Math 60850: Sample Exam 2
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Problem 1 (20 points) *A plane has one hundred people on it. Assume that the plane is full and seats 50 people on the left side and 50 on the right side. Assume that the weights of the people are independent random variables with a pdf having a mean of 90 kg and a standard deviation of 17 kg. Using the Central Limit Theorem, what is the probability that the weight on the left side of the plane and the right side of the plane differ by more than 400 kg.? I.e., with* $L_1, \ldots, L_{50}$ *the weights of the people on the left hand side and* $R_1, \ldots, R_{50}$ *the weights of the people on the right hand side of the plane, what is*

$$1 - P\left(-400 \leq \sum_{i=1}^{50} L_i - \sum_{j=1}^{50} R_i \leq 400\right)?$$

Problem 2 (30 points) *A particle forms a random walk on a graph with* 5 *vertices labeled* $1, 2, 3, 4, 5$ *and probability transition matrix*

$$P := \begin{bmatrix} 1/4 & 1/4 & 1/4 & 1/4 & 0 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 0 & 1/4 & 1/4 & 1/4 & 1/4 \end{bmatrix}$$

1. *discuss the equivalence classes of intercommunicating states, i.e., what are they and which are transient.*
2. *Starting at vertex* 1*, what is the mean number of steps until the particle's first return to vertex* 1*?*
3. *Starting at vertex* 1*, what is the mean number of steps until the particle's first visit to vertex* 5*?*

Problem 3 (50 points) *Consider an homogeneous Markov chain with state space* $\mathcal{S} := \{1, 2, 3, 4, 5, 6\}$ *with transition matrix*

$$P := \begin{bmatrix} 0 & 1/4 & 1/4 & 1/4 & 0 & 1/4 \\ 1/5 & 1/5 & 0 & 1/5 & 0 & 2/5 \\ 0 & 0 & 1/4 & 3/4 & 0 & 0 \\ 0 & 0 & 1/3 & 2/3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3/8 & 5/8 \\ 0 & 0 & 0 & 0 & 3/4 & 1/4 \end{bmatrix}.$$

1. *Using the communication equivalence relation, break up the states into equivalence classes.*

2. *Which classes are transient and which are persistent?*

3. *Compute exactly all the stationary distributions for each irreducible class of* $\mathcal{S}$*. If there is only one for any of the classes explain why this is so.*

4. *Compute exactly, for each transient state* i *and each irreducible persistent class* C*, the probability that starting at* i *you are eventually in* C*.*

5. *Using the information in parts 1 to 4, write down* $\lim_{n\to\infty} P^n$*.*

6. *Compute* P^{100} *numerically. Does it agree approximately with the result in part 5?*

Note in parts 3, 4 and 5 exact answers are required, i.e., if this was a test, you would not receive much credit for an approximate answer.