

Homework Problems 5
Due Wednesday, October 1, 2008

Problem 1 A plane has one hundred people on it. Assume that the plane is full and seats 50 people on the left side and 50 on the right side. Assume that the weights of the people are independent random variables with a pdf having a mean of 90 kg and a standard deviation of 17 kg. Using the Central Limit Theorem, what is the probability that the weight on the left side of the plane and the right side of the plane differ by more than 400 kg.? I.e., with L_1, \dots, L_{50} the weights of the people on the left hand side and R_1, \dots, R_{50} the weights of the people on the right hand side of the plane, what is

$$1 - P \left(-400 \leq \sum_{i=1}^{50} L_i - \sum_{j=1}^{50} R_j \leq 400 \right)?$$

Problem 2 You have two lamps with bulbs in your room. Let T_1, T_2 be the times to failure of the first and second bulbs respectively. Assume that these random variables are independent with exponential probability density functions f_1 and f_2 with parameters $\lambda_1 = 1/10$ and $\lambda_2 = 1/20$ respectively. Thus

$$f_i(x) = \begin{cases} \lambda_i \exp(-\lambda_i x) & \text{for } x \geq 0 \\ 0 & \text{for } x < 0 \end{cases}$$

Assume both lamps are on continuously. Let X denote the time until a bulb burns out and let Y denote the time until both bulbs burn out, i.e., let $X = \min\{T_1, T_2\}$ and $Y = \max\{T_1, T_2\}$. Compute $E(X)$ and $E(Y)$.

Problem 3 Consider the function

$$f(x) = \begin{cases} c(x - x^2) & \text{for } 0 < x < 1 \\ 0 & \text{otherwise,} \end{cases}$$

where c is a constant. Assume that $f(x)$ is a density function for a random variable X . What must c be?

Problem 4 X and Y are random variables with density function

$$f(x, y) = \begin{cases} x + y & \text{for } 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

1. Find the marginal probability density functions $f_X(x)$ for X and $f_Y(y)$ for Y .
2. Find the conditional density $f_{X|Y}(x|y)$.
3. Are X and Y independent?

Problem 5 If X_1 and X_2 are independent exponential random variables each having parameter λ , find the joint density of $Y_1 = X_1 + X_2$ and $Y_2 = e^{X_1}$.