_w}{\dot{m}_a}$$

$$\boxed{\frac{K_y\, a\, Z}{\dot{m}_w} = \beta \left[\frac{\dot{m}_w}{\dot{m}_a}\right]^n} \quad * \qquad \text{---} \quad 22$$

Where β & n are empirical constants specific for particular tower design.

However if we multiply equ. 22 by $\left(\frac{\dot{m}_w}{\dot{m}_a}\right)$

then will have

$$\boxed{\frac{K_y\, a\, Z}{\dot{m}_a} = \beta \left[\frac{\dot{m}_w}{\dot{m}_a}\right]^{n+1} = N_{tOG}}$$

See equ. (18)

* Over all Cooling tower effectiveness ε

ε = is the ratio of the actual energy transfer to the maximum posible energy transfer

$$\varepsilon = \frac{h_2 - h_1}{h_2^* - h_1}$$

see fig 4

Murphree gas-phase stage efficiency

$$E_{MG} = \frac{Y_1 - Y_2}{Y_{as} - Y_1} = 1 - \frac{Y_{as} - Y_2}{Y_{as} - Y_1} = 1 - e^{-K_y a Z / \dot{M}_a}$$

$$E_{MG} = 1 - e^{-N_{tG}}$$

this is spacial case when inlet water temp. equal to the adiapatic - Saturation temperature of inlet gas (t_{as}).

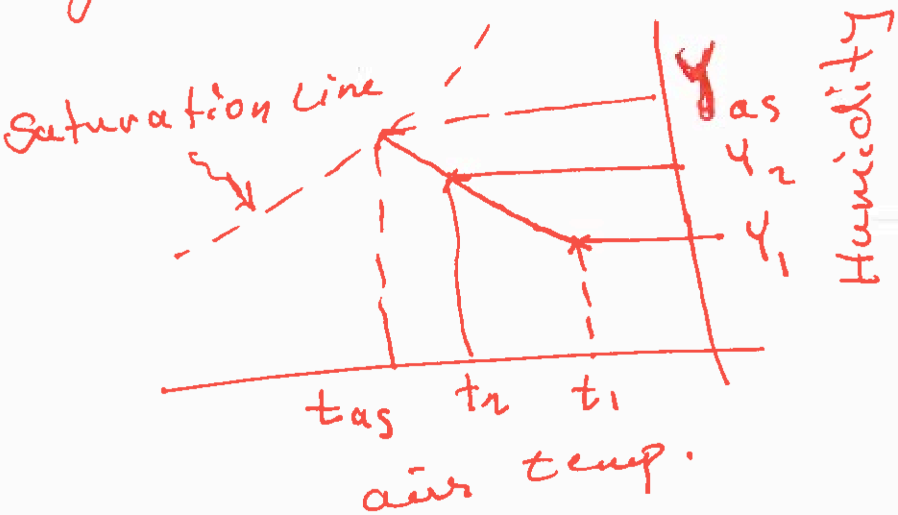
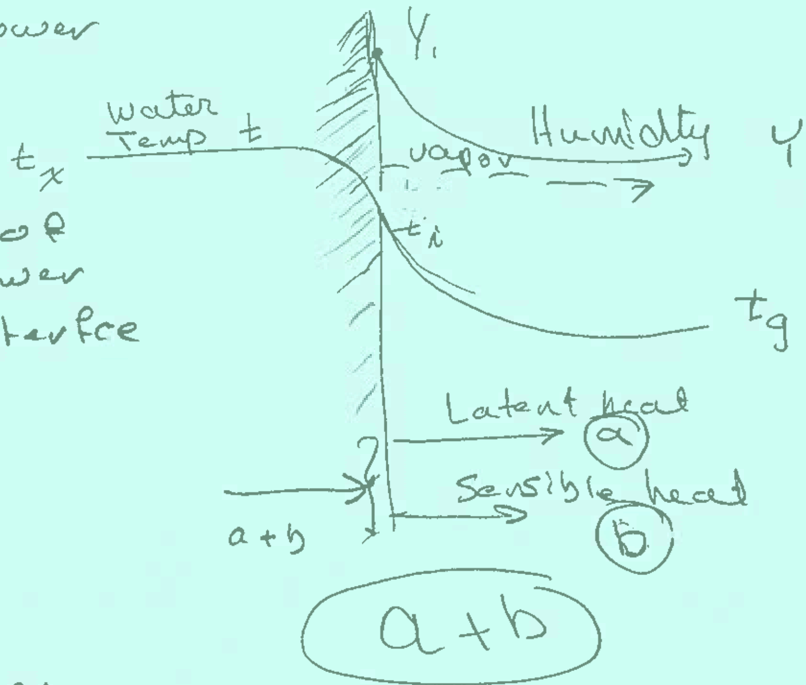


Fig 5

Conditions in Cooling Tower

Top of tower



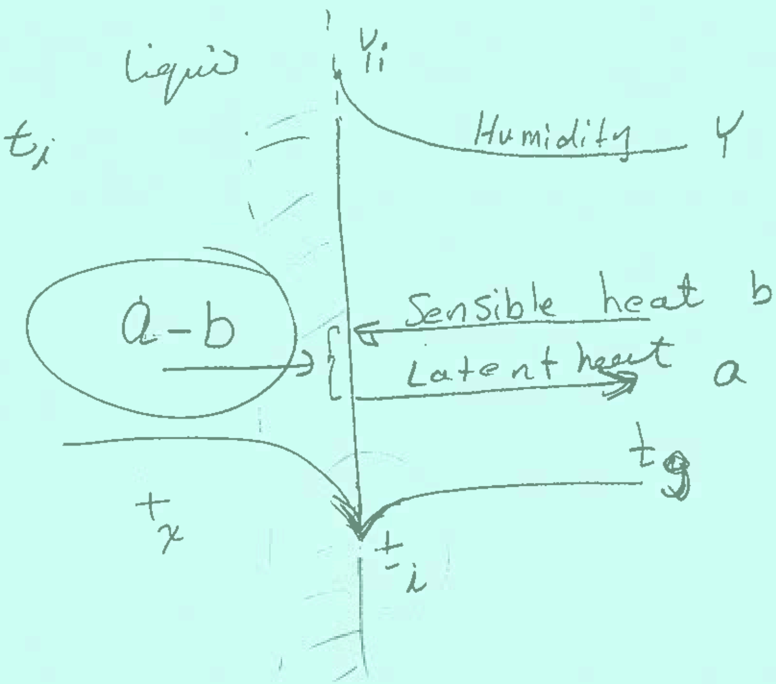
Temperature of the gas lower than the interface

$t_i > t_g$

Bottom of tower

$t_i < t_g$

t_g grater than t_i



$\gamma = (h^{*} - h)$

$\gamma_m = (h^{*}_m - h_m)$