

Homework Problems 6
Due Wednesday, October 15, 2008

Problem 1 Let X and Y be independent normal random variables. Assume that $E(X) = 1$, $E(Y) = 2$, $\text{Var}(X) = 2$, and $\text{Var}(Y) = 2$. Compute $P(X + Y < 5)$.

Problem 2 Assume that the time until burnout of a certain type of lamp is an exponential random variable with mean equal to 40 hours. Use the Central Limit Theorem to compute the probability that the average of the times until burnout of N lamps for $N = 1, \dots, 25$ is at most 48 hours.

Problem 3 Assume that the time until burnout of a certain type of lamp is an exponential random variable with mean equal to 40 hours. Use the appropriate Gamma Distribution to compute the probability that the average of the times until burnout of N lamps for $N = 1, \dots, 25$ is at most 48 hours.

Problem 4 For each N from 1 to 25, let x_N^* be the answer by the approximate method of Problem 2 and let x_N be the answer by the exact method of Problem 3.

1. Plot the relative error, $\frac{|x_N - x_N^*|}{|x_N|}$, of the methods as a function of N .
2. What conclusions do you draw regarding the methods?
3. Redo this problem with $N = 1$ to 40. What is going on?

Problem 5 Aladdin is lost in the Forest of Forgetfulness. Standing in the glade of decision Aladdin sees two paths. One leads out of the forest in two days while the other returns Aladdin to the glade in three days. Aladdin takes a path. If Aladdin returns to the glade he gamely sets off down one of the two paths immediately. There is no fear of Aladdin starving since the forest is full of apple trees, the apples from which he munches merrily every day. Unfortunately the apples make Aladdin forget his past decisions and if he returns to the glade, he is equally likely to take either path. What is expected number of days until Aladdin gets out of the forest.