Math 60220, Basic Algebra, Problem Set 8, Spring 2010

INSTRUCTIONS: Do at least 6 of these problems. You may assume that k is a field. Due Wed, April 21.

(1) Let R be a commutative ring, and let S be the collection of zero divisors of R (for the purpose of this problem, 0 is counted as a zero divisor). Prove that there exists a prime ideal P of R such that $P \subset S$.
(2) (D+F, # 16, 15.1): Let V be a finite algebraic set in k^n. If V consists of m points, prove that $k[V] \cong k^m$.
(3) (D+F, # 22, 15.1): Let $SL_n(k)$ consists of the $n \times n$ matrices A with coefficients in k such that the determinant of A is 1. Show that $SL_n(k)$ is an affine algebraic subset of k^{n^2}. Show that the map $A \mapsto A^{-1}$ on $SL_n(k)$ is a morphism.
(4) (D+F, # 24, 15.1): Let $V = V(xy - z) \subset k^3$. Prove that the affine algebraic set V is isomorphic to $\mathbf{A}^2$ as affine algebraic sets.
(5) (D+F, # 25, 15.1)
(6) D+F, # 26, 15.1.
(7) D+F, # 28, 15.1.
(8) D+F, # 2, 15.2.
(9) D+F, # 3, 15.2.
(10) D+F, # 3, 15.3: Let k be a field and let i, j be relatively prime positive integers. Let $R = k[x, y]/(x^i - y^j)$, and let K be the fraction field of the integral domain R. Prove that $K \cong k(t)$ and $k[t]$ is the integral closure of R in K.
(11) D+F, # 4, 15.3.