Homework Problems 10
Due Monday, December 8, 2008

Problem 1 *Let $g(x, y)$ be a harmonic function, i.e.,*

$$\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} = 0.$$

Let $(B_1(s), B_2(s))$ be a two-dimensional Brownian motion. Show that

$$dg(B_1(s), B_2(s)) = \frac{\partial g}{\partial x}(B_1(s), B_2(s))\,\mathrm{dB_1(s)} + \frac{\partial \mathrm{g}}{\partial \mathrm{y}}(\mathrm{B_1(s), B_2(s)})\,\mathrm{dB_2(s)},$$

which shows, in particular, that $g(B_1(s), B_2(s))$ is a martingale.

Problem 2 *Assume B_t is a Brownian motion for $t \geq 0$.*

1. *Compute dX_t where*
$$X_t := \left(e^{\frac{t}{2}} \cos(B_t)\right)$$
for $t \geq 0$.
2. *Is X_t a martingale?*

Problem 3 *Assume B_t is a Brownian motion for $t \geq 0$.*

1. *Compute dX_t where*
$$X_t := \left(-\frac{B_t^4}{6} - \frac{B_t^3}{3} + tB_t^2 + tB_t - \frac{t^2}{2}\right)$$
for $t \geq 0$.
2. *Is X_t a martingale?*