

Math 60210, Basic Algebra, Study questions for Exam 1, Fall 2009
Exam 1 is Tues, Oct. 6, in-class
Do 4 of these 6

1. Let G be a finite group of order 606. Prove that G has a normal Sylow subgroup.
2. Let $\sigma = (1\ 2) \in S_n$, and let C_σ be the conjugacy class of σ .
 - (A) Compute $|C_\sigma|$.
 - (B) Compute the subgroup $C_{S_n}(\sigma)$, i.e., determine the elements of S_n which centralize σ .
3. Let G be a nonabelian group with center $Z(G)$. Show that there exists an abelian subgroup H of G such that $Z(G) \subset H$ but $Z(G) \neq H$.
4. Let G be a group of order p^2 . Prove that G is abelian (note: this is a result we proved in class. On the qualifying exam, it's not a good idea to say we proved it in class; instead recall that the center of a p -group is nontrivial, and use this to prove the assertion).
5. Let G be an infinite simple group. Show that if $H \subset G$ is a proper subgroup, then $[G : H]$ is infinite.
6.
 - (A) Let G be a finite group and let H be a subgroup of G . If $G = \cup_{x \in G} xHx^{-1}$, prove that $H = G$.
 - (B) Let $G = GL(2, \mathbf{C})$ and let $H = T(2, \mathbf{C})$. Prove that $G = \cup_{x \in G} xHx^{-1}$.
 - (C) Let $G = GL(2, F)$, where $F = \mathbf{Z}/3\mathbf{Z}$. Find an element of G that is not in $xT(2, F)x^{-1}$ for all $x \in G$.
7. Let G be a finite group of order $3^{10} \cdot 7$. Prove that G has a proper normal subgroup contained in a 3-Sylow subgroup of G .
8. Let G be a group of order 221. Prove that G is abelian. Is G necessarily cyclic?
9. For each of the following integers k , either find an element of S_{10} of order k , or explain why there is no such element.
 - (A) $k = 7$
 - (B) $k = 21$
 - (C) $k = 25$
 - (D) $k = 35$.