

**MATH 13150: Freshman Seminar**Answers to practice problems for second exam Mar., 2008

1. (a) 160 numbers.  
(b) 12 numbers divide both.  
(c)  $2^4 \cdot 3^2 \cdot 5^7 \cdot 7 \cdot 13$  is the least common multiple.
2. (a)  $11 \cdot 14 \equiv 2 \pmod{19}$   
(b)  $340 + 340 \equiv 339 \pmod{341}$   
(c)  $10 + 10 + 10 \equiv 8 \pmod{11}$ .
3. 40 numbers divide 26,000.
4. 2 numbers divide 211.
5.  $n$  and  $n + 2$  are relatively prime if and only if  $n$  is odd.
6. 135 and 209 are relatively prime.
7. 720 and 1309 are relatively prime.
8. (a)  $\phi(48) = 16$   
(b)  $\phi(210) = 48$   
(c)  $\phi(111) = 72$   
(d)  $\phi(83) = 82$
9. (a)  $\phi(200) = 80$ ,  $\phi(400) = 160$ ,  $\phi(800) = 320$ ,  $\phi(1600) = 640$ .  
(b) 160
10. (a)  $6 \cdot 7 \equiv 2 \pmod{8}$   
(b)  $6 \cdot 7 \equiv 6 \pmod{9}$   
(c)  $9 \cdot 7 \equiv 8 \pmod{11}$
11.  $-8 \equiv 45 \pmod{53}$
12. (a) 3 is the smallest prime dividing  $\binom{101}{5}$ .  
(b) 15 is a composite number dividing  $\binom{101}{5}$ .
13. No, because 7 does not divide  $\binom{48}{18}$ .
14. Yes, because 19 divides 38 in the numerator of  $\binom{48}{18}$  and is not cancelled by a 19 in the denominator.

15. (a)  $180 = 2^2 \cdot 3^2 \cdot 5$  and  $944 = 2^4 \cdot 59$ .  
(b) The gcd is  $2^2$  and the lcm is  $2^4 \cdot 3^2 \cdot 5 \cdot 59$ .
16. (a) 17 and 13.  
(b)  $221 = 17 \cdot 13$ .
17. 223, 227, 229, 233, 239
18.  $211 = 211^1$  is the prime factorization.
19. 521 is the smallest prime larger than 510.
20.  $420 = 2^2 \cdot 3 \cdot 5 \cdot 7$ .
21.  $\phi(201) = 132$  and  $\phi(256) = 128$ , so  $\phi(201)$  is larger.
22. 11 divides  $\phi(23) = 22$ .