

Homework Problems 8

Due Wednesday, November 5, 2008

Problems like this (or in one case identical to this) appeared on previous year's tests.

Problem 1 *A particle forms a random walk on a graph with 6 vertices labeled 1, 2, 3, 4, 5, 6 and probability transition matrix*

$$P := \begin{bmatrix} 1/5 & 1/5 & 1/5 & 1/5 & 1/5 & 0 \\ 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 0 & 1/4 & 1/4 & 1/4 & 1/4 & 0 \\ 0 & 0 & 1/4 & 1/4 & 1/4 & 1/4 \end{bmatrix}$$

1. Starting at vertex 1, what is the mean number of steps until the particle's first return to vertex 1?
2. Starting at vertex 1, what is the mean number of steps until the particle's first visit to vertex 5?

Problem 2 *Consider an homogeneous Markov chain with state space $S := \{1, 2, 3, 4, 5, 6\}$ with transition matrix*

$$P := \begin{bmatrix} 0 & 1/4 & 1/4 & 1/4 & 0 & 1/4 \\ 1/5 & 1/5 & 0 & 1/5 & 0 & 2/5 \\ 0 & 0 & 1/4 & 3/4 & 0 & 0 \\ 0 & 0 & 1/3 & 2/3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3/8 & 5/8 \\ 0 & 0 & 0 & 0 & 3/4 & 1/4 \end{bmatrix}.$$

1. Using the communication equivalence relation, break up the states into equivalence classes.
2. Which classes are transient and which are recurrent?
3. Compute exactly all the stationary distributions for each irreducible class of S . If there is only one for any of the classes explain why this is so.
4. Compute exactly, for each transient state i and each irreducible recurrent class C , the probability that starting at i you are eventually in C .
5. Using the information in parts 1 to 4, write down $\lim_{n \rightarrow \infty} P^n$.
6. Compute P^{100} numerically. Does it agree approximately with the result in part 5?

Problem 3 *Using the Markov Chain set up for the game of craps:*

1. what are the mean number of steps for the game to end?
2. what are the odds of winning and of losing?