MATH 13150: Freshman Seminar *2007*

Practice Problem Solutions

1. Suppose the olympic basketball coach has 1 center, 5 forwards, and 3 guards on her team. Her starting line-up must have 1 center, 2 forwards, and 2 guards. How many different starting line-ups can she choose?

   Answer:
   $$\binom{1}{1} \times \binom{5}{2} \times \binom{3}{2} = 1 \times \frac{5 \times 4}{2} \times 3 = 30$$

2. **Five** contestants must be chosen for a mathematically themed game show. There are **ten** audience members to choose from, **three** of which are mathematicians. If exactly **one** mathematician is to be on the show, count how many ways contestants may be choosen.

   Answer:
   $$\binom{3}{1} \times \binom{7}{4} = 3 \times \frac{7 \times 6 \times 5}{3 \times 2} = 3 \times 35 = 105$$

3. How many integers are there between 1 and 122 that are not divisible by 6? How many are not divisible 15? How many numbers are there between 1 and 122 which are divisible by 6, but not by 15 ?

   Answer:
   Total is 122. Those divisible by 6 are $6, 12, \ldots, 120$. Dividing by 6 gives $1, 2, \ldots, 20$ so there are 20 numbers divisible by 6. Of the 20 numbers $6, 12, \ldots, 120$ those which are also divisible by 15 are the multiples of $\text{lcm}(6, 15) = 30$. We want to remove these from the list of 20 and count. The multiples of 30 in the list $6, 12, \ldots, 120$ are $30, 60, \ldots, 120$. Dividing by 30 you get $1, 2, \ldots, \frac{120}{30} = 4$. Thus there are $20-4 = 16$ numbers between 1 and 122 which are divisible by 6, but not by 15.

4. There are 4 toppings at the salad bar for you to put (or not put) on the bowl of lettuce you hold in your hands.

   (a) Assuming that you like every topping, how many possible salads could you create?

   Answer:
   $$\binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 2^4 = 16.$$

   (b) How many possible salads could you create that have at least 2 toppings?

Answer:
Use the subtraction principle and part (a): $2^4 - \binom{4}{0} - \binom{4}{1} = 16 - 1 - 4 = 11$.

5. (a) Find $\mathbf{gcd}(20, 48)$ and $\mathbf{lcm}(20, 48)$.

Answer:
Use the Euclidean Algorithm: $\gcd(20, 48) = 4$, Use the formula that relates gcd and lcm: $\text{lcm}(20, 48) = \frac{20 \times 48}{4} = 240$.

(b) Is it possible to write 4 as a combination of 20 and 48? If yes, show a way of doing it. If no, explain why not.

Answer:
Yes, since 4 is a multiple of 4. One possible combination is: $4 = (5) \cdot 20 + (-2) \cdot 48$.

(c) Is it possible to write 6 as a combination of 20 and 48? If yes, show a way of doing it. If no, explain why not.

Answer:
No, since 6 is not a multiple of 4.

6. Find $\mathbf{lcm}(271, 670)$. How does it compare to 271×670 ?

Answer:
Euclidean Algorithm gives $\gcd(271, 670) = 1$ after 5 steps. Thus $\text{lcm}(271, 670) = \frac{271 \times 670}{1} = 181,570$. They are the same.

7. Can you write 1 as a combination of 11 and 29 ? If so, find X, Y so that $1 = X \cdot 11 + Y \cdot 29$.

Answer:
Yes since their gcd is 1. One possible combination is $1 = (8) \times 11 + (-3) \times 29$

8. Express 3 as a combination of 351 and 854.

Answer:
This one takes a while.. One possible answer is $1 = (309) \cdot 351 + (-127) \cdot 854$. Thus $3 = (3 \times 309) \cdot 351 + (3 \times -127) \cdot 854$ so $3 = (927) \cdot 351 + (381) \cdot 854$.