**MATH 13150: Freshman Seminar** due April 7, 2010

**Group Work # 9** Names:____________________________________

**Instructions:** Clearly explain the answers to the following questions.

1. In this problem compute the following powers in mod p arithmetic.
   (a) Compute $3^4 \pmod{13}$, $3^8 \pmod{13}$, and $3^{12} \pmod{13}$. Does your computation confirm Fermat's theorem?
   (b) Compute $2^4 \pmod{17}$ and $2^8 \pmod{17}$ and $2^{16} \pmod{17}$. Does your answer confirm Fermat's theorem?
2. (a) Compute $10^{22} \pmod{23}$.
   (b) Compute $10^{464} \pmod{23}$.
3. Compute $3^{423} \pmod{7}$.
4. Compute $3^{1324} \pmod{7}$.

5. Compute $5^{343} \pmod{11}$ and $5^{12345678903} \pmod{11}$.

6. (a) Compute $7^{14} \pmod{23}$

   (b) Compute $7^{476} \pmod{23}$.

7. Does Fermat's theorem imply that $3^9 \equiv 1 \pmod{10}$? Explain your answer.

8. Look at the partially completed table of powers mod 11.

MOD 11 TABLES OF POWERS

| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $1^n$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $2^n$ | 2 | 4 | 8 | 5 | | | | | | | | |
| $3^n$ | 3 | 9 | 5 | 4 | 1 | | | | | | | |
| $4^n$ | 4 | 5 | 9 | | | | | | | | | |
| $5^n$ | 5 | 3 | 4 | 9 | 1 | 5 | 3 | 4 | 9 | 1 | 5 | 3 |
| $6^n$ | 6 | 3 | 7 | 9 | 10 | 5 | 8 | 4 | 2 | 1 | 6 | 3 |
| $7^n$ | | | | | | | | | | | | |
| $8^n$ | | | | | | | | | | | | |
| $9^n$ | | | | | | | | | | | | |
| $10^n$ | | | | | | | | | | | | |

How is the 5-row related to the 6-row? Note that some elements in the same column are the same in both rows. For which columns is this true? When $5^k$ is not congruent to $6^k$ (mod 11), how do they differ? Does the fact that $6 \equiv -5$ (mod 11) help you explain this pattern? Complete the table. It'll be much faster if you use patterns like the relation between the 5 and the 6-row.

9. Compute $7^{156}$ (mod 11).