Essentials of Investments (BKM 7th Ed.)
Answers to Suggested Problems – Lecture 2

Chapter 5:

6. #3 For each portfolio: Utility = E(r) – (½ × 4 × σ^2).

Investment	E(r)	σ	U
1	0.12	0.30	-0.0600
2	0.15	0.50	-0.3500
3	0.21	0.16	0.1588
4	0.24	0.21	0.1518

The portfolio with the highest utility value is #3.

7. #4 When an investor is risk neutral, A = 0, so that the portfolio with the highest utility is the portfolio with the highest expected return. This is investment #4, with 24%.

9. $E(R_X)$ = [0.2 × (–20%)] + [0.5 × 18%] + [0.3 × 50%)] = 20%

$E(R_Y)$ = [0.2 × (–15%)] + [0.5 × 20%] + [0.3 × 10%)] = 10%

10. $\sigma_X^2 = [0.2 \times (-.20 - .20)^2] + [0.5 \times (.18 - .20)^2] + [0.3 \times (.50 - .20)^2] = .0592$

σ_X = 24.33%

$\sigma_Y = [0.2 \times (-.15 - .10)^2] + [0.5 \times (.20 - .10)^2] + [0.3 \times (.10 - .10)^2] = .0175$

σ_Y = 13.23%

15.

a. $E(R_P) - R_f = ½A\sigma_P^2 = ½ \times 4 \times (0.20)^2 = 0.08 = 8.0\%$

b. $0.09 = ½A\sigma_P^2 = ½ \times A \times (0.20)^2 \Rightarrow A = 0.09/(½ \times 0.04) = 4.5$

c. Increased risk tolerance means decreased risk aversion (A), which results in a decline in risk premiums.

18. a) Expected cash flow:

0.5($50,000) + 0.5($150,000) = $100,000

HPR = $(P_1 - P_0)/P_0$

0.15 = $(100,000 - P_0)/P_0$ P_0 = $86,956.52

b) HPR = (100,000 - 86956.52)/86956.52 = 15%

c) A risk-premium of 15%, leads to an expected return of 15%+5%=20%.

$0.20 = (100,000 - P_0)/P_0$ $P_0 = \$83,333.00$

d) There is an inverse relationship: price decreases as the risk premium increases. In order to earn a higher risk-premium (assuming the cash flows stay the same), you must be able to buy the security at a lower price. The investor requiring the 15% risk premium (20% HPR) is requiring a larger discount as compensation for risk.

19. a) $E(R_P) = 0.3(7\%) + 0.7(17\%) = 14\%$ $\sigma_P = 0.7(27\%) = 18.9\%$

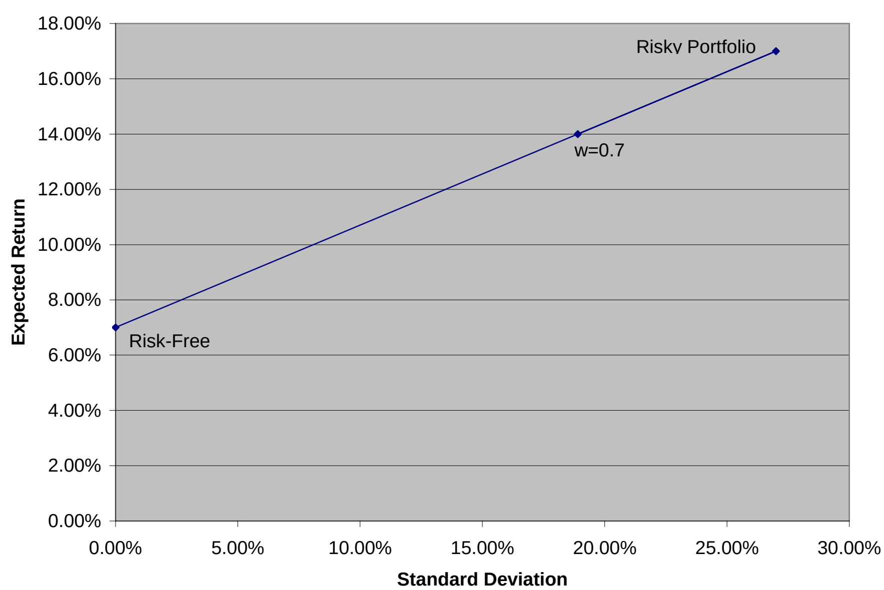
b)

T-Bills	= 30.0%
Stock A = 0.7(27%)	= 18.9%
Stock B = 0.7(33%)	= 23.1%
Stock C = 0.7(40%)	= 28.0%
Total Portfolio	= 100%

(The total weight in the portfolio is 70%, and the portfolio consists of 27% A, 33% B, and 40% C)

c) Your Reward-to-variability = $(R_P - R_F)/\sigma_P$ = (17% - 7%)/27% = 0.3704
Client's Reward-to-variability = (14% - 7%)/18.9% = 0.3704

d) The slope of the capital allocation line equals the reward-to-variability ratio (0.3704). Note that this is the same at any point you choose on the CAL.



20. a. Rule 1: $E(R_C) = R_F + y(R_P - R_F)$

$0.15 = 0.07 + y(0.17 - 0.07)$ $y = 0.80$

b.

T-Bills	= 20.0%	
Stock A	= 21.6%	(0.8*27%)
Stock B	= 26.4%	(0.8*33%)
Stock C	= 32.0%	(0.8*40%)

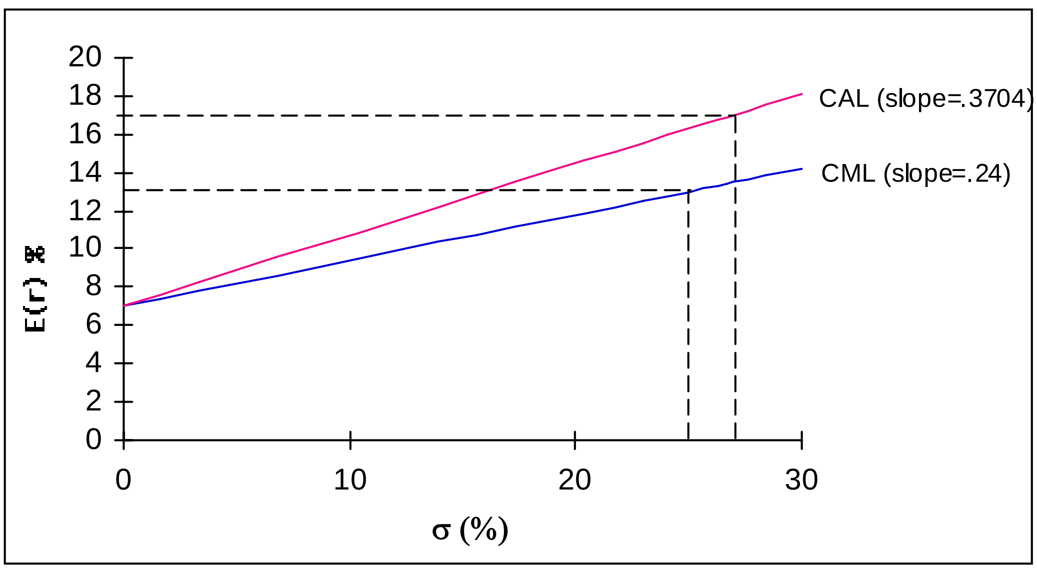
c. Rule 2: $\sigma_C = 0.80(0.27) = 0.216 = 21.6\%$

21. a. Portfolio standard deviation = y × 27%.

If the client wants a standard deviation of 20%, then y = 20/27 = .7407 = 74.07% in the risky portfolio.

b. Rule 2: E(R) = 7 + 10y = 7 + .7407 × 10 = 7 + 7.407 = 14.407%

22. a. Slope of the CML = $\frac{13-7}{25}$ = .24 The diagram is shown below.



b. My fund has a higher reward-to-variability (or a steeper CAL). This allows an investor in my fund to achieve a higher expected return for any given standard deviation than they would earn on the passive S&P fund. In other words, my fund provides a higher return at any given level of risk.

Chapter 6:

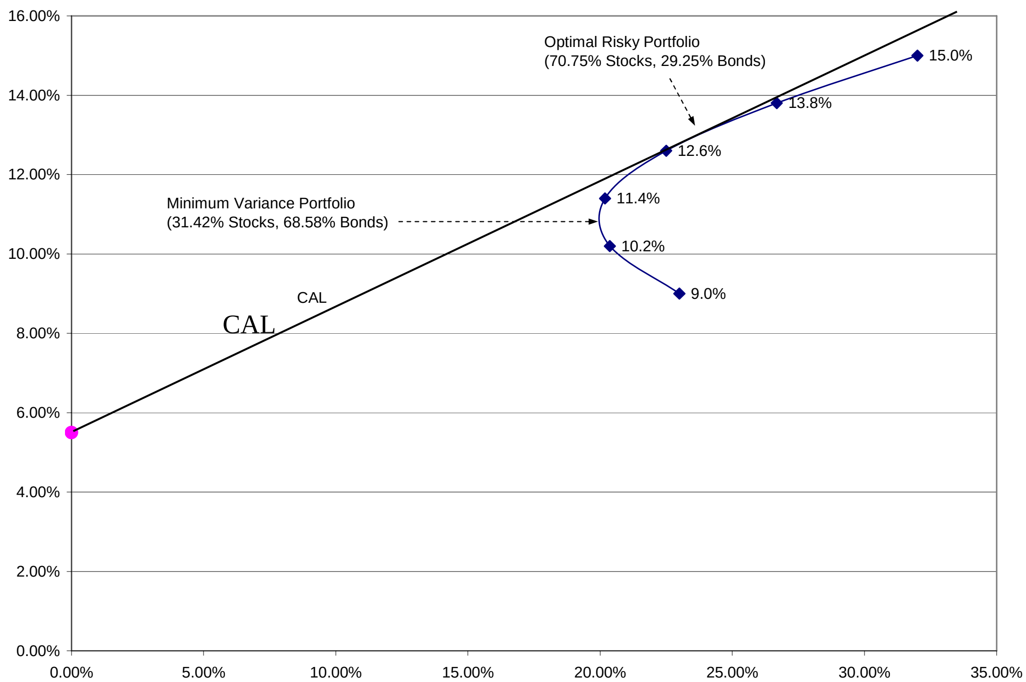
6. The parameters of the opportunity set are:

$E(R_S) = 15\%$, $E(R_B) = 9\%$, $\sigma_S = 32\%$, $\sigma_B = 23\%$, $\rho = 0.15$, $R_f = 5.5\%$

This gives $Cov(R_S, R_B) = \rho\sigma_S\sigma_B = .01104$

Using Rule 1* and Rule 2*, you can calculate the return and standard deviation on each of the six portfolio combinations (in 20% increments):

Weight S	***Weight B***	***E(R)***	***Std.Dev.***
100%	0%	15.0%	32.00%
80	20	13.8%	26.68
70.75	**29.25**	13.25	24.57
60	40	12.6%	22.50
40	60	11.4%	20.18
31.42	**68.58**	10.89	19.94
20	80	10.2%	20.37
0	100	9.0%	23.00



Of these six possible portfolios, the minimum variance portfolio is the combination with 40% in stocks and 60% in bonds.

Note: The true minimum variance portfolio is actually between the 40% stock and 20% stock choices - the exact minimum variance portfolio has a weight of 31.42% in stocks and 68.58% in bonds. The result can be found by minimizing the variance formula, Rule 2*, as we discussed in class. The minimum-variance portfolio proportions are:

$$w_{Min}(S) = \frac{\sigma_B^2 - Cov(r_S, r_B)}{\sigma_S^2 + \sigma_B^2 - 2Cov(r_S, r_B)} = \frac{0.0529 - 0.01104}{0.1024 + 0.0529 - (2 \times 0.01104)} = 0.3142$$

$$w_{Min}(B) = 1 - 0.3142 = 0.6858$$

Plugging these weights into Rule 1* and Rule 2* gives the expected return and standard deviation shown in the table above.

7. If you consider only the six portfolios you created at 20% increments, the optimal risky portfolio is the portfolio that includes 60% stocks and 40% bonds. This portfolio has an expected return of 12.6% and a standard deviation of 22.5%. You can use this portfolio to answer questions 8 through 10.

(Note: The true optimal risky portfolio (P) is somewhere between the 60% stock and 80% stock choices. The portfolio with 70% stocks and 30% bonds is close to the optimal risky portfolio. This portfolio has an expected return of 13.2% and a standard deviation of approximately 24.41%. Although I will not require you to do this calculation, you can solve for the exact weights in the optimal portfolio using the formula in footnote 3 on page 185 of the text: the weights are 70.75% stock and 29.25% bonds. This portfolio has an expected return of 13.25% and a standard deviation of 24.57%.)

8. The reward-to-variability ratio of the optimal CAL (using the 60/40 portfolio) is:

$$\frac{E(r_p) - r_f}{\sigma_p} = \frac{12.6 - 5.5}{22.5} = 0.3156$$

9. a) The equation for the CAL (using the 60/40 portfolio) is:

$$E(r_C) = r_f + \frac{E(r_p) - r_f}{\sigma_p}\sigma_C = 5.5\% + 0.3156\sigma_C$$

Setting $E(r_C)$ equal to 12% yields a standard deviation of 20.6%.

b. The mean of the complete portfolio as a function of the proportion invested in the risky portfolio (y) is:

$$E(r_C) = (l - y)r_f + yE(r_P) = r_f + y[E(r_P) - r_f] = 5.5 + y(12.6 - 5.5)$$

Setting $E(r_C) = 12\% \Rightarrow y = 0.9155$ (91.55% in the risky portfolio)

$1 - y = 0.0845$ (8.45% in T-bills)

From the composition of the optimal risky portfolio:
Proportion of stocks in complete portfolio = $0.9155 \times 0.60 = 0.5493$
Proportion of bonds in complete portfolio = $0.9155 \times 0.40 = 0.4507$

10. Using only the stock and bond funds to achieve a mean of 12% we solve:

$$12 = 15w_S + 9(1 - w_S) = 9 + 6w_S \Rightarrow w_S = 0.5$$

Investing 50% in stocks and 50% in bonds yields a mean of 12% and standard deviation of:

$$\sigma_P = [(0.50^2 \times 1024) + (0.50^2 \times 529) + (2 \times 0.50 \times 0.50 \times 110.4)]^{1/2} = 21.06\%$$

The efficient portfolio with a mean of 12% has a standard deviation of only 20.6%. Using the CAL reduces the SD by 46 basis points (0.46%).

12. If $\rho = -1$, a zero-risk portfolio can be created:

To find the answer, substitute $\rho = -1$ into Rule 2* and then set the equation equal to zero. This gives the minimum variance portfolio when $\rho = -1$. We discussed the solution to this equation in class. Here, the weight in stock A that gives a perfect hedge (zero risk) equals $\sigma_B/(\sigma_A+\sigma_B)=0.6/(0.4+0.6) = 60\%$. In other words, a portfolio with $w_A = 0.60$ and $w_B = 0.40$ would have $\sigma = 0$.

This portfolio has a return of: $E(R) = 0.6(.08) + 0.4(0.13) = 10\%$

Since this portfolio is riskless, the risk-free rate must be exactly equal to the expected return on this portfolio or 10%.

14. All investors will have the same optimal risky portfolio, since the optimal risky portfolio is independent of investor preferences. However, investors will adjust portfolios to obtain the appropriate levels of risk by combining the risky portfolio with a risk-free asset such as T-Bills. *(Note: This answer assumes that there is a risk-free security in the economy and investors can borrow and lend at this risk-free rate. If there is no risk-free security, then the optimal risky portfolio may depend on the risk-aversion level of the individual).*

15. No. It isn't possible to get such a diagram. With a correlation between –1.0 and +1.0, the graph should be a smooth curve passing through points A and B and bowing toward the Y-axis. (Even if the correlation between A and B were exactly +1.0, the frontier would be a straight line connecting A and B.)

16. The two measures are equal if $\rho=+1$. Otherwise, the portfolio standard deviation will be less than the weighted average of the standard deviations of the component assets. In other words, for any $\rho<1$, you can reduce the portfolio variance by diversifying.