

MATH 13150: Freshman Seminar
Exam #2 Practice Problems

for exam on April 4, 2007

1. Does 35 divide $\binom{48}{18}$? (Hint: write the formula for $\binom{48}{18}$)
2. Does 19 divide $\binom{48}{18}$?
3. (a) Find the prime factorizations of 180 and 944.
(b) Without using the Euclidean algorithm, find $\gcd(180, 944)$ and $\text{lcm}(180, 944)$.
4. How many numbers divide the number 26,000?
5. How many numbers divide 211?
6. (a) How many numbers divide $2^4 \cdot 3 \cdot 5^7 \cdot 13$?
(b) How many numbers divide both $2^2 \cdot 3^2 \cdot 5 \cdot 7$ and $2^4 \cdot 3 \cdot 5^7 \cdot 13$?
(c) Find the prime factorization of the least common multiple of $2^2 \cdot 3^2 \cdot 5 \cdot 7$ and $2^4 \cdot 3 \cdot 5^7 \cdot 13$.
7. Which of these numbers is the square of a fraction? If it is, find a fraction whose square is the number:
 - (a) $\frac{3^4 \cdot 5^3}{7^4 \cdot 5^5 \cdot 11^4}$
 - (b) $\frac{3^6 \cdot 5^8}{3^3}$
 - (c) $\frac{5^4}{5^2}$
8. Are 135 and 209 relatively prime?
9. Are 720 and 1309 relatively prime?
10. Using the formula for $\phi(n)$, find the following:
 - (a) $\phi(48)$
 - (b) $\phi(210)$
 - (c) $\phi(111)$
 - (d) $\phi(83)$
11. (a) Compute $\phi(200)$ and $\phi(400)$. What do you think $\phi(800)$ and $\phi(1600)$ are?
(b) How many numbers from 1 to 400 are relatively prime to 200?
(c) How many numbers from 1 to 50 are relatively prime to 200?
12. Compute the following any way you like:
 - (a) $6 \cdot 7 \pmod{8}$
 - (b) $6 \cdot 7 \pmod{9}$

- (c) $9 \cdot 7 \pmod{11}$
13. Compute the following $\pmod{40}$ without actually multiplying any of the numbers shown. You should also do this without your calculator.
- (a) $39 \cdot 41 \pmod{40}$
(b) $38 \cdot 42 \pmod{40}$
(c) $37 \cdot 43 \pmod{40}$
14. Find a number larger than 10 which is congruent to 4 $\pmod{9}$. Also, find a negative number which is congruent to 4 $\pmod{9}$.
15. Compute the following in the given arithmetic system. Your answer should be in the range $\{0, 1, \dots, m - 1\}$ where m is the modulus.
- (a) $7 \cdot 9 \pmod{36}$
(b) $8 - 21 \pmod{31}$
(c) $68 \times 69 \times 71 \pmod{72}$
(d) $108! \pmod{84}$
(e) $\frac{1}{2} \pmod{17}$
(f) $\frac{1}{11} \pmod{43}$
16. (a) Does $\frac{1}{4}$ exist $\pmod{6}$ and why or why not? If it exists, compute it.
(b) Does $\frac{1}{12}$ exist $\pmod{17}$ and why or why not? If it exists, compute it.
17. Compute $34 \cdot 35 \pmod{36}$.
18. Compute $100000000034 \cdot 100000000035 \pmod{100000000036}$.
19. Compute $23476523 \cdot 23476524 \pmod{23476525}$.
20. Compute $21 - 17 \pmod{25}$.
21. Compute $1845 - 1846 \pmod{1853}$.
22. Compute $\frac{5}{11} \pmod{13}$.
23. Suppose that a crow speaks every 37 minutes, and speaks at 12:37. When does the crow next speak 3 minutes after the top of the hour (e.g., at 1:03, 2:03, 3:03, etc)?
24. Suppose that a species of cicada swarms every 17 years, and swarms in 2017. When do the cicadas next swarm in a year ending in 3.