MATH 13150: Freshman Seminar
Exam #2 Practice Problems **Answers**

1. 35 does not divide $\binom{48}{18}$.

2. 19 does divide $\binom{48}{18}$.

3. (a) $180 = 2^2 \cdot 3^2 \cdot 5$. $944 = 2^4 \cdot 59$
   (b) $gcd(180, 944) = 2^2$, $lcm(180, 944) = 2^4 \cdot 3^2 \cdot 5 \cdot 59$

4. $26,000 = 2^4 \cdot 5^3 \cdot 13$ has $5 \cdot 4 \cdot 2 = 40$ divisors

5. Only two numbers divide 211 since 211 is prime.

6. (a) $5 \cdot 2 \cdot 8 \cdot 2 = 160$.
   (b) The gcd of the numbers is $2^2 \cdot 3 \cdot 5$, so the answer is $3 \cdot 2 \cdot 2 = 12$.
   (c) $2^4 \cdot 3^2 \cdot 5^7 \cdot 7 \cdot 13$.

7. (a) Yes: $\frac{3^2}{7^2 \cdot 5 \cdot 11^2}$.
   (b) No, reduced form is $\frac{3^3 \cdot 5^8}{1}$, and 3 occurs to an odd power.
   (c) Yes. 5.

8. Yes, $gcd(135, 209) = gcd(74, 135) = 1$ since $74 = 2 \cdot 37$, and neither 2 nor 37 divide 135.

9. Yes, 720 and 1309 are relatively prime.

10. (a) $\phi(48) = 16$
    (b) $\phi(210) = 48$
    (c) $\phi(111) = 72$
    (d) $\phi(83) = 82$

11. (a) $\phi(200) = 80$, $\phi(400) = 160$, $\phi(800) = 320$, $\phi(1600) = 640$.
    (b) $\phi(400) = 160$.
    (c) $\phi(50) = 20$.

12. (a) $6 \cdot 7 \equiv 2 \pmod 8$
    (b) $6 \cdot 7 \equiv 6 \pmod 9$
    (c) $9 \cdot 7 \equiv 8 \pmod{11}$

13. (a) $39 \equiv -1 \pmod{40}$

(b) $36 \equiv -4 \pmod{40}$

(c) $31 \equiv -9 \pmod{40}$

14. $13 \equiv 4 \pmod 9$ and $-5 \equiv 4 \pmod 9$

15. (a) $7 \cdot 9 \equiv 27 \pmod{36}$

    (b) $8 - 21 \equiv 18 \pmod{31}$

    (c) $68 \cdot 69 \cdot 71 \equiv 60 \pmod{72}$

    (d) $108! \equiv 0 \pmod{84}$

    (e) $\frac{1}{2} \equiv 9 \pmod{17}$

    (f) $\frac{1}{11} \equiv 4 \pmod{43}$

16. (a) No, because $\gcd(4, 6) = 2 \neq 1$

    (b) Yes, because 12 and 17 are relatively prime. $\frac{1}{12} \equiv 10 \pmod{17}$

17. $2 \pmod{36}$

18. $2 \pmod{100000000036}$

19. $2 \pmod{23476525}$.

20. $4 \pmod{25}$

21. $-1 \equiv 1852 \pmod{1853}$.

22. $\frac{5}{11} \equiv 4 \pmod{13}$

23. To solve this, let x be the number of times the crow has spoken. The number of minutes after the crow has spoken x times is $37 \cdot x$. We want to know when $37 \cdot x \equiv 3 \pmod{60}$, or when $x \equiv \frac{3}{37} \pmod{60}$. Compute $\frac{1}{37} \equiv 13 \pmod{60}$, so $\frac{3}{37} \equiv 3 \cdot 13 \equiv 39 \pmod{60}$. When $x = 39$, $37 \cdot 39 = 1443$ minutes have elapsed. This problem is too hard!!!

24. To solve this, let x be the number of times the cicadas have swarmed. When the cicadas have swarmed x times, it is $17 \cdot x$ years after 2000. The last digit in the year is 3 when $17 \cdot x \equiv 3 \pmod{10}$, so when $x \equiv \frac{3}{17} \pmod{10}$. We solve $\frac{1}{17} \equiv 3 \pmod{10}$, so $\frac{3}{17} \equiv 3 \cdot \frac{1}{17} \equiv 3 \cdot 3 \equiv 9 \pmod{17}$, so $x \equiv 9 \pmod{17}$. At that time, $9 \cdot 17 = 153$ years have elapsed, so it is 2153.