Suggested Problem Solutions
Investment Valuation – Damodaran

Lecture 5 – Final Steps in DCF Analysis

CHAPTER 16 – ESTIMATING EQUITY VALUE PER SHARE

1. Value of operating assets = $CF_1/(R\text{-}g)$ = 250(1.05)(1-.4)(1-.3333)/(.10-.05) = $2,100
   + Cash = $500.00
   – Debt = $750.00
   = Value of equity = $1,850 or Value per share = 1850/200 = $9.25

2. Value of equity (from question #1) = $ 1,850 million
   – (Equity Options)(1-T) = $ 250 million x (1-0.4) = $150.0 million
   = Value of equity in common stock = $ 1,700 million

   Value per share = 1700/200 = $ 8.50

6. Value of Genome Sciences = 300 (1.06)(1-.40)/(.12 - .06) = $3,180 million
   + Value of Gene Therapies = .10 (100 * 50) = $ 500 million
   = Value of Genome Sciences (with minority holding) = $ 3,680 million

   – Debt = $ 800 million
   = Value of Equity = $ 2,880 million

   Value per share = 2880/50 = $57.60

8. To get back to the analyst's original estimate of equity value, we take their estimated price per share multiplied by the diluted number of shares, or $11x1.4 million = $15.40 million

   a) Estimated value with treasury stock approach

      Value per share = (15.40 + 0.4*5)/1.4 = $12.43 per share

   b) With option pricing approach

      Value per option based on Black-Scholes = $9.32
      (inputs: S= 12.5, K=5,t=3,r=5%,Std dev=80%)

      Value of Equity = $15.40 million
      – (Value of options)(1-T) = (0.4)(9.32)(1-.4) = $ 2.24 million
      = Value of equity in common stock = $ 13.16 million

      Value per share = $ 13.16m/1m = $ 13.16/share

c) You could re-estimate the value of the options using the estimated value per share of $ 13.16 to arrive at a value of each option of $9.93. This would change the value per share to a slightly higher value. You could continue until you converge on a value per share.

CHAPTER 13 – DIVIDEND DISCOUNT MODELS

Note: Damodaran provides a lot of information in some of the Chapter 13 problems and probably makes them more complicated than necessary. However, if you can work through the problem (using the solution if necessary), it should improve your understanding of valuation inputs.

1.

a) False. The dividend discount model can still be used to value the dividends that the company will pay after the high growth eases.

b) False. It depends upon the assumptions made about expected future growth and risk.

c) False. This will be true only if the stock market falls more than merited by changes in the fundamentals (such as growth and cash flows).

d) True. Portfolios of stocks that are undervalued using the dividend discount model seem to earn excess returns over long time periods.

e) True. The model is biased towards these stocks because of its emphasis on dividends.

2.

a) Cost of Equity = 6.25% + 0.90 * 5.5% = 11.20%

Value Per Share = $3.56 * 1.055/(.1120 - .055) = $65.89

b) $3.56 (1 + g)/(.1120 - g) = $80

Solving for g:
g = (80 * .112 - 3.56)/(80 + 3.56) = 6.46%

3.

a) Retention Ratio = 1 - Payout Ratio = 1 - 0.42/1.50 = 72%
Return on Capital = (Net Income + Int Exp (1-t))/(BV of Debt + BV of Equity)
= (30 + 0.8 * (1 - 0.385))/(7.6 + 160) = 18.19%
Debt/Equity Ratio = 7.6/160 = .0475
Interest Rate on Debt = 0.8/7.6 = 10.53%
Expected Growth Rate = 0.72 [.1819 + .0475 (.1819 - .1053 * (1 - 0.385))] = 13.5%

Alternatively, and much more simply:

Return on Equity = 30/160 = .1875
Expected Growth Rate = 0.72 * .1875 = 13.5%

b) Note that payout ratio equals 1 minus the retention ratio. In addition, the retention ratio can be estimated as g/ROE. Damodaran requires a complicated calculation here to get ROE. Questions that I write would generally be more straight forward in this respect.

Expected payout ratio after 1998 = 1 – g/ROE = 1 - g/[ROC + D/E (ROC - i (1-t))]
= 1 - .06/(.125+.25(.125 - .07(1-.385)) = 0.5876

c) Beta in 1993 = 0.85
Unlevered Beta = 0.85/(1 + (1 - 0.385) * 0.05) = 0.8246
Beta After 1998 = 0.8246 * (1 + (1 - 0.385) * 0.25) = 0.95

d) Note that to get cost of equity, Damodaran expects you to use the same market risk premium (5.5%) that he provided in question 2.

Cost of Equity in 1999 = 7% + 0.95 * 5.5% = 12.23%
Expected Dividend in 1999 = (\$1.50 * $(1.135)^5$ * 1.06) * 0.5876 = \$1.76
Expected Price at End of 1998 = \$1.76/(.1223 - .06) = \$28.25

e)

Year	*EPS*	*DPS*	*Price*
1994	\$1.70	\$0.48	
1995	\$1.93	\$0.54	
1996	\$2.19	\$0.61	
1997	\$2.49	\$0.70	
1998	\$2.83	\$0.79	\$28.25

Cost of Equity = 7% + 0.85 * 5.5% = 11.68%

PV of Dividends and Terminal Price (@ 11.68%) = \$18.47

f) Total Value per Share = \$18.47

Value Per Share With No Growth = \$1.50 * 0.5876/.1223 = \$7.21

Value Per Share Using Gordon Growth Model (constant growth)
= \$1.50 * 1.06 * 0.5876/(.1223 - .06) = \$15.00

Value of Stable Growth = \$15.00 - \$7.21 = \$7.79

Value of Extraordinary Growth = \$18.47 - \$15.00 = \$3.47

CHAPTER 14 – FREE CASH FLOW TO EQUITY DISCOUNT MODELS

Note: Damodaran provides a lot of information in some of the Chapter 14 problems and probably makes them more complicated than necessary. However, if you can work through the problem (using the solution if necessary), it should improve your understanding of valuation inputs.

1.

a) True. Dividends are generally smoothed out. Free cash flows to equity reflect the variability of the underlying earnings as well as the variability in capital expenditures.

b) False. Firms can have negative free cash flows to equity. Dividends cannot be less than zero.

c) False. Firms with high capital expenditures, relative to depreciation, may have lower FCFE than net income.

d) False. The free cash flow to equity can be negative for companies, which either have negative net income and/or high capital expenditures, relative to depreciation. This implies that new stock has to be issued.

4.

a) You can analyze this question either on an aggregate or a per share basis. I provide both answers below. Note that aggregate values are simply equal to per share values multiplied by 7 million shares.

Damodaran makes conflicting assumptions regarding the stable growth stage. He originally states that capex and depreciation grow at the same rate as earnings (14% for five years and 7% thereafter). However, he states implies in part (b) that capex should be 150% of revenues during the stable growth stage. Please ignore the 150% assumption. Also, I will assume that working capital equals 50% of revenues during the high growth period and 25% of revenues during the stable growth period.

The cost of equity during high growth equals: 0.07 + 1.2(0.055) = 13.60%
The cost of equity during stable growth equals: 0.07 + 1.1(0.055) = 13.05%

Aggregate Analysis:

Year	Total Revenues	EPS	Total Earnings	Capex	Depr	ChgWC	FCFE
1993	106.00	2.02	14.14	4.20	2.00		
1994	112.36	2.30	16.12	4.79	2.28	3.18	11.00
1995	119.10	2.63	18.38	5.46	2.60	3.37	12.77
1996	126.25	2.99	20.95	6.22	2.96	3.57	14.80
1997	133.82	3.41	23.88	7.09	3.38	3.79	17.13
1998	141.85	3.89	27.23	8.09	3.85	4.01	19.80
1999	147.53	4.16	29.13	8.65	4.12	2.84	22.50

Per Share Analysis:

Year	Revenues	EPS	Capex	Depr	ChgWC	FCFE
1993	15.14	2.02	0.60	0.29		
1994	16.05	2.30	0.68	0.33	0.45	1.57
1995	17.01	2.63	0.78	0.37	0.48	1.82
1996	18.04	2.99	0.89	0.42	0.51	2.11
1997	19.12	3.41	1.01	0.48	0.54	2.45
1998	20.26	3.89	1.16	0.55	0.57	2.83
1999	21.08	4.16	1.24	0.59	0.41	3.21

The net capital expenditures (Cap Ex - Depreciation) and working capital change is funded partially by debt (10%). The balance comes from equity. For instance, in year 1, FCFE = \$2.30 - (\$0.68 - \$0.33 + \$0.45) * (1 - 0.10) = \$1.57

b) Terminal Equity Value$_{1998}$ = \$22.50/ (.1305 - .07) = \$371.88
Terminal Pice$_{1998}$ = \$3.21/ (.1305 - .07) = \$53.13

c) Present Value Per Share
= $1.57/(1.136) + 1.82/(1.136)^2 + 2.11/(1.136)^3 + 2.45/(1.136)^4 + (2.83+53.13)/(1.136)^5$
= \$35.28

This can also be obtained by taking the aggregate present value divided by 7 million shares, or: 246.99/7 = \$35.28

7.

a) Equity Reinvestment rate
= (Cap Ex – Deprec'n + Chg in WC- Net Debt Issued)/ Net Income
= (50 –20 + 20 - 10)/ 80 = 50%

Return on Equity = Net Income/ Book value of equity = 80/ 400 = 20%
Expected growth rate = ROE * Equity Reinv. Rate = 20% * .5 = 10%

b) Equity reinvestment rate after year 5 = g/ ROE = 4/12 = 33.33%

Year	Net Income	Equity Reinvestment	FCFE	Terminal Value	PV
1	\$88.00	\$44.00	\$44.00		\$40.00
2	\$96.80	\$48.40	\$48.40		\$40.00
3	\$106.48	\$53.24	\$53.24		\$40.00
4	\$117.13	\$58.56	\$58.56		\$40.00
5	\$128.84	\$64.42	\$64.42	\$1,488.83	\$964.44
6	\$133.99	\$44.66	\$89.33		

Value of Equity today = \$1,124.44 million

CHAPTER 15 – FIRM VALUATION: COST OF CAPITAL AND APV APPROACHES

Note: Damodaran provides a lot of information in some of the Chapter 15 problems and probably makes them more complicated than necessary. However, if you can work through the problem (using the solution if necessary), it should improve your understanding of valuation inputs.

1.

a) False. It can be equal to the FCFE if the firm has no debt.

b) True.

c) False. It is pre-debt, but after-tax.

d) False. It is after-tax, but pre-debt.

e) False. The free cash flow to firm can be estimated directly from the earnings before interest and taxes.

3.

a)

Yr	*EBITDA*	*Depr*	*EBIT*	*EBIT(1-t)*	*CapEx*	*WC*	*FCFF*	*Terminal Value*
0	$1,290	$400	$890	$534	$450	$82	$402	
1	$1,413	$438	$975	$585	$493	$90	$440	
2	$1,547	$480	$1,067	$640	$540	$98	$482	
3	$1,694	$525	$1,169	$701	$591	$108	$528	
4	$1,855	$575	$1,280	$768	$647	$118	$578	
5	$2,031	$630	$1,401	$841	$708	$129	$633	$13,468.07

	'93-97	*After 1998*
Levered Beta	1.10	0.9637
Unlevered Beta	0.7413	0.7413
Cost of Equity (Based on CAPM)	13.05%	12.30%
Pre-tax Cost of Debt	8.00%	7.50%
After-tax Cost of Debt	4.80%	4.50%
Cost of Capital	9.37%	9.70%

*Note: Damodaran provides a different solution here. His solution lists the cost of equity after 1998 as 11.89% and the associated WACC as 9.45%. I have not been able to figure out where he gets these results and have edited the solution.

To get the Terminal Value, we need to estimate cash flows in year 6:

After-tax operating income in year 6 = $EBIT_5(1\text{-}T)(1+g) = 841(1.04) = 874.64$

Damodaran notes that CapEx and Depr both grow at 4% in the stable growth stage, giving:

Net Capex in year 6 = (708 – 630)(1.04) = $81.12

Damodaran also notes that Capex equals 120% of Depr in the stable growth stage, but this is inconsistent with his assumption above, so you can ignore this statement.

Damodaran does not tell us how much working capital is expected to grow during the stable growth stage, but he does note that working capital equals 7% of revenues. We can therefore estimate the change in working capital by finding the change in revenues from 1997 to 1998 and multiplying by 7%.

$\text{Revenue}_{1997} = 13500(1.095)^5 = \$21{,}252.223$
$\text{Revenue}_{1998} = 13500(1.095)^5(1.04) = \$22{,}102.312$
Change in Revenue = 22102.312 – 21,252.223 = $850.089

Change in working capital = 850.089(.07) = $59.51

Given these inputs for after-tax operating income, net capital expenditures, and change in working capital, we can then estimate FCFF for 1998 as follows:

FCFF_{1998} = 874.64 – 81.12 – 59.51 = $734.01

This gives a terminal value of:

Terminal value = 734.01/(.097 - .04) = $12,878.95

And, finally, we can estimate firm value by taking the present value of free cash flows during the high growth period plus the present value of the terminal value:

Firm Value =

$$\frac{440}{1.0937}+\frac{482}{1.0937^2}+\frac{528}{1.0937^3}+\frac{578}{1.0937^4}+\frac{(633+12879)}{1.0937^5}=\$10,247$$

b) Value of Equity in the Firm = ($10247 - Market Value of Debt)
= 10247 - 3200 = $7047

Value Per Share = $7047/62 = $113.66