Immunization Example – Life Insurance

A life insurance company expects to pay the beneficiary of a policy $1,000,000 10 years from today. The manager of the life insurance company wants to make an investment today that will provide the necessary funding to make this payment. However, the manager also wants to immunize her firm against interest rate risk. The market yield is currently 7%.

One possible solution to this problem is to purchase 1,000 10-year zero coupon bonds. Each of these bonds will pay $1,000 at the maturity date, thus providing you with the required $1,000,000. At a yield of 7%, these zero coupon bonds would cost $508.35 each, so the total investment would cost you $508,350.

The zero coupon bond investment provides a hedge against interest rate risk, because the duration is exactly equal to the 10 year investment horizon. However, we can do the same thing even if zero coupon bonds are not available.

Suppose there are two fixed income securities available for investment: a 30-year 6% (annual) coupon bond and a 10-year 5% (annual) coupon bond. What combination of these two bonds would immunize the insurance company against interest rate risk and at the same time provide a value of $1,000,000 ten years from today.

Solution:

We must first recognize that the duration of our liability equals 10 years. The key is to immunization is to create a portfolio of these two fixed-income securities that has an overall duration of 10 years.

The duration of the 30-year bond equals: $D = \left(\frac{1.07}{.07}\right) - \left(\frac{1.07 + 30(.06 - .07)}{.06(1.07^{30} - 1) + .07}\right) = 13.636 \ \ years$

The duration of the 10-year bond equals: $D = \left(\frac{1.07}{.07}\right) - \left(\frac{1.07 + 10(.05 - .07)}{.05(1.07^{10} - 1) + .07}\right) = 7.935 \ \ years$

We need to choose portfolio weights in the two bonds such that: $w_{30}(D_{30})+w_{10}(D_{10})=10$ years

Since the two weights must sum to one, we get: $10 = w_{30}(13.636) + (1 - w_{30})(7.935)$

or $w_{30} = 36.22\%$ and $w_{10} = 63.78\%$

We now know that we need to invest 36.22% of our portfolio in the 30-year bond and 63.78% of our portfolio in the 10-year bond. To get the actual bond positions, note that we must make a total investment equal to the present value of the $1,000,000 liability. In other words, we need to invest:

$$\frac{1{,}000{,}000}{(1.07)^{10}} = \$508{,}349.29$$

We therefore need to invest (508,349.29)(.3622)=$184,124.11 in the 30-year bonds and (508,349.29(.6378)=$324,225.18 in the 10-year bonds.

We can also show that the price of the 30-year bond is $875.91 and the price of the 10-year bond is $859.53.

$$B_{30} = 60\left[\frac{1-(1.07)^{-30}}{.07}\right] + 1000\left[\frac{1}{(1.07)^{30}}\right] = \$875.91$$

$$B_{10} = 50\left[\frac{1-(1.07)^{-10}}{.07}\right] + 1000\left[\frac{1}{(1.07)^{10}}\right] = \$859.53$$

We therefore need to purchase 210.2 30-year bonds (or $184,124.11/$875.91) and 377.2 10-year bonds (or $324,225.18/$859.53). This position will provide us with a value of approximately $1,000,000 ten years from today regardless of what happens to interest rates.

**Note that hedge positions must be adjusted over time. For example, we might want to adjust our positions in the 30-year and 10-year bonds every year during the 10-year investment horizon.

Demonstrating that immunization works:

As an additional illustration, we can show that this position returns approximately $1,000,000 ten years from today regardless of what happens to interest rates. For example, calculate the value of this investment in ten years assuming market interest rates (yields) immediately increases to 8%.

30-year bonds:

In ten years, we will need to sell the 30-year bond. If interest rates rise to 8%, the bond will be worth:

$$B_{30} = 60\left[\frac{1-(1.07)^{-20}}{.07}\right] + 1000\left[\frac{1}{(1.07)^{20}}\right] = \$803.64$$

Since we own 210.2 of these bonds, we can sell our position for $168,925.13 (or 210.2 x 803.64).

10-year bonds:

In ten years, the 10-year bond will reach maturity and we will be paid the $1,000 principal amount on each bond. Since we own 377.2 of these bonds, we will receive a total of $377,200 (or 377.2 x 1000).

Coupon payments and reinvestment:

We also earn interest payments on each of the bonds and we can reinvest each coupon payment during our ten-year investment horizon. Each stream of coupon payments can be valued as an annuity, where the future value of a $1 annuity equals:

$$\frac{(1+R)^{N} - 1}{R} = \frac{1.08^{10} - 1}{.08} = 14.4866$$

The stream of $60 coupon payments from the 30-year bond is therefore equal to $60(14.4866)=$869.20. Since we own 210.2 of these bonds, we have a total future value of $182,704.99 (or 210.2 x 869.20).

The stream of $50 coupon payments from the 10-year bond is equal to $50(14.4866)=$724.33. Since we own 377.2 of these bonds, we have a total future value of $273,217.28 (or 377.2 x 724.33).

In total, the future value of coupon payments from all of our bond positions equals:

$182,704.99 + $273,217.28 = $455,922.27

Total Value:

The total value of our investment is the sum of our 30-year bond position, our 10-year bond position, and the future value of our coupon payments. This gives:

$168,925.13 + $377,200.00 + $455,922.27 = $1,002,047.40

Note that this is extremely close to the $1million goal. We could repeat this exercise assuming interest rates stay 7% or decrease to 6%. The results would be very similar. In other words, our position earns approximately $1 million, regardless of what happens to interest rates during the 10 year horizon.