

Math 60210, Basic Algebra, HInt for Problem Set 4, Fall 2009

Hint for Problem 2: It would be nice to find a set S with 3 elements that S_4 acts on. Consider the set

$$S = \{(1, 2)(3, 4), (1, 3)(2, 4), (1, 4)(2, 3)\},$$

consisting of products of pairs of disjoint transpositions. Show that S_4 acts on S by conjugation (Problem 1 is helpful here). Interpret this action as a group homomorphism $\phi : S_4 \rightarrow A_S = S_3$. Note that $\phi(S)$ is a subgroup of S_3 , so to show $\phi(S) = S_3$, it suffices to show that $\phi(S)$ contains a set of generators of S_3 . Now use the fact that ϕ is surjective to compute the cardinality of $\text{Ker}(\phi)$, and use this to compute $\text{Ker}(\phi)$.