MATH 13150: Freshman Seminar, spring 2010 *Practice for Exam 1*

Instructions: Clearly explain the answers to the following questions. If appropriate, you may do this by indicating steps clearly without many words. Showing your work may help you get partial credit. Note: this practice exam is about twice the length of the exam you will take on Friday, February 19.

1. Suppose the olympic basketball coach has 1 center, 5 forwards, and 3 guards on her team. Her starting line-up must have 1 center, 2 forwards, and 2 guards. How many different starting line-ups can she choose?

   Brief answer: $\binom{1}{1} \times \binom{5}{2} \times \binom{3}{2} = 30$.

2. (a) Find $\mathbf{gcd}(20, 48)$ and $\mathbf{lcm}(20, 48)$.

   Brief answer: $\gcd(20, 48) = 4$. $\text{lcm}(20, 48) = \frac{20 \cdot 48}{4} = 240$.

   (b) Is it possible to write 4 as a combination of 20 and 48? If yes, show a way of doing it. If no, explain why not.

   Brief answer: yes, because 4 is a multiple of $\gcd(20, 48)$. In fact, $4 = 3 \cdot 48 - 7 \cdot 20$, which you can find by listing multiples of 20 and 48.

   (c) Is it possible to write 6 as a combination of 20 and 48? If yes, show a way of doing it. If no, explain why not.

   Brief answer: no, because 6 is not a multiple of $\gcd(20, 48)$.

3. (a) Give the prime factorization of 198.

   Brief answer: $198 = 2 \cdot 3^2 \cdot 11$.

   (b) Decide whether 199 is prime.

   Brief answer: 199 is prime.

4. **Five** contestants must be chosen for a mathematically themed game show. There are **ten** audience members to choose from, **three** of which are mathematicians.

   (a) If exactly **one** mathematician is to be on the show, count how many ways contestants may be choosen.

   Brief answer: $\binom{3}{1} \times \binom{7}{4} = 105$.

   (b) How many ways are there to choose the contestants if at least one mathematician must be chosen?

   Brief answer: $\binom{10}{5} - \binom{7}{5}$.

5. There are 4 toppings at the salad bar for you to put (or not put) on the bowl of lettuce you hold in your hands.

   (a) Assuming that you like every topping, how many possible salads could you create?

   Brief answer: $2^4 = 16$.

   (b) How many possible salads could you create that have at least 2 toppings?

   Brief answer: $2^4 - \binom{4}{0} - \binom{4}{1} = 11$.

6. How many numbers are there between 20 and 120 that are not divisible by 6? How many numbers are there between 20 and !20 that are divisible by 6 but not by 15?

   Brief answer: $121 - 17 = 104$ are not divisible by 6. $17 - 3 = 14$ are divisible by 6 but not by 15.

7. Find $\text{lcm}(271, 620)$. How does it compare to 271×620?

   Brief answer: By the Euclidean algorithm $\gcd(271, 620) = 1$, so $\text{lcm}(271, 620) = 271 \cdot 620 = 168020$.

8. Write 1 as a combination of 11 and 29.

   Brief answer: $1 = 8 \cdot 11 - 3 \cdot 29$.

9. Write 1 as a combination of 351 and 854. Write 3 as a combination of 351 and 854.

   Brief answer: $1 = 309 \cdot 351 - 127 \cdot 854$. Multiply both sides by 3 to get $3 = 927 \cdot 351 - 381 \cdot 854$.

10. Give a prime factorization of 844.

    Brief answer: $844 = 2^2 \cdot 211$.

11. Find the smallest prime bigger than 300.

    Brief answer: 307 (use the sieve of Eratosthenes from 300 to 320).

12. Express $\binom{43}{13} + \binom{43}{14} + \binom{44}{13}$ as a binomial coefficient $\binom{m}{k}$.

    Brief answer: $\binom{45}{14}$.

13. Suppose Cathy is choosing a committee of three kids from a class with 14 boys and 16 girls.

(a) How many ways can she choose the committee?

Brief answer: $\binom{30}{3}$.

(b) How many ways can she choose the committee if it must have 2 boys and 1 girl?

Brief answer: $\binom{14}{2} \cdot \binom{16}{1}$.

(c) How many ways can she choose the committee if it must have at least one girl?

Brief answer: $\binom{30}{3} - \binom{14}{3}$.

14. (a) How many ways are there to pick four different numbers between 5 and 43 if the order in which they are chosen does not matter?

Brief answer: $\binom{39}{4}$.

(b) How many ways are there to pick four different numbers between 5 and 43 if at least one is a multiple of 4 and the order in which they are chosen does not matter?

Brief answer: $\binom{39}{4} - \binom{30}{4}$.