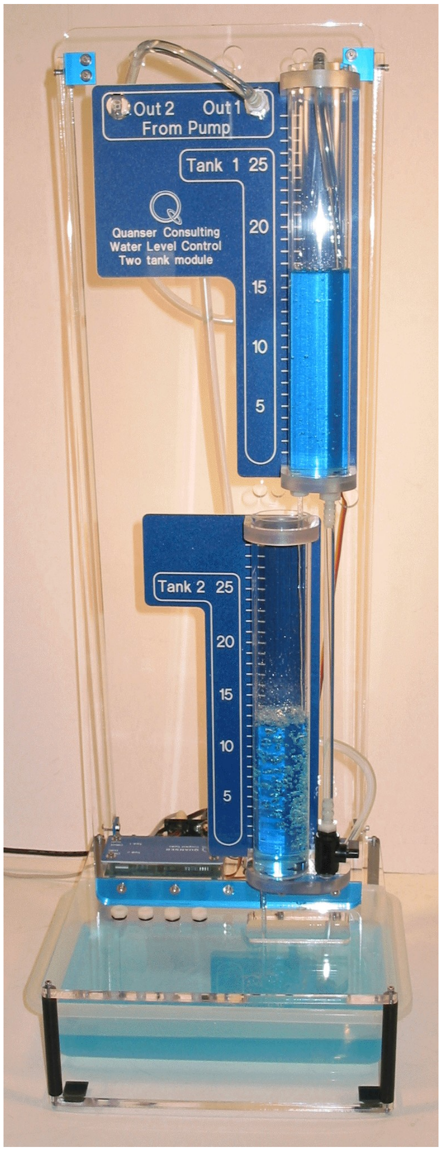
Specialty Plants

Specialty Experiment: PI-plus-Feedforward Water Level Control

Coupled Water Tanks



Student Handout

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1. Objectives

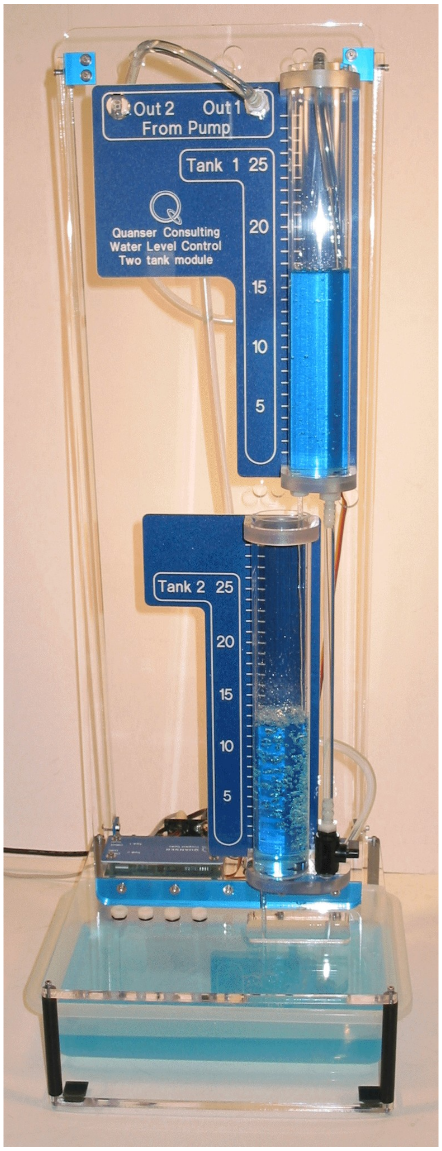

Figure 1 The Coupled-Tank Experiment

The Coupled-Tank plant is a "Two-Tank" module consisting of a pump with a water basin and two tanks. The two tanks are mounted on the front plate such that flow from the first (i.e. upper) tank can flow, through an outlet orifice located at the bottom of the tank, into the second (i.e. lower) tank. Flow from the second tank flows into the main water reservoir. The pump thrusts water vertically to two quick-connect orifices "Out1" and "Out2". The two system variables are directly measured on the Coupled-Tank rig by pressure sensors and available for feedback. They are namely the water levels in tanks 1 and 2. A more detailed description is provided in Reference [1]. To name a few, industrial applications of such Coupled-Tank configurations can be found in the processing system of petro-chemical, paper making, and/or water treatment plants. During the course of this experiment, you will become familiar with the design and pole placement tuning of Proportional-plus-Integral-plus-Feedforward-based water level controllers. In the present laboratory, the Coupled-Tank system is used in two different configurations, namely configuration #1 and configuration #2, as described in Reference [1]. In configuration #1, the control challenge is to track to a desired trajectory the water level in the top tank (i.e. tank #1) from the voltage applied to the pump. The coupled-tank system in configuration #2 is an example of state coupling. In configuration #2, the control challenge is to track to a desired trajectory the water level in the bottom tank (i.e. tank #2) from the water flow coming out of the top tank (i.e. tank #1).

At the end of the session, you should know the following:

- How to mathematically model the Coupled-Tank plant from first principles in order to obtain the two open-loop transfer functions characterizing the system, in the Laplace domain.

- How to linearize the obtained non-linear equation of motion about the quiescent point of operation.
- How to design, through pole placement, a Proportional-plus-Integral-plus-Feedforward-based controller for the Coupled-Tank system in order for it to meet the required design specifications for each configuration.
- How to implement each configuration controller(s) in real-time and evaluate its/their actual performance.

2. Prerequisites

To successfully carry out this laboratory, the prerequisites are:

i) To be familiar with your Coupled-Tank plant main components (e.g. mechanical design, actuator, sensors), your power amplifier (e.g. VoltPAQ), and your data acquisition card (e.g. MultiQ), as described in References [1], [2], and [3].
ii) To be familiar in using QUARC to control and monitor the plant in real-time and in designing a controller through Simulink, as detailed in Reference [4].
iii) To be familiar with the complete wiring of your Coupled-Tank specialty plant, as per dictated in Reference [1].

3. References

[1] *Coupled Tanks User Manual.*
[2] *Data Acquisition Card User Manual.*
[3] *Power Amplifier User Manual.*
[4] *QUARC User Manual* (type doc quarc in Matlab to access).
[5] *QUARC Installation Guide.*

4. Experimental Setup

4.1. Main Components

To setup this experiment, the following hardware and software are required:

- **Power Module:** Quanser VoltPAQ, or equivalent.
- **Data Acquisition Board:** Quanser Q2-USB, Q8-USB, QPID, or equivalent.
- **Coupled-Tank Plant:** Quanser Coupled Tanks, as represented in Figure 1.
- **Control Software:** The QUARC-Simulink configuration, as detailed in Reference [4], or equivalent.

For a complete and detailed description of the main components comprising this setup, please refer to the manuals corresponding to your configuration.

4.2. Wiring

To wire up the system, please follow the default wiring procedure for your Coupled Tanks as fully described in Reference [1]. When you are confident with your connections, you can power up the amplifier.

5. Controller Design Specifications

In the present laboratory (i.e. the pre-lab and in-lab sessions), you will design and implement two control strategies corresponding to configuration #1 and configuration #2 of the Coupled Tanks. Depending on the tanks' configuration and coupling, the purpose of the laboratory session is to regulate and track the water level in either tank #1 and/or tank #2.

5.1. Configuration #1: Tank #1 Level Specifications

In configuration #1, a single-tank system, consisting of the top tank (i.e. tank 1), is considered. The designed closed-loop system is to control the water level (or height) inside tank 1 via the commanded pump voltage. It is based on a Proportional-plus-Integral-plus-Feedforward scheme.

In response to a desired ±1-cm square wave level setpoint from tank 1 operating level position, the water height behaviour should satisfy the following design performance requirements:

1. The operating level (a.k.a. equilibrium height), L_{10}, in tank 1 should be as follows:
   $L_{10} = 15\ [cm]$
2. The Percent Overshoot should be less than 1%, i.e.:
   $PO_1 \leq 11.0\ [\text{"\%"}]$
3. The 2% Settling Time should be less than 5 seconds, i.e.:
   $t_{s\_1} \leq 5.0\ [s]$
4. The response should have no steady-state error.

5.2. Configuration #2: Tank #2 Level Specifications

In configuration #2, the pump feeds tank 1 and tank 1 feeds tank 2. The designed closed-loop system is to control the water level in tank 2 (i.e. the bottom tank) from the water flow coming out of tank 1, located above it. Similarly to configuration #1, the control scheme is based on a Proportional-plus-Integral-plus-Feedforward law.

In response to a desired ±1-cm square wave level setpoint from tank 2 equilibrium level position, the water height behaviour should satisfy the following design performance requirements:

1. The operating level (a.k.a. equilibrium height), L_{20}, in tank 2 should be as follows:
   $L_{20} = 15 \; [cm]$
2. The Percent Overshoot should be less than 2%, i.e.:
   $PO_2 \leq 10.0 \; [\text{"\%"}]$
3. The 2% Settling Time should be less than 20 seconds, i.e.:
   $t_{s\_2} \leq 20.0 \; [s]$
4. The response should have no steady-state error.

6. Pre-Lab Assignments

6.1. Coupled-Tank System Representation and Notations

A schematic of the Coupled-Tank plant is represented in Figure 2, below. The Coupled-Tank system's nomenclature is provided in Appendix A. As illustrated in Figure 2, the positive direction of vertical level displacement is upwards, with the origin at the bottom of each tank (i.e. corresponding to an empty tank), as represented in Figure 2.

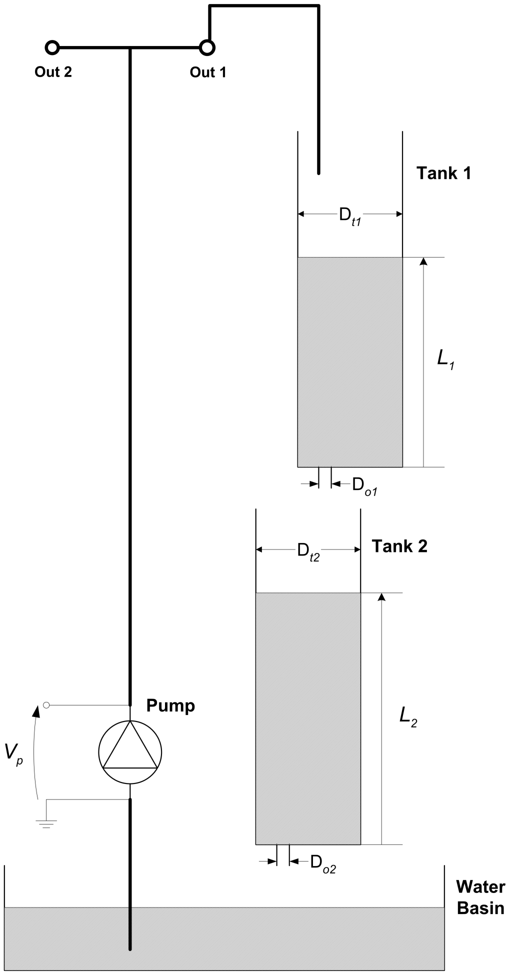

Figure 2 Schematic of the Coupled-Tank Plant

6.2. Assignment #1: Tank 1 Level Modelling - Non-Linear Equation Of Motion (EOM)

Assignment #1 derives the mathematical model of your Coupled-Tank system in configuration #1, as described in Reference [1]. It is reminded that in configuration #1, the pump feeds into Tank 1 and that tank 2 is not considered at all. Therefore, the input to the process is the voltage to the pump and its output is the water level in tank 1 (i.e. top tank).
The purpose of the present modelling session is to provide you with the system's open-loop transfer function, $G_1(s)$, which in turn will be used to design an appropriate level controller.

Answer the following questions:

1. Using the notations and conventions described in Figure 2, above, derive the Equation Of Motion (EOM) characterizing the dynamics of tank 1. Is the tank 1 system's EOM linear?
   Hint #1:
   The obtained EOM should be a function of the system's input and output, as previously defined. Therefore, you should express the resulting EOM under the following format:

   $$\frac{\partial}{\partial t} L_1 = f(L_1, V_p) \qquad [1]$$

   where *f* denotes a function.
   Hint #2:
   The mass balance principle can be applied to the water level in tank 1.
   Hint #3:
   The volumetric inflow rate to tank 1 is assumed to be directly proportional to the applied pump voltage, such that:

   $$F_{i1} = K_p V_p \qquad [2]$$

   Hint #4:
   Applying Bernoulli's equation for small orifices, the outflow velocity from tank 1, v_{o1}, can be expressed by the following relationship:

   $$v_{o1} = \sqrt{2}\sqrt{g L_1} \qquad [3]$$

2. The nominal pump voltage V_{p0} for the pump-tank 1 pair can be determined at the system's static equilibrium. By definition, static equilibrium at a nominal operating point (V_{p0}, L_{10}) is characterized by the water in tank 1 being at a constant position level L_{10} due to the constant inflow rate generated by V_{p0}. Express the static equilibrium voltage V_{p0} as a function of the system's desired equilibrium level L_{10} and the pump flow constant K_p. Using the system's specifications given in Reference [1] and the desired design requirements, evaluate V_{p0}.

6.3. Assignment #2: Tank 1 Level Modelling - EOM Linearization and Transfer Function

In order to design and implement a linear level controller for the tank 1 system, the Laplace open-loop transfer function should be derived. However by definition, such a transfer function can only represent the system's dynamics from a linear differential equation. Therefore, the EOM found in Assignment #1 should be linearized around a quiescent point of operation.

In the case of the water level in tank 1, the operating range corresponds to small departure heights, L_{11}, and small departure voltages, V_{p1}, from the desired equilibrium point (L_{10}, V_{p0}). Therefore, L_1 and V_p can be expressed as the sum of two quantities, as shown below:

$$L_1 = L_{10} + L_{11} \quad \text{and} \quad V_p = V_{p0} + V_{p1} \qquad [4]$$

Answer the following questions:

1. Linearize tank 1 water level's EOM found in Assignment #1 about the quiescent operating point (L_{10}, V_{p0}).
   Hint #1:
   For a function, *f*, of two variables, L_1 and V_p, a first-order approximation for small variations at a point (L_1, V_p) = (L_{10}, V_{p0}) is given by the following Taylor's series approximation:

   $$f(L_1, V_p) = f(L_{10}, V_{p0}) + \left(\frac{\partial}{\partial L_1} f(L_{10}, V_{p0})\right)(L_1 - L_{10}) + \left(\frac{\partial}{\partial V_p} f(L_{10}, V_{p0})\right)(V_p - V_{p0}) \qquad [5a]$$

   Hint #2:
   The obtained linearized EOM should be a function of the system's small deviations about its equilibrium point (L_{10}, V_{p0}). Therefore, you should express the resulting linear EOM under the following format:

   $$\frac{\partial}{\partial t} L_{11} = f(L_{11}, V_{p1}) \qquad [5b]$$

   where *f* denotes a function.

2. Determine from the previously obtained linear equation of motion, the system's open-loop transfer function in the Laplace domain, as defined by the following relationship:

   $$G_1(s) = \frac{L_{11}(s)}{V_{p1}(s)} \qquad [6]$$

Express the open-loop transfer function DC gain, K_{dc_1}, and time constant, τ_1, as functions of L_{10} and the system parameters. What are the order and type of the system? Is it stable? Evaluate K_{dc_1} and τ_1 accordingly to the system's parameters and the desired design requirements.

As a remark, it is obvious that linearized models, such as the Coupled-Tank tank 1's voltage-to-level transfer function, are only approximate models. Therefore, they should be treated as such and used with appropriate caution, that is to say within the valid operating range and/or conditions. However for the scope of this lab, Equation [6] is assumed valid over the pump voltage and tank 1 water level entire operating range, V_{p_peak} and L_{1_max}, respectively.

6.4. Assignment #3 – Tank 1 Level Controller Design: Pole Placement

For zero steady-state error, tank 1 water level is controlled by means of a Proportional-plus-Integral (PI) closed-loop scheme with the addition of a feedforward action, as illustrated in Figure 3, below.

As depicted in Figure 3, the voltage feedforward action is characterized by:

$$V_{p\_ff} = K_{ff\_1} \sqrt{L_{r\_1}} \tag{7}$$

and:

$$V_p = V_{p1} + V_{p\_ff} \tag{8}$$

As it can be seen in Figure 3, the feedforward action is necessary since the PI control system is designed to compensate for small variations (a.k.a. disturbances) from the linearized operating point (L_{10}, V_{p0}). In other words, while the feedforward action compensates for the water withdrawal (due to gravity) through tank 1 bottom outlet orifice, the PI controller compensates for dynamic disturbances.

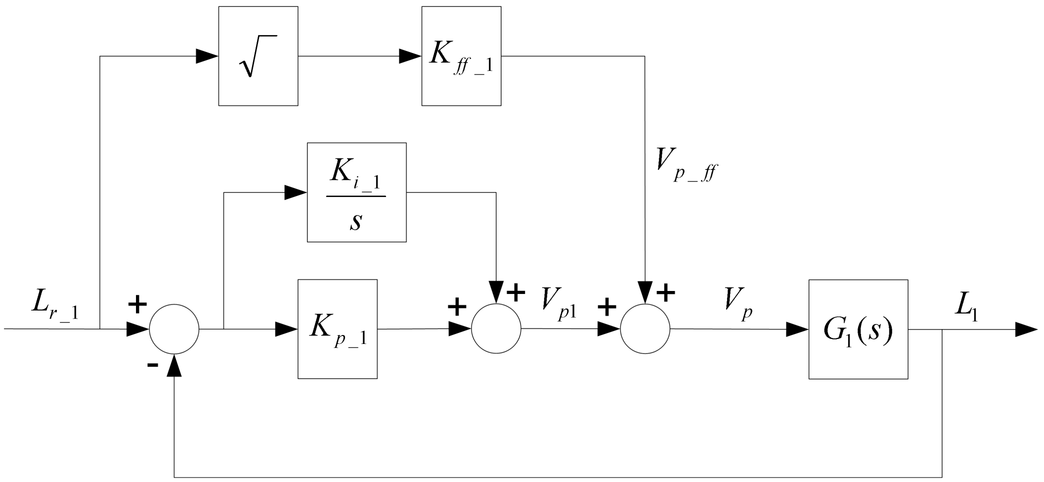

Figure 3 Tank 1 Water Level PI-plus-Feedforward Control Loop

The open-loop transfer function $G_1(s)$ takes into account the dynamics of the tank 1 water level loop, as characterized by Equation [6] in Assignment #2. However due to the presence of the feedforward loop, $G_1(s)$ can also be written as follows:

$$G_1(s) = \frac{L_1(s)}{V_p(s)} \tag{9}$$

Answer the following questions:

1. Analyze tank 1 water level closed-loop system at the static equilibrium point (L_{10}, V_{p0}) and determine and evaluate the voltage feedforward gain, K_{ff_1}, as defined by Equation [7].

2. Using tank 1 voltage-to-level transfer function $G_1(s)$ determined in Assignment #2 and the control scheme block diagram illustrated in Figure 3, derive the normalized characteristic equation of the water level closed-loop system.
   Hint #1:
   The feedforward gain K_{ff_1} does not influence the system characteristic equation. Therefore, the feedforward action can be neglected for the purpose of determining the denominator of the closed-loop transfer function. Block diagram reduction can be carried out.
   Hint #2:
   The system's normalized characteristic equation should be a function of the PI level controller gains, K_{p_1}, and K_{i_1}, and system's parameters, K_{dc_1} and τ_1.

3. By identifying the controller gains K_{p_1} and K_{i_1}, fit the obtained characteristic equation

to the second-order standard form expressed below:

$$s^2 + 2\,\zeta_1\,\omega_{n1}\,s + \omega_{n1}^2 = 0 \qquad [10]$$

Determine K_{p_1} and K_{i_1} as functions of the parameters ω_{n1}, ζ_1, K_{dc_1}, and τ_1.

4. Determine the numerical values for K_{p_1} and K_{i_1} in order for the tank 1 system to meet the closed-loop desired specifications, as previously stated.
   Hint #1:
   Tank 1 level response Percent Overshoot can be expressed as follows:

$$PO_1 = 100\,\mathbf{e}^{\left(-\frac{\zeta_1\,\pi}{\sqrt{1-\zeta_1^2}}\right)} \qquad [11]$$

   Hint #2:
   Tank 1 level response 2% Settling Time can be expressed as follows:

$$t_{s\_1} = \frac{4}{\zeta_1\,\omega_{n1}} \qquad [12]$$

6.5. Assignment #4: Tank 2 Level Modelling - Non-Linear Equation Of Motion (EOM)

Assignment #4 derives the mathematical model of your Coupled-Tank system in configuration #2, as described in Reference [1]. It is reminded that in configuration #2, the pump feeds into tank 1, which in turn feeds into tank 2. As far as tank 1 is concerned, the same equations as the ones previously developed in Assignments #1, #2, and #3 apply. However, the water level Equation Of Motion (EOM) in tank 2 still needs to be derived. The input to the tank 2 process is the water level, L_1, in tank 1 (generating the outflow feeding tank 2) and its output is the water level, L_2, in tank 2 (i.e. bottom tank).
The purpose of the present modelling session is to provide you with the system's open-loop transfer function, $G_2(s)$, which in turn will be used to design an appropriate level controller.

Answer the following questions:

1. Using the notations and conventions described in Figure 2, above, derive the Equation Of Motion (EOM) characterizing the dynamics of tank 2. Is the tank 2 system's EOM linear?
   Hint #1:
   The obtained EOM should be a function of the system's input and output, as previously defined. Therefore, you should express the resulting EOM under the following format:

   $$\frac{\partial}{\partial t} L_2 = \mathrm{f}(L_2, L_1) \quad [13]$$

   where *f* denotes a function.
   Hint #2:
   The mass balance principle can be applied to the water level in tank 2.
   Hint #3:
   The volumetric inflow rate to tank 2 is equal to the volumetric outflow rate from tank 1, that is to say:

   $$F_{i2} = F_{o1} \quad [14]$$

   Hint #4:
   Applying Bernoulli's equation through small orifices, the outflow velocity from tank 2, v_{o2}, can be expressed by the following relationship:

   $$v_{o2} = \sqrt{2}\sqrt{g L_2} \quad [15]$$

2. The nominal water level L_{10} for the tank1-tank2 pair can be determined at the system's static equilibrium. By definition, static equilibrium at a nominal operating point (L_{10}, L_{20}) is characterized by the water in tank 2 being at a constant position level L_{20} due to the

constant inflow rate generated from the top tank by L_{10}. Express the static equilibrium level L_{10} as a function of the system's desired equilibrium level L_{20} and the system's parameters. Using the system's specifications given in Reference [1] and the desired design requirements, evaluate L_{10}.

6.6. Assignment #5: Tank 2 Level Modelling - EOM Linearization and Transfer Function

In order to design and implement a linear level controller for the tank 2 system, the Laplace open-loop transfer function should be derived. However by definition, such a transfer function can only represent the system's dynamics from a linear differential equation. Therefore, the EOM found in Assignment #4 should be linearized around a quiescent point of operation.

In the case of the water level in tank 2, the operating range corresponds to small departure heights, L_{11} and L_{21}, from the desired equilibrium point (L_{20}, L_{10}). Therefore, L_2 and L_1 can be expressed as the sum of two quantities, as shown below:

$$L_2 = L_{20} + L_{21} \qquad \text{and} \qquad L_1 = L_{10} + L_{11} \tag{16}$$

Answer the following questions:

1. Linearize tank 2 water level's EOM found in Assignment #4 about the quiescent operating point (L_{20}, L_{10}).
   Hint #1:
   For a function, *f*, of two variables, L_2 and L_1, a first-order approximation for small variations at a point (L_2, L_1) = (L_{20}, L_{10}) is given by the following Taylor's series approximation:
   $$f(L_2, L_1) = f(L_{20}, L_{10}) + \left(\frac{\partial}{\partial L_2} f(L_{20}, L_{10})\right)(L_2 - L_{20}) + \left(\frac{\partial}{\partial L_1} f(L_{20}, L_{10})\right)(L_1 - L_{10}) \tag{17a}$$
   Hint #2:
   The obtained linearized EOM should be a function of the system's small deviations about its equilibrium point (L_{20}, L_{10}). Therefore, you should express the resulting linear EOM under the following format:
   $$\frac{\partial}{\partial t} L_{21} = f(L_{21}, L_{11}) \tag{17b}$$
   where *f* denotes a function.

2. Determine from the previously obtained linear equation of motion, the system's open-

loop transfer function in the Laplace domain, as defined by the following relationship:

$$G_2(s) = \frac{L_{21}(s)}{L_{11}(s)} \quad [18]$$

Express the open-loop transfer function DC gain, K_{dc_2}, and time constant, τ_2, as functions of L_{20}, L_{10}, and the system's parameters. What are the order and type of the system? Is it stable? Evaluate K_{dc_2} and τ_2 accordingly to the system's parameters and the desired design requirements.

As a remark, it is obvious that linearized models, such as the Coupled-Tank's tank 2 level-to-level transfer function, are only approximate models. Therefore, they should be treated as such and used with appropriate caution, that is to say within the valid operating range and/or conditions. However for the scope of this lab, Equation [18] is assumed valid over tank 1 and tank 2 water level entire range of motion, L_{1_max} and L_{2_max}, respectively.

6.7. Assignment #6 – Tank 2 Level Controller Design: Pole Placement

For zero steady-state error, tank 2 water level is controlled by means of a Proportional-plus-Integral (PI) closed-loop scheme with the addition of a feedforward action, as illustrated in Figure 4, below.

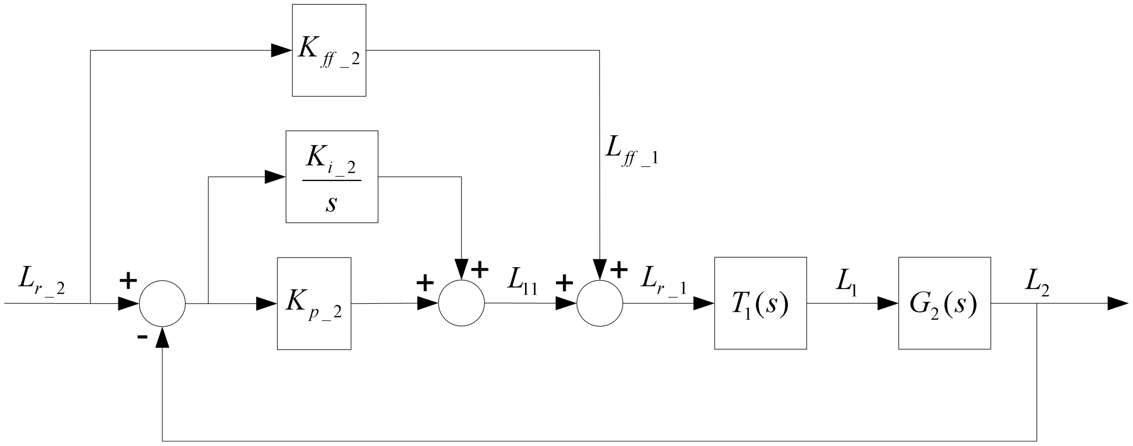

Figure 4 Tank 2 Water Level PI-plus-Feedforward Control Loop

In the block diagram depicted in Figure 4, the water level in tank 1 is controlled by means of the closed-loop system previously designed in Assignment #3. This is represented by the tank 1 closed-loop transfer function defined below:

$$T_1(s) = \frac{L_1(s)}{L_{r\_1}(s)} \qquad [19]$$

Such a subsystem represents an inner (or nested) level loop. In order to achieve a good overall stability with such a configuration, the inner level loop (i.e. tank 1 closed-loop system) must be much faster than the outer level loop. This constraint is met by the previously stated controller design specifications, where $t_{s\_1} << t_{s\_2}$.

However for the sake of simplicity in the present analysis, the water level dynamics in tank 1 are neglected. Therefore, it is assumed hereafter that:

$$L_1(t) = L_{r\_1}(t) \qquad \text{i.e.} \qquad T_1(s) = 1 \qquad [20]$$

Furthermore as depicted in Figure 4, the level feedforward action is characterized by:

$$L_{ff\_1} = K_{ff\_2}\, L_{r\_2} \qquad [21]$$

and:

$$L_{r\_1} = L_{11} + L_{ff\_1} \qquad [22]$$

The level feedforward action, as seen in Figure 4, is necessary since the PI control system is only designed to compensate for small variations (a.k.a. disturbances) from the linearized operating point (L_{20}, L_{10}). In other words, while the feedforward action compensates for the water withdrawal (due to gravity) through tank 2's bottom outlet orifice, the PI controller compensates for dynamic disturbances.

The open-loop transfer function $G_2(s)$ takes into account the dynamics of the tank 2 water level loop, as characterized by Equation [18] in Assignment #5. However due to the presence of the feedforward loop and the simplifying assumption expressed by Equation [20], $G_2(s)$ can also be written as follows:

$$G_2(s) = \frac{L_2(s)}{L_1(s)} \qquad [23]$$

Answer the following questions:

1. Analyze tank 2 water level closed-loop system at the static equilibrium point (L_{20}, L_{10}) and determine and evaluate the level feedforward gain, K_{ff_2}, as defined by Equation [21].

2. Using tank 2 level-to-level transfer function *$G_2(s)$* determined in Assignment #5 and the control scheme block diagram illustrated in Figure 4, derive the normalized characteristic equation of the water level closed-loop system.
   Hint #1:
   Block diagram reduction can be carried out.
   Hint #2:
   The system's normalized characteristic equation should be a function of the PI level controller gains, K_{p_2} and K_{i_2}, and system's parameters, K_{dc_2} and τ_2.

3. By identifying the controller gains K_{p_2} and K_{i_2}, fit the obtained characteristic equation to the second-order standard form expressed below:

   $$s^2 + 2\zeta_2\omega_{n2}s + \omega_{n2}^2 = 0 \tag{24}$$

   Determine K_{p_2} and K_{i_2} as functions of the parameters ω_{n2}, ζ_2, K_{dc_2}, and τ_2.

4. Determine the numerical values for K_{p_2} and K_{i_2} in order for the tank 2 system to meet the closed-loop desired specifications, as previously stated.
   Hint #1:
   Tank 2 level response Percent Overshoot can be expressed as follows:

   $$PO_2 = 100\,\mathbf{e}^{\left(-\frac{\zeta_2\pi}{\sqrt{1-\zeta_2^2}}\right)} \tag{25}$$

   Hint #2:
   Tank 2 level response 2% Settling Time can be expressed as follows:

   $$t_{s\_2} = \frac{4}{\zeta_2\omega_{n2}} \tag{26}$$

7. In-Lab Procedure

7.1. Experimental Setup And Wiring

Even if you do not configure the experimental setup entirely yourself, you should be at least completely familiar with it and understand it. If in doubt, refer to References [1], [2], [3], and/or [4].

The first task upon entering the lab is to ensure that the complete system is wired as fully described in Reference [1]. You should be familiar with the complete wiring and connections of your Coupled-Tank system. If you are still unsure of the wiring, please ask for assistance from the Teaching Assistant assigned to the lab. When you are confident with your connections, you can power up the amplifier. You are now ready to begin the lab.

7.2. Real-Time Implementation – Configuration #1: Tank 1 PI-plus-Feedforward Level Control Loop

7.2.1. Objectives

- To tune through pole placement the PI-plus-feedforward controller for the actual water level in tank 1 of the Coupled-Tank system.
- To implement with QUARC the PI-plus-feedforward control loop for the actual Coupled-Tank's tank 1 level.
- To run the obtained PI-plus-feedforward level controller and compare the actual response against the controller design specifications.
- To run the system's simulation simultaneously, at every sampling period, in order to compare the actual and simulated level responses.

7.2.2. Experimental Procedure

Please follow the steps described below:

Step 1. Load Matlab and set the Current Directory to your folder with the Coupled Tanks lab files.

Step 2. Open the *q_tanks1.mdl* Simulink model file shown in Figure 5, below. The model implements the system's actual Proportional-plus-Integral (PI) closed-loop with feedforward action, as studied in Assignment #3.

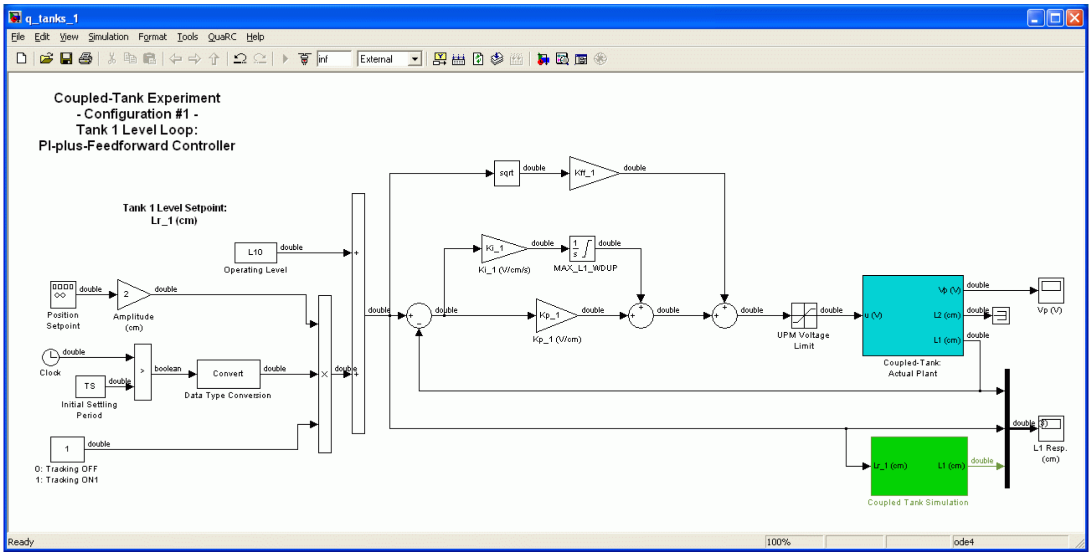

Figure 5 Real-Time Implementation of the Tank 1 Level Control Loop: Configuration #1

Step 3. In order to use your actual coupled-tank system, the controller diagram directly interfaces with your system hardware in the *Coupled-Tank: Actual Plant block*, as shown in Figure 6, below.

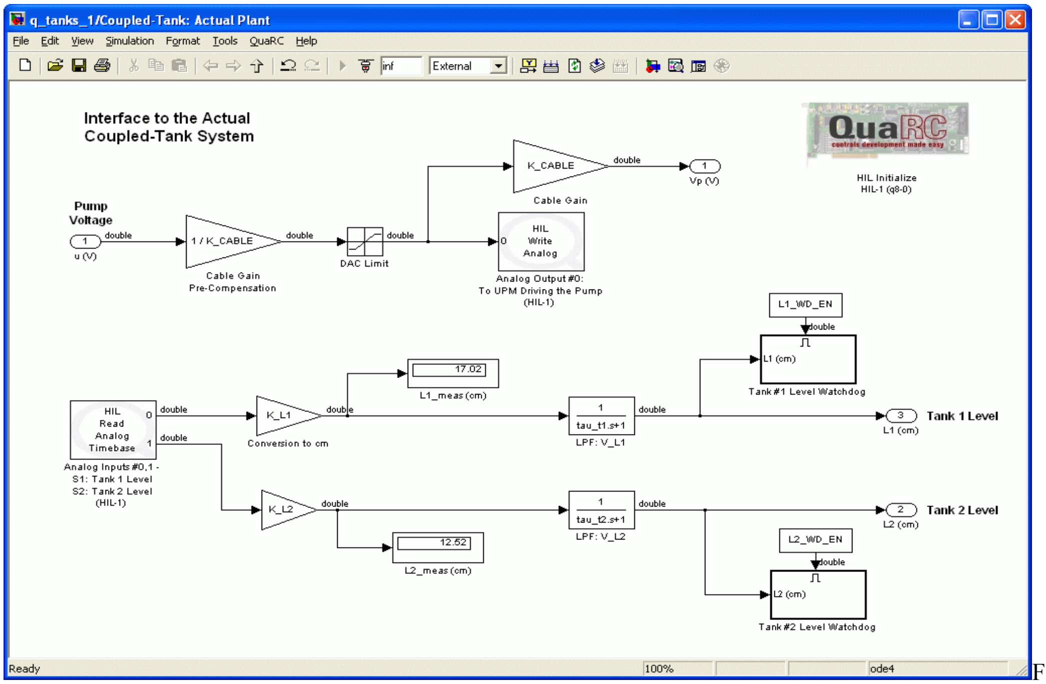

Figure 6 Interface Subsystem to the Actual Coupled-Tank Plant Using the Q8 Card

To familiarize yourself with the diagram, it is suggested that you open the model subsystems to get a better idea of their composing blocks as well as take note of the I/O connections. You should also check that the signal generator block properties are properly set to output a square wave signal, of amplitude 1 and of frequency 0.05 Hz. The total level setpoint for tank 1 should result to be a square wave of ±1cm around the desired equilibrium level L_{10}. It should be noted that a simple low-pass filter of cut-off frequency 2.5 Hz (set by *'tau_t1'*) is added to the output signal of the tank 1 level pressure sensor. This filter is necessary to attenuate the high-frequency noise content of the level measurement. Such a measurement noise is mostly created by the sensor's environment consisting of turbulent flow and circulating air bubbles. Although introducing a short delay in the signals, low-pass filtering allows for higher controller gains in the closed-loop system, and therefore for higher performance. Moreover, as a safety watchdog, the real-time controller will stop if the water level in either tank 1 or tank 2 goes beyond 30 cm (set by *'L1_MAX'*) or 25 cm (set by *'L2_MAX'*), respectively. This is implemented in Figure 6 through the *Dead Zone* and *Stop With Error* blocks.

Step 4. In the *Coupled-Tanks: Actual Plant* subsystem, click on the HIL Initialize block and set the *Board type* field to the data-acquisition board that is connected to the Coupled Tanks system, e.g. Q4 HIL device.

Step 5. Before being able to run the actual control loop, the PI-plus-feedforward controller gains must be initialized in the Matlab workspace, since they are to be used by the Simulink controller diagram. Start by running the Matlab script called *setup_lab_tanks.m*. However, ensure beforehand that the *CONTROLLER_TYPE* flag is set to *'MANUAL'*. This file initializes all the Coupled-Tank model parameters and user-defined configuration variables needed by the Simulink diagram. As seen in pre-lab assignment #3, the quiescent voltage feedforward term, V_{p_ff}, is added to V_{p1} to compensate for the known water withdrawal bias from the bottom of tank 1 as well as to help bringing the water level, L_1, to its operating position. You can now initialize in the Matlab workspace the controller and feedforward gains as calculated in Assignment #3. **Have your lab assistant check your values**. With his or her approval you can now enter your calculated values for K_{p_1}, K_{i_1}, and K_{ff_1} in the Matlab workspace by following the Matlab notations used for the controller gains as presented in Table A.2 of Appendix A. You are now ready to go ahead with compiling and running your actual level controller for tank 1.

Step 6. Build the real-time code corresponding to your diagram, by using the *QUARC | Build* option from the Simulink menu bar.

Step 7. Clicking on QUARC | Start should start the gear pump thrusting water filling tank 1 up to its operating level L_{10}. Then after a 15-second settling delay (in order to stabilize the system at its operating point), the water level in tank 1 should start tracking the desired ±1-cm square wave setpoint around the desired operating level L_{10}. As a remark, the initial settling time for the system to reach its operating point is defined in Matlab by the parameter *'TS'*.

Step 8. In order to observe the system's responses from the actual system, double-click on the following scopes in the Simulink model: *L1 Resp. (cm)* and *Vp (V)*. You should now be able to monitor, as the water flows through the Coupled-Tank system, the actual water level in tank 1 as it track its reference input. The corresponding commanded pump voltage, which is proportional to the control effort spent, is sent to the power amplifier and can also be monitored and plotted on-line.

Step 9. Assess the actual performance of the level response and compare it to the design requirements. Measure your response actual percent overshoot and settling time. Are the design specifications satisfied? Explain. If your level response does not meet the desired design specifications of Section Controller Design Specifications on page 4, review your PI-plus-Feedforward gain calculations and/or alter the closed-loop pole locations (i.e. PO_1 and t_{s_1}) until they do. If you are still unable to achieve the required performance level, ask your T.A. for advice.

Hint:

The *L1 Resp. (cm)* and the *Vp (V)* scopes automatically save the last 30 seconds of data to the Matlab workspace under the variables *data_L1* and *data_Vp*, respectively. These variables can be used with Matlab to accurately measure the percent overshoot and settling time. See Reference [4] for more information on saving data for offline analysis.

Step 10. Specifically discuss in your lab report the following points:

How does your actual tank 1 level compare to the simulated response?

i) Is there a discrepancy in the results? If so, discuss some of the possible reasons.

ii) From the plot of the actual level response, measure your system t_{s_1} and PO_1. Are the values in agreement with the design specifications? If not exactly, find some of the possible reasons.

Step 11. Once your results are as closely as possible in agreement with the closed-loop requirements of configuration #1, your tank 1 level response should look similar to the one displayed in Figure 7, below.

Step 12. Include in your lab report your final values for K_{p_1}, K_{i_1}, and K_{ff_1} as well as the resulting response plot of the actual and theoretical L_1 versus L_{r_1}. Also include from the same run the corresponding plot of V_p. Ensure to properly document all your results and observations before moving on the the next section.

Step 13. You can now proceed to the next section, which deals with the actual implementation in real-time of your PI-plus-Feedforward level controller for tank 2 of the Coupled-Tank system in configuration #2.

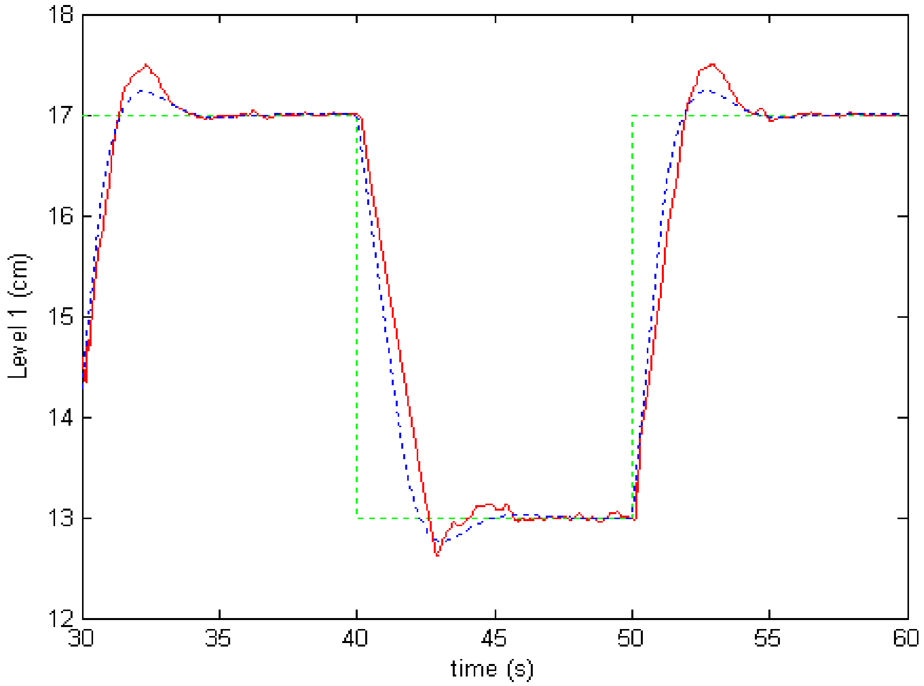

Figure 7 Actual And Theoretical Tank 1 Level Tracking Response: Configuration #1

7.3. Real-Time Implementation – Configuration #2: Tank 2 PI-plus-Feedforward Level Control Loop

7.3.1. Objectives

To tune through pole placement the PI-plus-Feedforward controller for the actual water level of the Coupled-Tank system's tank 2.

To implement in real-time with QUARC the PI-plus-Feedforward control loop for the actual tank 2 water level.

To run the obtained Feedforward-plus-PI level controller and compare the actual response against the controller design specifications.

To run the system's simulation simultaneously, at every sampling period, in order to compare the actual and simulated level responses.

To investigate the effect of the nested PI-plus-Feedforward level control loop implemented for tank 1.

7.3.2. Experimental Procedure

Please follow the steps described below:

Step 1.Load Matlab and set the Current Directory to your folder with the Coupled Tanks lab files

Step 2.Open the *q_tanks2.mdl* Simulink model file shown in Figure 8, below. The model implements a Proportional-plus-Integral-plus-Feedforward closed-loop, as studied in Assignment # 6.

As mentioned in the pre-lab assignments, the tank 2 water level control loop is based on top of tank 1 level controller, as developed and tuned in the previous sections. The nested actual tank 1 level control scheme is depicted in Figure 9, below. Similarly, the level controller diagram for the Coupled-Tank in configuration #2 also interfaces directly with your Coupled-Tank hardware, as shown in Figure 6, above. To familiarize yourself with the diagram, it is suggested that you open the model subsystems to get a better idea of their composing blocks as well as take note of the I/O connections. You should also check that the signal generator block properties are properly set to output a square wave signal, of amplitude 1, and of frequency 0.018 Hz. The total level setpoint for tank 2 should result to be a square wave of ±1cm around the desired equilibrium level L_{20}. Also, your model sampling time should be set to 1 ms, i.e. $T_s = 10^{-3}$ s and the solver type to *'ode4 (Runge-Kutta)'*.

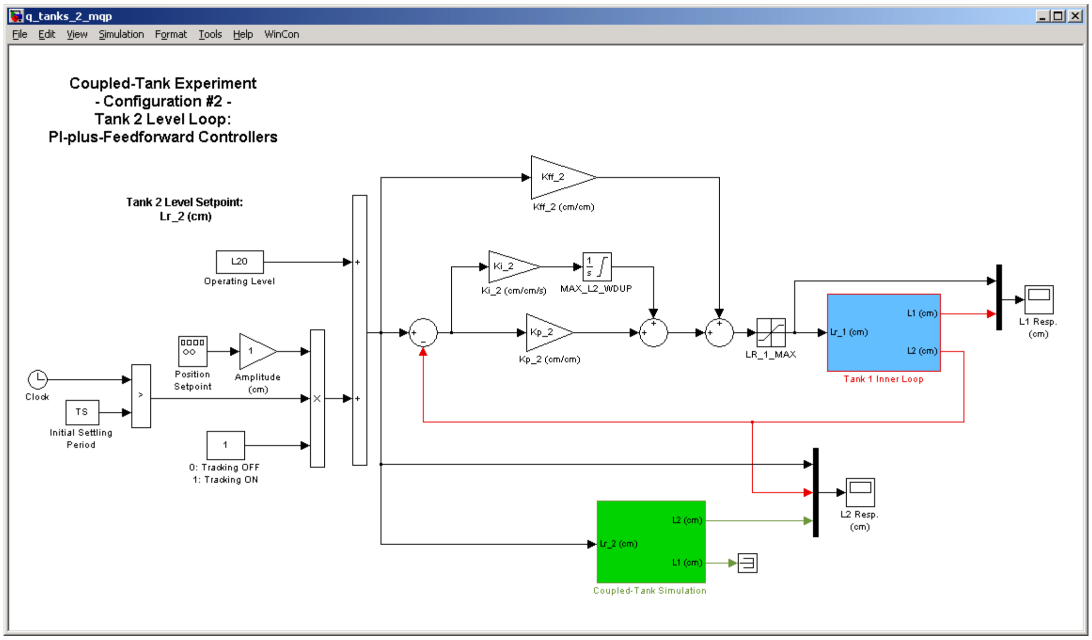

Figure 8 Real-Time Implementation of the Tank 2 Level Control Loop: Configuration #2.

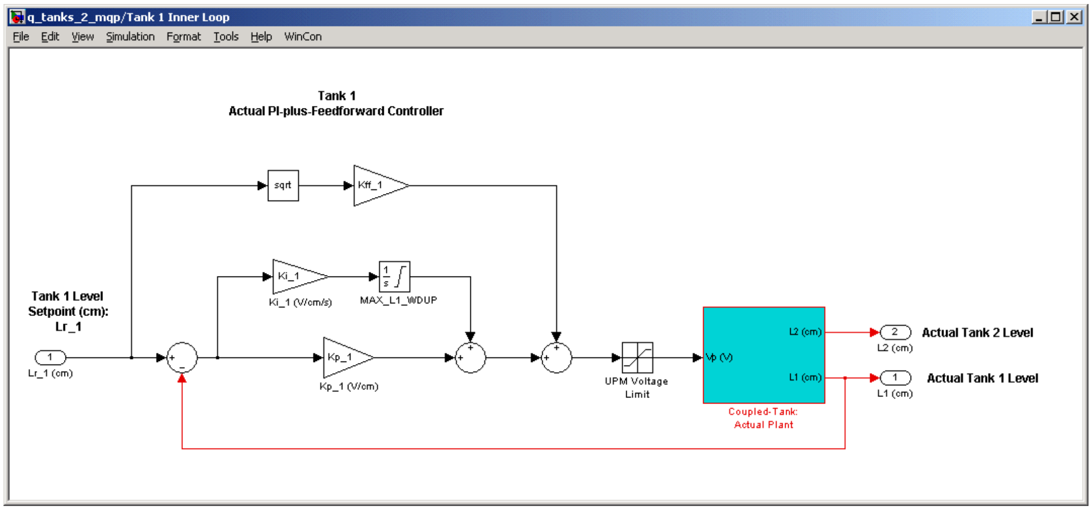

Figure 9 Real-Time Implementation of the Nested Tank 1 Level Control Loop: Configuration #2

It should be noted that two simple low-pass filters of cut-off frequency 2.5 Hz (set by *'tau_t1'*) and 0.33 Hz (set by *'tau_t2'*) are added to the output signals of the tank 1 and tank 2 level pressure sensors, respectively. These filters are necessary to attenuate the high-frequency noise content of the level measurements. Such a measurement noise is mostly created by the sensors environment made of turbulent flow and circulating air bubbles. Although introducing a short delay in the signals, low-pass filtering allows for higher controller gains in the closed-loop system, and therefore for higher performance.

Moreover, as a safety watchdog, the real-time controller will stop if the water level in either tank 1 or tank 2 goes beyond 30 cm (set by *'L1_MAX'*) or 25 cm (set by *'L2_MAX'*), respectively. This is implemented in Figure 6 through the *Dead Zone* and *Stop With Error* blocks.

Step 3.In the *Coupled-Tanks: Actual Plant* subsystem, click on the HIL Initialize block and set the *Board type* field to the data-acquisition board that is connected to the Coupled Tanks system, e.g. Q4 HIL device.

Step 4.Before being able to run the actual control loop, the PI-plus-feedforward controller gains for tank 2 must also be initialized in the Matlab workspace, since they are to be used by the Simulink controller diagram. However, keep in the Matlab workspace the PI-plus-feedforward controller gains for tank 1 of the Coupled-Tank system in configuration #1, as previously implemented. As seen in pre-lab assignment #6, the quiescent level feedforward term, L_{ff_1}, is added to L_{11} to compensate for the known water withdrawal bias from the bottom of tank 2 as well as to help bringing the water level, L_2, to its operating position. You can now initialize in the Matlab workspace the controller and feedforward gains as calculated in Assignment #6. **Have your lab assistant check your values**. With his or her approval you can now enter your calculated values for K_{p_2}, K_{i_2}, and K_{ff_2} in the Matlab workspace by following the Matlab notations used for the controller gains as presented in Table A.2 of Appendix A. You are now ready to go ahead with compiling and running your actual level controller for tank 2 of the Coupled-Tank system in configuration #2.

Step 5.Build the real-time code corresponding to your diagram, by using the *QUARC | Build* option from the Simulink menu bar.

Step 6.Clicking on QUARC | Start should start the gear pump thrusting water filling up both tank 1 and tank 2 up to their operating levels, L_{10} and L_{20}, respectively. Then after a 35-second settling delay (in order to stabilize the system at its operating point), the water level in tank 2 should start tracking the desired ±1-cm square wave setpoint around the operating level L_{20}. As a remark, the initial settling time for the system to reach its operating point is defined in Matlab by the parameter *'TS'*.

Step 7.In order to observe the system's real-time responses from the actual system, double-click on the following scopes in the Simulink model: *L2 Resp. (cm)*, *L1 Resp. (cm)*, and *Vp (V)*. You should now be able to monitor on-the-fly, as the water flows through the Coupled-Tank system, the actual water levels in tanks 1 and 2 as they track their respective reference inputs. The corresponding commanded pump voltage, which is proportional to the control effort spent, is sent to the power amplifier and can also be monitored and plotted on-line.

Step 8.Assess the actual performance of the level response in tank 2 and compare it to the design requirements. Measure your response actual percent overshoot and settling time. Are the design specifications satisfied? Explain. If your level response does not meet the desired design specifications of Section Controller Design Specifications on page 4, review your PI-plus-Feedforward gain calculations and/or alter the closed-loop pole locations (i.e. PO_2 and t_{s_2}) until they do. If you are still unable to achieve the required performance level, ask your T.A. for advice.

Hint:

The *L1 Resp. (cm), L2 Resp. (cm)*, and the *Vp (V)* scopes automatically save the last 60 seconds of data to the Matlab workspace under the variables *data_L1, data_L2,* and *data_Vp*, respectively. These variables can be used with Matlab to accurately measure the percent overshoot and settling time. See Reference [4] for more information on saving data for offline analysis.

Step 9.Specifically discuss in your lab report the following points:

i). How does your actual tank 2 level compare to the simulated response?

ii). Is there a discrepancy in the results? If so, discuss some of the possible reasons.

iii). From the plot of the actual level response, measure your system t_{s_2} and PO_2. Are the values in agreement with the design specifications? If not exactly, find some of the possible reasons.

Step 10.Once your results are as closely as possible in agreement with the closed-loop requirements of configuration #2, your level response in tank 1 should look similar to the one displayed in Figure 10, below.

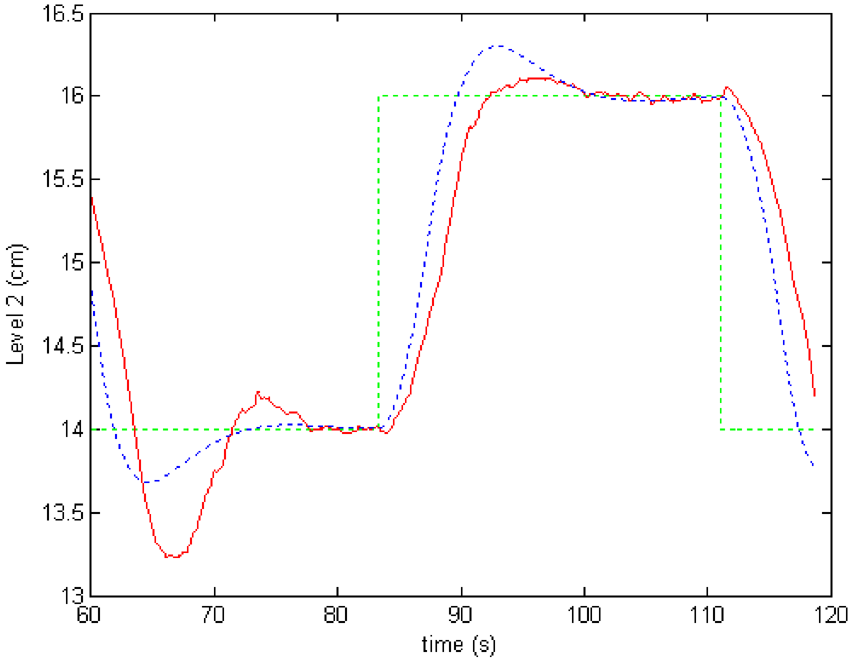

Figure 10 Actual And Theoretical Tank 2 Level Tracking Response: Configuration #2

Step 11. From the same run, the corresponding water level in tank 1 is displayed in Figure 11, below.

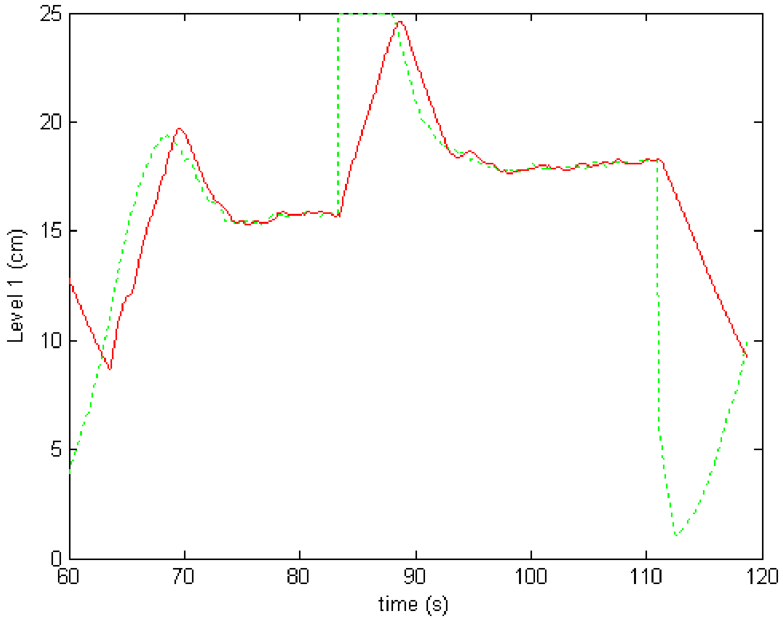

Figure 11 Actual Tank 1 Level Tracking Response: Configuration #2

Step 12. Include in your lab report your final values for K_{p_2}, K_{i_2}, and K_{ff_2} as well as the resulting response plot of the actual and theoretical L_2 versus L_{r_2}. Also include from the same run the corresponding plots of L_1 and V_p.

Step 13. You can now move on to writing your lab report. Ensure to properly document all your results and observations before leaving the laboratory session.

Appendix A. Nomenclature

Table A.1, below, provides a complete listing of the symbols and notations used in the Coupled-Tank system mathematical modelling, as presented in this laboratory. The numerical values of the system parameters can be found in Reference [1].

Symbol	***Description***	***Units***	***Matlab Notations***
K_p	Pump Volumetric Flow Constant	$cm^3/s/V$	Kp
V_p	Actual Pump Input Voltage	V	Vp
V_{p0}	Steady-State Pump Voltage	V	Vp0
V_{p1}	Small Variation Around V_{p0}	V	Vp11
V_{p_ff}	Feedforward Pump Voltage	V	Vp_ff
D_{t1}	Tank 1 Inside Diameter	cm	Dt1
D_{o1}	Tank 1 Outlet Diameter	cm	Do1
A_{t1}	Tank 1 Inside Cross-Section Area	cm^2	At1
A_{o1}	Tank 1 Outlet Cross-Section Area	cm^2	Ao1
F_{i1}	Volumetric Inflow Rate To Tank 1	cm^3/s	
F_{o1}	Volumetric Outflow Rate From Tank 1	cm^3/s	
L_1	Tank 1 Water Level	cm	L1
L_{10}	Steady-State Water Level in Tank 1	cm	L10
L_{11}	Small Variation Around L_{10}	cm	L11
L_{r_1}	Tank 1 Reference (a.k.a. Desired) Level	cm	Lr_1
K_{L1}	Tank 1 Water Level Sensor Sensitivity	cm/V	K_L1
D_{t2}	Tank 2 Inside Diameter	cm	Dt2
D_{o2}	Tank 2 Outlet Diameter	cm	Do2
A_{t2}	Tank 2 Inside Cross-Section Area	cm^2	At2
A_{o2}	Tank 2 Outlet Cross-Section Area	cm^2	Ao2
F_{i2}	Volumetric Inflow Rate To Tank 2	cm^3/s	
F_{o2}	Volumetric Outflow Rate From Tank 2	cm^3/s	
L_2	Tank 2 Water Level	cm	L2

Symbol	***Description***	***Units***	***Matlab Notations***
L_{20}	Steady-State Water Level in Tank 2	cm	L20
L_{21}	Small Variation Around L_{20}	cm	L21
L_{r_2}	Tank 2 Reference (a.k.a. Desired) Level	cm	Lr_2
K_{L2}	Tank 2 Water Level Sensor Sensitivity	cm/V	K_L2
g	Gravitational Constant on Earth	cm/s^2	g

Table A.1 Coupled-Tank System Model Nomenclature

Table A.2, below, provides a complete listing of the symbols and notations used in the design of both control loops (i.e. the PI-plus-Feedforward loops for the water levels in tank 1 and tank 2), as presented in this laboratory.

Symbol	***Description***	***Units***	***Matlab / Simulink Notation***
PO_1	Tank 1 Level Percent Overshoot	%	PO_1
t_{s_1}	Tank 1 Level 2% Settling Time	s	ts_1
K_{p_1}	Tank 1 Level Proportional Gain	V/cm	Kp_1
K_{i_1}	Tank 1 Level Integral Gain	V/s/cm	Ki_1
G_1	Open-Loop Tank 1 Level Transfer Function	cm/V	G1
K_{dc_1}	Tank 1 Level Open-Loop DC Gain	V/cm	Kdc_1
τ_1	Tank 1 Level Open-Loop Time Constant	s	tau_1
ζ_1	Tank 1 Level Damping Ratio		zeta_1
ω_{n1}	Tank 1 Level Undamped Natural Frequency	rad/s	wn_1
T_1	Closed-Loop Tank 1 Level Transfer Function	cm/cm	T1
PO_2	Tank 2 Level Percent Overshoot	%	PO_2
t_{s_2}	Tank 2 Level 2% Settling Time	s	ts_2
K_{p_2}	Tank 2 Level Proportional Gain	cm/cm	Kp_2
K_{i_2}	Tank 2 Level Integral Gain	1/s	Ki_2
G_2	Open-Loop Tank 2 Level Transfer Function	cm/cm	G2
K_{dc_2}	Tank 2 Level Open-Loop DC Gain	cm/cm	Kdc_2

Symbol	***Description***	***Units***	***Matlab / Simulink Notation***
τ_2	Tank 2 Level Open-Loop Time Constant	s	tau_2
ζ_2	Tank 2 Level Damping Ratio		zeta_2
ω_{n2}	Tank 2 Level Undamped Natural Frequency	rad/s	wn_2
K_{ff_1}	Feedforward Pump Voltage Gain (Configuration #1)	$V/cm^{1/2}$	Kff_1
K_{ff_2}	Feedforward Tank 1 Level Gain (Configuration #2)	cm/cm	Kff_2
s	Laplace Operator	rad/s	
t	Continuous Time	s	

Table A.2 Control Loops Nomenclature