

Math 60210, Basic Algebra, Hints for Problem Set 6, Fall 2009

Hints for Problem 6-7: Let  $p = 2$  in problem 5 and solve the following problems.

(A) Show there exists  $a \in G$  such that  $|a| = 4$ .

(B) If  $g \in G$  and  $|g| = 4$ , show that  $g^2 = z$ , where  $Z = \{e, z\}$ . If  $a \in G$  has order 4, and  $x \in G - \langle a \rangle$ , show that  $xax^{-1} = a^{-1}$  using 5(B).

(C) For  $g \in G$ , let  $\bar{g} = gZ$ . Suppose there exists  $b \in G$  such that  $|b| = 2$  and  $\bar{b} \notin \{\bar{e}, \bar{a}\}$ . Prove that  $G \cong D_8$ .

(D) Suppose there exists  $b \in G$  such that  $|b| = 4$  and  $\bar{b} \notin \{\bar{e}, \bar{a}\}$ . Prove that  $G \cong Q_8$ .