

Exam #1

Name: _____

Instructions: Clearly explain the answers to the following questions. If appropriate, you may do this by indicating steps clearly without many words. Showing your work may help you get partial credit. You may use a calculator, but you may only use usual arithmetic keys, and you may not use special keys to compute gcd and lcm. No books or notes are allowed.

Problem	Score
1	
2	
3	
4	
5	
6	
7	
Total	

1. (a) How many ways are there to line up 8 kids in order?

soln is $8!$

(b) Suppose two of the kids are Fred and Joe. How many ways are there to line up the 8 kids in order, if any line-up with Fred first and Joe second is not allowed?

soln is $8! - 6!$

(c) How many ways are there to line up the 8 kids if Fred and Joe must have at least **three** kids between them?

Soln is $8! - 36 \cdot 6! = 20 \cdot 6!$

2. Suppose a class has 11 kids.

(a) How many ways are there to form a committee with 5 kids?

$$\text{sol'n } \binom{11}{5}$$

(b) How many ways are there to divide the class into two committees, if the first committee has 5 kids, the second committee has 6 kids, and two the kids, Alea and Daniel, must be on the first committee?

$$\text{sol'n } \binom{9}{3} = 84$$

3. (a) How many numbers from 17 to 357 are divisible by 4?

$$\text{sol'n } 89 - 5 + 1 = 85$$

(b) How many numbers from 17 to 357 are not divisible by 4 or 6?

$$\text{sol'n } 227 = 341 - 114, 114 = 85 + 57 - 28$$

4. (a) Express $\binom{100}{7} + \binom{100}{8}$ as a binomial number $\binom{n}{k}$.

sol'n $\binom{101}{8}$

- (b) Compute $\binom{100}{7} - \binom{100}{93}$. Is it positive, negative, or zero?

soln is 0

5. (a) Find the greatest common divisor of 91 and 325 using the Euclidean algorithm. Find the least common multiple of 91 and 325.

soln $13 = \gcd(91, 325)$, $7 \cdot 325 = 2275$ is the lcm

(b) Which of the following numbers can be written as a combination of 14 and 21? Explain your answers. If it is possible to write the number as a combination of 14 and 21, then do it.

(i) 7

soln: $7 = 1 \cdot 21 - 1 \cdot 14$ is a combination of 21 and 14.

(ii) 20

soln: 20 is not a combination of 21 and 14 since $7 = \gcd(21, 14)$ does not divide 20 evenly.

(iii) 707

soln: $707 = 101 \cdot 7 = 101 \cdot 21 - 101 \cdot 14$ is a combination of 21 and 14

6. Factor each of the following numbers as a product of primes, or explain why the number is prime.

(a) 329

$$329 = 7 \cdot 47$$

(b) 319

$$11 \cdot 29$$

(c) 3080

$$3080 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 7 \cdot 11$$

(d) 349

soln: 349 is prime. To see this, compute $\sqrt{349} = 18.6\dots$, and check that none of the primes less than 18 divide 349.

7. (a) Write 1 as a combination of 58 and 3.

$$1 = 58 - 19 \cdot 3$$

(b) Write 1 as a combination of 177 and 119

$1 = 39 \cdot 177 - 58 \cdot 119$ is the answer. The steps are:

$$177 = 119 + 58$$

$$119 = 2 \cdot 58 + 3$$

$$58 = 19 \cdot 3 + 1$$

so from last equation,

$$1 = 58 - 19 \cdot 3$$

$1 = 58 - 19 \cdot (119 - 2 \cdot 58) = 39 \cdot 58 - 19 \cdot 119$ by using the second equation, so

$$1 = 39 \cdot (177 - 119) - 19 \cdot 119 = 39 \cdot 177 - 58 \cdot 119.$$