

Spanwise Aero-Optical Measurements in a Subsonic Turbulent Boundary Layer to Study Velocity Field

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Aero-optical distortions of a subsonic boundary layer in a spanwise direction were experimentally investigated at different spatial resolutions. From these data, convective speeds were extracted as a function of distance from the wall and were found to agree with mean velocities, measured using a hot-wire. Deflection angle power spectra were also presented and analyzed. Several methods were introduced to mitigate corrupting effects, influencing deflection angle spectra. True OPD_{rms} and θ_{rms} profiles were computed and the influence of spatial resolution on the accuracy of the measurements was identified and discussed. A constant, A, was proposed to exist between OPD_{rms} and θ_{rms} that was found to decrease linearly from a value at the wall of A=15, to A=5 at y/ δ =0.4 where it remains constant for the remainder of the boundary layer. Finally, Sutton's linking equation was implemented to extract spanwise correlation lengths, Λ_z . The spanwise correlation length was found to linearly increase throughout the full boundary layer. With known Λ_z and the A-constant, θ_{rms} data can be used to non-intrusively extract u_{rms} profiles.

I. Introduction

FOR turbulent flows, the turbulent structures cause density fluctuations which in turn alter the speed of light passing through the region spatially as well as temporally. This issue is generally referred to as the aero-optic problem [1,2] and has been one of the hindrances in furthering our capacity for using lasers on airborne platforms. Understanding the temporal and spatial turbulent fluctuations in a given flow would allow using lasers for high-speed and secure communication across hundreds of miles or as precision targeting systems for military purposes. It also provides a valuable insight into fundamental dynamics and properties of turbulent flows, as information about turbulent structures is "imprinted" onto the laser beam.

The aero-optic problem is a direct result of the relationship between index-of-refraction, *n*, and density of air, ρ , via the Gladstone-Dale constant, K_{GD} (which is approximately 2.27×10⁻⁴ m³/kg in air for visible wavelengths of light) by,

$$n(\vec{x},t) - 1 = K_{GD}\rho(\vec{x},t) \tag{1}$$

where both index-of-refraction and density are functions of space and time. Unsteady turbulent structures in a flow will vary the index-of-refraction, deflect, distort and scatter the focused laser, and decrease the intensity of the beam at distances far away from the emitting device. To determine the extent to which turbulent density fluctuations effect the propagation of light, an Optical Path

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Length (OPL) is defined as the integral of the index-of-refraction of a medium along the physical length traversed by a ray of light. Since index-of-refraction and density are related via Equation (1), OPL can be expressed as

$$OPL(x, y, t) = \int_{a}^{b} n(x, y, z, t) dz = \int_{a}^{b} [K_{GD} \rho(x, y, z, t) + 1] dz.$$
(2)

where *z* is the direction of beam propagation. The resulting deviation from the space-average OPL can then be expressed as the Optical Path Difference (OPD),

$$OPD(\bar{x}, t) = OPL(\bar{x}, t) - \overline{OPL(\bar{x}, t)},$$
(3)

where the overbar denotes spatial averaging.

Turbulent boundary layers are always present on an airborne platform and affect the outgoing laser beam even in the absence of point-and-track turrets. They are also one of the important fundamental flows with complex dynamics. Aero-optical properties of boundary layers have been extensively studied both experimentally [3 and references therein, 4, 5], and numerically [6].

In previous studies [4, 5, and 7] the laser beam was propagated through the boundary layer either in a wall-normal direction or close to it, so the aero-optical effects were integrated along the beam, thus losing information about wall-normal variation of various boundary layer properties. In this study, the laser beam was transmitted in a spanwise direction in order to study this wall-normal variation.

A high-speed Shack-Hartmann wavefront sensor (WFS), was used to measure temporal-spatial variation of aero-optical distortions. As aero-optical distortion convect with turbulent structures, cross-correlating aero-optical aberration between two or more nearby spatial points provides a direct measurement of a convective speed of the structures [3,6,7,8]. By sending the laser beam in the spanwise direction, the flow velocity and density statistics of a turbulent boundary layer as a function of the wall normal distance can be directly and non-intrusively measured. Thus, wavefront sensors can be used as complimentary sensors, which, combined with traditional sensors like hotwires and PIV, can gain a deeper understanding of the nature of turbulent structures. In this study, hot-wire measurements were taken at the same location as the beam, for comparison purposes.

For any wavefront sensor, integrated density is essentially the variable that is measured. There are very few other direct methods of obtaining density profiles besides measuring temperature and pressure at every point of interest. One of these is the use of acetone seeding and laser induced fluorescence [9]. This method has been proven to be effective yet carries with it a number of issues regarding calibration. Because of sensitivity to the density field only, any optical-based sensors, including wavefront sensors have a unique potential. Sutton [10] derived a theoretical formulation of a "linking equation" between turbulence quantities and OPD_{rms}. In a simplified form, it is given as,

$$OPD_{rms}^{2} = 2K_{GD}^{2} \int_{0}^{L} \rho_{rms}^{2}(s) \Lambda_{\rho}(s) \, ds, \tag{4}$$

where OPD_{rms} is the spatial root-mean square of OPD over an aperture, ρ_{rms} is the root-meansquare density fluctuations, and Λ_{ρ} is the density correlation length along the beam propagation, *s*. This equation has been validated both experimentally [3,5] and numerically [6,8]. Typically, the laser beam passes the boundary layer in the wall-normal direction and some estimates should be made to calculate the density variation along the beam [3,5]. However, by passing the beam in the wall-parallel direction, where the wall-normal direction, y, is constant, both the density fluctuations and the correlation length in the spanwise direction, $\Lambda_Z(y)$, are constant along the beam and Eq. (4) becomes

$$OPD_{rms}^{2}(y) = 2K_{GD}^{2}\rho_{rms}^{2}(y)\Lambda_{Z}(y)L.$$
(5)

This allows for direct measurements of the product $\rho_{rms}^2(y)\Lambda_Z(y)$ as a function of the distance from the wall and gaining a further insight into the density structure in turbulent boundary layers.

II. Classical Convective Velocity

Convective velocity is typically computed using a time-delayed two point correlation between flow parameters at two spatial locations in the streamwise direction [18, 19, 20] The resulting time delay, coupled with the known separation between points, yields a velocity describing how flow parameters convect downstream. Embedded in this calculation is the assumption that the individual flow parameters remain coherent enough to be correlated. This assumption, known as Taylor's Frozen Flow hypothesis, has been the topic of much scrutiny and examination [21, 22]

It has been assumed in the past that the convective velocity and the local mean velocity are interchangeable. While this may work as a first guess, recent studies [21] have revealed that this is not strictly the case. Two factors have been linked to the discrepancy between the convective and the local mean velocities, namely a wall normal location and a streamwise wavelength, λ_x . It is likely that these two factors are connected or at the least, it is difficult to separate their influence, however we will attempt to discuss them separately.

Del Alamo and Jimenez [21, 23], in their computational work on channel flows, found that the convective velocity near the wall appears to asymptote to a non-zero constant. In addition, while decomposing the convective velocity into large and small wavelengths revealed that the largest wavelengths convect at a nearly constant velocity independent of wall normal location and the small wavelengths generally follow the mean velocity everywhere except near the wall. In a similar study using PIV, LeHew et al [24] found agreement with the results of Del Alamo and Jimenez. Even though there is a wavelength dependence, there remains a discrepancy between the convective and the local mean velocities right next to the wall. This near wall deviation begins near the bottom of the log region, and interestingly, is still left unexplained.

As to the wavenumber dependence, the results of Del Alamo and Jimenez show that the convective velocity increases with λ_x near the wall and decreases with λ_x away from it. The point of this transition where the convective velocity matches the local mean velocity was suggested by Chung and McKeon [25] to be coincident with the large scale streamwise energy peak in the log region. Jimenez [26] showed that the correlation height of eddies in a channel is roughly proportional to the magnitude of their wall-parallel wavelength, and that it reaches the half-width of the channel for the global modes. This is consistent with both observations that small eddies move with the local mean velocity and that the convection velocity of the global modes is roughly proportional to the bulk velocity U_b.

As was mentioned at the outset of this section, Taylor's frozen flow hypothesis establishes a link between the frequency, f, and streamwise wave number, $k_x = 2\pi /\lambda_x$. In essence, this requires that the turbulence is frozen or evolves on a timescale much larger than the advective one, such that the conversion from space to time (or vice versa) can be made by using the local mean velocity as the convection velocity. To correct the velocity discrepancy, Del Alamo and Jimenez [21] and Moin [22] suggest using resolved spectral information in either space or time and on a local derivative in the other. This approach seeks to minimize the variance in the total derivative and as a result retains the frequency/wavenumber dependence of the convective velocity.

III. Experimental Setup

Wind tunnel tests were conducted to firstly understand the optimal operating parameters for spanwise optical deflection angle measurements. What is the necessary sampling frequency and spatial resolution to properly resolve optical distortions? What types of corrupting influences should one expect when collecting data? Can those corrupting influences be mitigated or removed? Second, these tests were conducted to understand what types of information can be learned about the flow from optical measurements, for example, the skin friction coefficient or spanwise correlation length. Lastly, the main purpose of this paper is to illustrate how to directly relate non-intrusive optical measurements to velocity measurements.

All measurements were conducted in the Whitefield facility at University of Notre Dame. The Whitefield facility is a Mach 0.6 closed-circuit wind tunnel powered by a 1750 horsepower variable RPM AC motor. It can be run at variable speeds at very low turbulence levels (< 0.4%). The tunnel has three interchangeable test sections that are each 9 m in length with 1 m² square cross sections. Each of the four sides have three windows that are 0.61 m² which are designed to give flexible optical access to the tunnel. The windows have the additional flexibility of being interchanged with aluminum inserts that can be used for mounting test models or other test section geometry. The boundary layer development plate used in this research is constructed of aluminum and elevated into the freestream using a series of 12 inch tall leg supports. The downstream end of the plate has an adjustable angle flap to establish a desired pressure gradient (in this study, a zero pressure gradient was used). The plate has a 0.5 inch gap between it and the wall on either side in an effort to minimize the influence of corner vortices on optical measurements [17].

A CTA hot-wire was used to map the boundary layer created by the plate and establish a baseline for further optical measurements. The hot-wire was calibrated in the range of Mach numbers 0.1 - 0.4, and the data was found to be in good agreement with a standard King's Law Fit [11]. To calibrate the hot-wire, the tunnel was run at known speeds and voltage data from anemometer were collected. A best fit of the data when plotting voltage versus the known velocity yields the King's Law constants A and B for n = 0.5. After the calibration, time series of the streamwise velocity were measured at 51 wall-normal locations with the sampling frequency of 30 kHz for 30 sec. From the velocity data, both mean and fluctuating velocity profiles were extracted. The skin friction velocity, u_{τ} , needed to compute the inner units, was calculated from C_f as $u_{\tau} = U_{\infty} \sqrt{C_f / 2}$. The skin friction coefficient, C_f , in turn, was computed from Re $_{\Theta}$ using the

second Coles-Fernholz 2 relation with modified constants $C_f = 2[2.604 \ln(\text{Re}_{\theta}) + 4.127]^2 [13]$. Several characteristics of the studied boundary layer are provided in Table 1.

Mach Number	Boundary Layer Thickness (δ)	Free Stream Velocity, U_{∞}	Friction Velocity, u _t	Reθ	Ret	H factor
0.3	25 mm	100.8 m/s	3.2 m/s	19,975	5,517	1.33

Table 1: Boundary layer parameters.

The Shack-Hartmann WFS was used to measure spatially and temporally resolved deflection angles. WFS uses a collimated beam to interrogate the flow. The laser beam was expanded to a 50-mm beam and forwarded through the boundary layer in the spanwise direction, as shown in Figure 1. After being reflected back by the return mirror along the same way it came and increasing the signal by a factor of two, the returned beam was split off and forwarded onto the high-speed camera, a Phantom v1610. Data were collected by the camera at a variety of sampling frequencies from 130 kHz up to 311 kHz. To provide different spatial resolutions, different sets of re-imaging lenses were used. A full list of all test cases and related parameters is shown in Table 2. The camera had a 38 mm focal length, with a 70 x 60 lenslet array attached, which splits the incoming beam into a rectangular array of smaller beams and focuses each of them onto the sensor. Knowing the instantaneous dot position's displacement from the center, Δx or Δy , and the focal length of the lenslet array, the temporal deflection angle at different wall-normal locations, $\theta(x, y, t) = \Delta x/f$, were reconstructed using a small angle approximation as shown in Figure 2.

Test Case	Mach Number	Wall-normal Extent and Spatial Resolution (in parenthesis) in Outer Units (δ)	Wall-normal Extent and Spatial Resolution (in parenthesis) in Inner Units (+)	Sampling Frequency
1 (high resolution)	0.35	0.12δ (Δ=0.012δ)	$680+(\Delta^+=68)$	311 kHz
2 (medium resolution)	0.3	0.19δ (Δ=0.019δ)	$1090+(\Delta^+=109)$	311 kHz
3 (low resolution)	0.3	0.48δ (Δ=0.048δ)	$2740+(\Delta^+=274)$	311 kHz
4 (low resolution)	0.3	1.06δ (Δ=0.048δ)	$6042+(\Delta^+=274)$	130 kHz
Hotwire	0.3	1.2δ (Δ=0.01δ)	6840+ (Δ+=57)	30 kHz

a labe 2: Experimental test cases and associated paramete



Figure 1. Schematic of a Shack-Hartmann wavefront sensor in a double-pass measurement configuration.



Figure 2. Calculating the deflection angles in a lenslet array due to abberated wavefronts.

In an effort to accurately measure the wall normal distance of the first row of dots, the beam was aligned with the lenslet array so that the peak intensity of the first row of dots was half the intensity of the row above. If each dot is produced from light entering a square aperture centered at the dot location, clipping half of the light from entering the aperture using the wall edge should result in the wall being located at exactly the location of the dot. Therefore, the row of half intensity dots is the wall location and the full intensity dots of the row above are the first row of data.

IV. Calculating Convective Velocity

Spatially-temporally-resolved optical wavefronts were used to compute convective speeds at different wall-normal locations using the two-point cross-correlation method. From the measured deflection angles, the spectra of the streamwise component of the deflection angles, $\hat{\theta}(f)$, were computed for each lenslet sub-aperture. A critical assumption is made in calculating the convective

velocity from $\hat{\theta}(f)$ spectra, namely the frozen flow assumption. This assumption allows the deflection angle spectra between two parallel beams, separated by a small distance, Δ , in the flow direction, to be correlated and a convective time delay, τ , between two signals to be calculated. Thus, the convective velocity can be experimentally calculated from the argument or the phase of the spectral cross correlation, $S(f) = \langle \hat{g}_1(f) \hat{g}_2^*(f) \rangle$, where $\hat{g}_1(f)$ and $\hat{g}_2(f)$ denote the Fourier transforms of the time series of deflection angles from the upstream and downstream beams, respectively, and the star denotes the complex conjugate. Knowing the phase slope, the convective speed can be calculated as $U_c = \Delta/\tau$, where the time delay τ is computed from the slope of the argument, $dArg[S(f)]/df = 2\pi\tau$. However, an advantage of the Shack-Hartmann sensor is that it has multiple points in the streamwise direction that can be correlated many times, the so-called multi-point cross-correlation method [7]. For example, a data set has 10 points in the stream wise direction. These 10 points can be each correlated with their neighbor for 9 unique correlations. In addition, each point can be correlated with the point two away for 8 more unique correlations. This process can be stretched as far as the number of available streamwise points but to not push the boundaries of the frozen flow assumption between points, this current analysis limits the multicorrelations to the first 4 seperations.

The phase slope however, is unfortunately not always uniquely defined. For example, take a sample of phase data from the low spatial resolution case plotted in Figure 3. It is almost always the case that below 1 kHz, mechanical vibrations dominate the signal and result in little to no coherence in the phase. As a result, the slope needs to be practically computed apart from this influence. However, small variations in slope in this region have led to large differences in the computed convective velocity. The cause behind these subtle variations is still not fully understood but it is suspected that different scale structures at a given height in the boundary layer convect at different speeds and these scales occupy particular frequency bands. This presents a difficulty when computing convective velocity because there is no obvious way to sort these subregions of the phase plot as they are neither consistent in the wall normal direction or in the streamwise direction. This leaves two options: either one manually computes convective velocity for each individual frequency band for 100 (at minimum) lenslet correlations, or one maximizes the frequency bounds of the slope fitting algorithm to include everything except the low end corruption and have a composite calculation that lumps together all the subtle slope variations. Option one would no doubt result in a very interesting analysis of density scale evolution in the boundary layer, but option two is the more robust due to averaging out the smaller variations. However, due to the inclusion of hotwire measurements in this study, the mean profile is known and can be used in the reverse to understand which frequency bands have a slope that is consistent with a convective velocity equal to the mean velocity.



Figure 3. Sample phase plot illustrating low end corrupting influences and subtle slope inconsistencies.

This analysis was used to generate the convective velocity profiles in Figure 4. In addition, this brought to light the influence of spatial resolution and the related aperture effects on the turbulence scales that are most visible. It has been shown in [3] that increasing aperture acts as a low-pass filter and the frequency bounds used to generate the profiles Figure 4 follow this trend. The high spatial resolution data required frequency bounds between 20 and 40 kHz to match the phase slope of the hotwire profile, whereas the low spatial resolution data (illustrated in Figure 3) needed bounds of 5 to 12 kHz.

The log region of the profile is of particular interest. If the slope of the log region matches the hotwire mean profile, the friction velocity, u_{τ} , can still be extracted non-intrusively. Figure 5 shows the comparison of mean and convective velocity data sets, along with the theoretical line in the inner units, $U^+ \equiv U/u_{\tau} = 1/\kappa \cdot \ln(y^+) + C$. The slope of the convective velocity profile matches that of the hotwire profile well. The tapering of the profiles to a constant near the wall has been observed by several other researchers [21, 22, 24] yet it would appear in our case to be dependent on the spatial resolution. The low spatial resolution data appears to taper to a u⁺ of 20 where the medium and high spatial resolution data tapers to a u⁺ of 18.5. It is suspected that the low spatial resolution case is unable to accurately resolve the small scales, which move slower than the large scales, and results in a higher convective velocity at the point nearest the wall.

Overall, convective speeds inside the log-region, extracted from the deflection angle data, are correctly measured. Thus, the velocity profiles, extracted non-intrusively using the spanwise optical set-up, can be used to extract the friction velocity, u_{τ} , and the skin friction coefficient, C_{f} , via the Clauser method.



Figure 4. Convective velocity profiles for all data sets (solid) and hotwire mean profile (dashed) for comparison.



Figure 5. Convective velocity profiles in inner units.

V. Analysis of Optical Spectra

Power spectra of the deflection angles at different wall-normal locations were calculated and presented in Figure 6. The power spectra of the local deflection angles can be used to provide an insight into the scale and magnitude of density structures as a function of wall normal location. Figure 6 shows how the shape and magnitude of the power spectra evolve throughout the boundary layer for the low resolution data. Very close to the wall, the amplitude is at a local minimum and the peak is centered at a higher frequency of $St_{\delta} = 2.5$. This peak occurs consistently at Strouhal number 2 to 2.5 near the wall up to $y/\delta = 0.15$, that is inside the log-region, with a power spectra magnitude roughly 20% of the maximum found in boundary layer at $y/\delta = 0.5$. Moving away from the wall into the outer region, for $y/\delta = 0.15$ to 0.5, the amplitude increases and the peak location shifts to lower frequencies, which corresponds to larger structures in this region. The high frequency end of the spectra, corresponding to the smaller structures, has a minimal effect on optical distortions due to the wavefront power spectrum and deflection angle power spectrum being related through the convective velocity and a factor of $(2\pi f)^2$. The bulk of the distortions are due to larger, lower frequency structures in the outer region of the boundary layer, above $y/\delta=0.5$ that all have a characteristic size of δ , corresponding to a Strouhal number of 1. The decrease in magnitude above $y/\delta=0.5$ is again due to the reduction in density fluctuations as the boundary layer location approaches the freestream. The trends in the spectra in the current study are consistent with those of previous findings [16,17]. It should be noted that the low end proportionality to $\sim f^2$, which results in a finite wavefront spectrum as frequency approaches 0, is not observed which indicates significant corrupting influences.



Figure 6. Deflection angle power spectra evolution through the boundary layer with peak locations.

As will be shown in the following section, deflection angle power spectra form the basis for computing true or uncorrupted OPD_{rms} . The significant low end corruption that was observed in Fig. 6, drastically changed computed OPD_{rms} . In an effort to mitigate this corruption, a method developed by Smith et al. [7] was utilized. The method uses several spatial correlations of data at varying separations, similar to the method mentioned in the previous section, to split the spectrum into a convective portion and a stationary portion. Equation 6 shows the decomposition of the cross correlation, $S(\Delta x, f)$:

$$S(\Delta \mathbf{x}, \mathbf{f}) = \langle \hat{\theta}(\mathbf{x}, f) \hat{\theta}^*(\mathbf{x} + \Delta \mathbf{x}, f) \rangle = \left| \hat{\theta}_S(f) \right|^2 + \left| \hat{\theta}_T(f) \right|^2 \exp\left(\frac{2\pi i f \Delta \mathbf{x}}{U_c}\right)$$
(6)

where $\hat{\theta}_S$ is the stationary spectra, $\hat{\theta}_T$ is the convective (traveling) spectra, and U_c is the convective velocity at that location in the boundary layer. With multiple correlations, this equation is overdetermined and a least squares solution can be obtained at each frequency for $\hat{\theta}_S$ and $\hat{\theta}_T$. For a detailed analysis of this method, see the discussion in [7].

In theory, the stationary portion would contain all the corrupting elements due to mechanical vibration, optical components, or electronic interference and could then be thrown out. The remaining convective spectrum would be the true spectrum due to the flow itself. This multipoint analysis has been used successfully even with severely undersampled data that had good spatial resolution.

In the current work, the multipoint decomposition was used with somewhat mixed success. A sample of this decomposition is shown in Figure 7. It can be seen that the low end build up was completely isolated from the convecting portion and the theoretical slope was recovered. However, the least squares solution consistently resulted in a convective spectrum that was larger in magnitude than the total spectrum. The reason for this is still not known and currently is under investigation. The potential consequences of this issue are discussed in the following sections.



Figure 7. Power spectra decomposition using Eq. 6.

Two additional methods were utilized in the current work to mitigate the deflection angle spectral corruption. The first was simply replacing the spectral content below a Strouhal number of 1 with the theoretical slope of f². While this greatly increased the consistency of the OPD_{rms} calculation, there still remains a significant drop off in spectral content at the high end of the spectrum compared to the theoretical f^{4/3} slope. This drop off most strongly impacts the calculation of $\theta_{\rm rms}$ and should be removed if possible. A simple model was used by Smith et al. [27] to approximate the shape of deflection angle power spectra. The model was adapted to the current study so that power spectra can be reconstructed with only the peak amplitude, the peak Strouhal number, and Re₀. The model is as follows,

$$\left|\hat{\theta}\right|_{fit}^{2} = \left|\hat{\theta}\right|_{peak}^{2} \frac{6.5}{St_{\delta,peak}^{2}} \left[\frac{St_{\delta}}{1 + \left(\frac{1.25}{St_{\delta,peak}}\right)^{5/3}}\right]^{2} exp\left[-\left(\frac{St_{\delta} - 8}{9.56 \, Re_{\theta}^{0.082}}\right)^{2}\right].$$
(7)

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The benefit of this equation is that it drastically reduces the complexity of removing the corrupting influences at both the high and low end of the spectrum and simply uses peak quantities in the middle of the spectrum. Figure 8 shows spectra from four different data sets at a constant height in the boundary layer as well as the Eq. 7 model fit. The model fit captures the central portion of the spectrum excellently but fills out the spectrum at both ends.



Figure 8. Convecting power spectra using Eq. 6 and model fit.

In addition to highlighting how the model performs, Figure 8 also shows the spectral influence of optical magnification. The medium and high spatial resolution data deviate slightly from each other at a Strouhal number of about 14 but are more or less collapsed. The low spatial resolution data sets drop off far sooner at a Strouhal number of 3. The effects of this will be discussed in more

depth in future sections but for the present, θ_{rms} is greatly reduced for the low spatial resolution cases. In addition, because this falloff is so drastic, it actually skews the spectral peak to lower Strouhal numbers and thus yields a slight increase in OPD_{rms}.

VI. Computing uncorrupted OPD_{rms} from Deflection Angles

One issue in analyzing directly the wavefronts is that they in practice are always affected or corrupted by the aperture size, the so-called aperture effects, see [3]. One way to go around this problem is to study the statistics of the local deflection angles instead. To compute uncorrupted OPD_{rms} , deflection angle spectra at each wall-normal location were integrated along the spanwise direction as,

$$OPD_{rms}^{2}(y) = 2U_{C}(y)\int_{0}^{\infty} \frac{\left|\hat{\theta}(f)\right|^{2}}{\left(2\pi f\right)^{2}} df.$$
 (8)

As was mentioned previously, due to the factor of f^2 in the denominator, this equation is highly sensitive to the shape of the low end spectrum. Any contamination is dramatically amplified and OPD_{rms} estimate becomes completely unreliable. Thankfully, the methods discussed in the previous section were able to significantly reduce the impact of low end contamination. In addition, the factor of f^2 also renders anything above a Strouhal number of 2 or 3 to be inconsequential towards OPD_{rms}. While it is important for other reasons to have a clean upper end of the spectrum, the OPD_{rms} calculation is largely insensitive to any high end contamination.

In Section V it was shown that increasing optical magnification skewed the spectral peak to lower Strouhal numbers as well as increased the high end spectral drop off rate. The high end spectral differences are inconsequential for the OPD_{rms} calculation but the amplitude of the lower half of the model fit is determined by the spectral peak location. Equation 8 was used to compute OPD_{rms} profiles throughout the boundary layer and Figure 9 shows these profiles for the various test cases. The influence of magnification rate can be seen clearly in the first 20% of the boundary layer where the low spatial resolution cases have slightly larger OPD_{rms} values. Apart from the near wall discrepancy, Figure 9 shows the bulk of optical distortions residing between $y/\delta=0.4$ and 0.8, consistent with previous studies [3, 4, 7].



Figure 9. OPD_{rms} profiles computed using Eq. 9.

VII. Link Between θ_{rms} and OPD_{rms}

One of the goals of quantifying optical distortions in the spanwise direction of the boundary layer is to establish a non-intrusive optical method, the results of which can be used to extract velocity-related statistics. As was mentioned in the Introduction, a link needs to be established between θ_{rms} and uncorrupted or true OPD_{rms} and from there to velocity statistics. By means of Equation 8, we have an independent calculation of OPD_{rms} that is free of aperture contamination that can be used to that end.

Using the spectral integral definition of r.m.s. and assuming spectral self-similarity in the deflection angle power spectra at a given height in the boundary layer, it can be shown that a constant exists linking θ_{rms} and OPD_{rms}. This constant is only dependent on the wall normal location, the boundary layer thickness, and the local convective velocity profile.

$$A(y) = \frac{\theta_{rms}(y)}{OPD_{rms}(y)} \frac{U_c(y)}{U_{\infty}} \delta$$
(9)

Using experimental data for different spatial resolution cases, wall normal profiles for θ_{rms} were computed and are plotted in Figure 10. Since the defect in the lower spatial resolution data is a shift in the location of the spectral peak rather than a change in amplitude, in Figure 10 there is better collapse across all test cases. This is due to the integral definition of r.m.s. being sensitive only to the "area under the curve" which in turn is determined by the peak amplitude and not the peak Strouhal value. In contrast to the OPD_{rms} profile in Figure 9, the larger values of the θ_{rms} profile are closer to the wall, centered on $y/\delta=0.2$. This means that smaller, near wall structures contribute most to θ_{rms} while larger outer structures contribute most to OPD_{rms}.





Combining Figures 9 and 10 with Eq. 9 yields the profiles for A-constant in Figure 11. With larger relative OPD_{rms} and smaller relative θ_{rms} , the low spatial resolution data has a much lower A-value than the medium and high spatial resolution data in the region closer to the wall, $y/\delta < 0.2$. However, because this effect can be traced back to the high end spectral drop off in the low spatial resolution data, the A-constant value near the wall is believed to be better estimated using the medium and high spatial resolution data. At the wall it reaches a value of A = 15 and decreases somewhat linearly to a value of A = 5 at $y/\delta=0.4$ where it remains constant for the upper half of the boundary layer. This profile will form the link to connect θ_{rms} to true OPD_{rms}.



Figure 11. A-constant profiles computed using Eq. 9.

VIII. Link Between OPD_{rms} and u_{rms}

Returning now to the spanwise form of Sutton's linking equation (Eq. 5), we have a relationship between true OPD_{rms} and ρ_{rms} connected through known constants and an unknown spanwise correlation length, Λ_Z . The link between ρ_{rms} and u_{rms} can be achieved using the Strong Reynold's Analogy. The accuracy and applicability of the SRA has been investigated previously [5] and at least for the case of an adiabatic boundary layer, it was demonstrated to work well. For a detailed derivation of the following relation, see the discussion in [5, 17] but ρ_{rms} can be computed from u_{rms} as:

$$\rho_{rms}^{2}(y) = \rho_{\infty}^{2}(\gamma - 1)^{2} r^{2} M_{\infty}^{4} \left(\frac{U}{U_{\infty}}\right)^{2} \left(\frac{u_{rms}}{u_{\tau}}\right)^{2}$$
(10)

where ρ_{∞} is the free stream density, γ is the ratio of specific heats, and r is the recovery constant taken to be 0.89. With Eq. 10, we now have the last piece to connect θ_{rms} and u_{rms} , however, the spanwise correlation length is still unknown. At this point, hotwire velocity data can be used from one end, spanwise optical deflection angles from the other, and where they meet in the middle should be the missing spanwise correlation length, $\Lambda_z(y)$.

Beginning with the hotwire data, pre-multiplied velocity power spectra are plotted in Figure 12. The peak location is consistent with other canonical boundary layers at a $y^+=200$, $\lambda_x^+=25,000$ [22] however the relatively small boundary layer thickness prevented from fully resolving the peak in the viscous sublayer. Confident in the validity of the hotwire measurements, u_{rms} profiles were computed and using Eq. 10, ρ_{rms} profiles were computed. Comparing the OPD_{rms} and ρ_{rms} profiles through the linking equation, Eq. 5, yields the spanwise correlation length plotted in Figure 13. The first observation is that despite the above-discussed problems with the low spatial resolution data that caused a lack of collapse previously (Figures 10 and 11), there still appears to be a decent

collapse between all four test cases. In addition, the profile appears to be linear for the majority of the boundary. It is consistent with the Townsend's attached eddy hypothesis, which states that the eddy size increases linearly with the distance from the wall, at least up to the end of the log-region [9].



Figure 12. Hotwire Pre-multiplied energy spectra.



Figure 13. Computed spanwise correlation lengths, $\Lambda_Z(y)$.

IX. Conclusions

Experimental measurements of aero-optical distortions in the subsonic boundary layer along the spanwise direction were presented and discussed. A Shack-Hartmann wavefront sensor was used to record temporal sequences of deflection angles imposed on a laser beam. By crosscorrelating time series of deflection angles in a streamwise direction, convective speeds of aerooptical structures were extracted at different distances from the wall. The convective speeds were found to vary significantly, depending on a selected frequency band. Using the hotwire velocity profile as a guide, frequency bands were identified where the convective velocity was equal to the mean velocity. It was found that lower spatial resolution data required lower frequency bands than higher spatial resolution data. Plotted in inner units and using the extracted frequency bands, the convective and hot-wire velocity slopes collapsed in the log-region. This indicates that the WFS can be used to measure the convective velocities inside the log-region and non-intrusively estimate the skin friction coefficient, using the Clauser method.

Deflection angle power spectra at different wall-normal locations were also presented and analyzed. It was shown that significant contamination existed at both the low and high end of the spectrum. Several methods were presented to remove these contaminating effects and a model was introduced to replicate the spectrum with only the peak Strouhal number, peak amplitude, and Re_{θ} as input parameters. It was then shown that low spatial resolution was responsible for the high end drop off in spectra and the effect was strong enough to shift the spectral peak to lower Strouhal numbers.

True OPD_{rms} profiles, free of corrupting aperture effects, were then computed from the corrected deflection angle power spectra. The computation was shown to be highly sensitive to the low end shape and magnitude of the spectrum. Because of this, the low spatial resolution data had an artificially high OPD_{rms} compared to the medium and high spatial resolution data. The peak in the OPD_{rms} profile was observed to occur between $y/\delta=0.4$ and 0.8. The same spatial resolution effect had less of an impact on θ_{rms} , however there was still a slight reduction in θ_{rms} for the low resolution data. To connect θ_{rms} and OPD_{rms}, the A-constant was introduced and computed. It was shown that the A-constant has a value of A=15 at the wall and decreases linearly to A=5 at $y/\delta=0.4$ where it remains constant for the remainder of the boundary layer.

Using the Strong Reynold's Analogy and Sutton's linking equation, it was shown that u_{rms} can be used to compute ρ_{rms} which can then be used to compute OPD_{rms}, and vice versa. However, the spanwise correlation length is an unknown quantity in the linking equation. Using both hotwire velocity data and optical OPD_{rms} data, the spanwise correlation length was extracted as a function of distance from the wall. Collapse was found across all test cases and the spanwise correlation length was observed to increase linearly moving away from the wall. With the A-constant and the spanwise correlation length, Λ_z , θ_{rms} can be used to compute OPD_{rms}, OPD_{rms} can be used to compute ρ_{rms} , and ρ_{rms} can finally be used to compute u_{rms} .

These experiments have shown that a non-intrusive wavefront sensor could be used as valuable complimentary sensor in measuring the properties of velocity and density structures in turbulent flows. One obvious limitation of this non-intrusive measurement technique is that it calculates the

velocity and density properties averaged along the laser beam, thus limiting flows of interest to spanwise-uniform flows. However, the non-intrusive nature of the technique might be very useful in conducting measurements at high supersonic and hypersonic speeds, where there are many difficulties of using intrusive-type sensors.

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